ON THE TRADE-OFF BETWEEN EFFICIENCY IN JOB ASSIGNMENT AND TURNOVER: THE ROLE OF BREAKUP FEES*

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Abstract. We highlight a novel trade-off with the use of breakup fees in employment contracts. Under asymmetric learning about the workers' productivity the market takes job-assignments (or, "promotions") as signal of quality and bids up the wages of a promoted worker, leading to inefficiently few promotions (Waldman, 1984). Breakup fees can mitigate such inefficiencies by shielding the firm from the labor market competition but thwart efficiency in turnover when there are firm-specific matching gains. We show that it is optimal to use breakup fees if and only if the difference between the worker's expected productivity in the pre- and post-promotion jobs is small. Also, the relationship between the optimality of breakup fees and the importance of firm-specific human capital is more nuanced than what the extant literature may suggest.

1. Introduction

Firms often stipulate breakup fees in their employment contracts in order to dissuade their workers from moving to competing employers. Such breakup fees, also known as "golden handcuffs," are a contractual obligation for the employee to pay back a part of his compensation (or to pay a "damage" fee) to the firm if he leaves to join a rival. For example, the deferred compensation plans such as retirement benefits and stock options with gradual vesting force the employee to forfeit a portion of his compensation if he quits sooner than later. Another common form of employment contract with a steep breakup fee is the so-called "noncompete clause" where, for a certain duration of time, the worker is contractually prohibited from taking up employment with a competitor. Should the worker decide to move while the clause is still in effect, he may make a buyout offer in order to absolve himself from any legal bindings.[1]

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[1]There is strong empirical evidence that breakup fees are effective in reducing employee turnover. Mehran and Yermack (1997) find that stock options can reduce CEO turnover (also see, Jackson and Lazear, 1991, and Scholes, 1991). Allen et al. (1993) find similar effects of a deferred compensation through pension plans. Manchester (2009) and Hoffman and Burks (2013) show the effectiveness of "training contracts" where an employee must reimburse her cost of training to the firm should she decide to leave. Analyzing the career patterns of top executives, Garmaise (2011) finds that noncompete clauses, too, help to reduce the turnover rate.

In recent years, the use of noncompete clauses has proliferated in a wide range of industries. A large majority of managerial and technical employees at all levels of organizational hierarchy are estimated to have signed contracts that include some form of noncompete clause (Lobel, 2013; p. 51). It is also interesting to note that the contractual restrictions on workers' mobility is becoming commonplace at a time when the recent growth in the recruiting networks (e.g., staffing agencies, social media sites such as LinkedIn, etc.) has made a worker's career progress within a firm—e.g., his job assignments or promotions—more visible to the outsiders.

The extant literature on the breakup fees (including noncompete covenants) argues for their effectiveness in protecting proprietary knowledge and in sharpening the firm's incentives for human capital investments. But it fails to explain the wide-spread use of such clauses in industries where such concerns are not relevant (Lobel, 2013). Also, it cannot justify the aforementioned contemporaneity between the use of such contracts and the increased visibility of the workers' career progress. In this article, we present a novel justification for the use of breakup fee that abstracts away from the issues of investment or knowledge protection and is also consistent with the simultaneity between the rise in the use of such fees and the increased visibility of the workers' career path.

We consider an environment with asymmetric learning on workers' productivity where the outside labor market takes the workers' job assignments (or promotions) as signal of their productivity. As shown by Waldman (1984), such a signaling role of job assignment leads to inefficiently few promotions. We argue that breakup fee can mitigate such inefficiencies. However, in the presence of firm-specific matching gains, breakup fees may also reduce efficiency in worker turnover. We analyze the optimality of the breakup fee in light of this trade-off.

To study this trade-off, we consider a simple two-period principal-agent model where the firm (principal) has two types of job, 1 and 2. In period one, the firm hires an agent with unknown ability and assigns him to job 1. The initial contract specifies a wage for period one and a breakup fee payable to the firm should the worker decide to leave for a competitor firm in the future. In period two, the firm privately observes the worker's ability and decides whether to promote him to job 2. The workers with higher ability are more productive in job 2 compared to job 1. Once the promotion decision is made, it is publicly observed and multiple raiding firms—where the worker might be better matched—compete in wages to bid away the worker. The initial employer can make a counteroffer upon observing the raiders' bids. The firm offers a period-two wage if it prefers to retain the worker. Otherwise, it lets the worker go and collects the breakup fee. However, the firm may renegotiate and lower the fee if it is profitable for the coalition of the firm and the worker to do so.

As the workers with higher ability are more likely to be promoted and a worker's promotion is more visible publicly than his the actual ability, job assignment becomes a signal of quality. However, such a signaling role of promotion distorts the efficiency in job assignment (Waldman, 1984). The outside labor market takes promotion as a signal of high quality of a worker and may try to bid him away by offering a higher wage. As competition bids up the wage of a promoted worker, the firm promotes a worker only if he is sufficiently more productive in job 2 and worth the higher wage that comes with promotion. Consequently, too few workers are promoted compared to what is socially efficient.[2] Breakup fees can mitigate

[2]Several authors have offered empirical evidence of the signaling role of job assignment and resulting distortions as predicted in Waldman (1984); e.g., see DeVaro and Waldman (2012), Bognanno and Melero

such inefficiencies by creating a wedge between what the market offers to a promoted worker and what the firm must pay to retain him—the worker would stay back as long as the market's bid *net of break-up fee* is dominated by his current wage offer. Consequently, promotion becomes less expensive (for the firm) and the firm has a stronger incentive to promote the worker.

But on the other hand, the use of breakup fee reduces the efficiency in turnover: When a breakup fee is in place, the firm is more likely to retain a worker even when he is better matched with the raiders. As the firm lowers its promotion threshold, promotion becomes a weaker signal of quality. As a result, the market reduces its bid for the worker and the firm may find it more profitable to retain him by making a counteroffer. The inefficiency in turnover is detrimental to the firm (ex-ante) since the firm could extract the matching gains up-front from the worker.

We show that the optimality of a break-up fee depends on the *relative size* of the worker's expected productivity in the two jobs. It is optimal to specify a break-up fee if and only if the difference between the worker's expected productivity in the two jobs is not too large. Moreover, when the use of breakup fee is optimal for the firm, it is also socially optimal (i.e., it increases the aggregate social surplus).

The intuition for this finding is as follows. When the difference in the worker's expected productivity in the two jobs is large, the firm already has a strong incentive to promote the workers as they are much more productive in job 2 than in job 1. The workers who are inefficiently kept in job 1 are of low ability and would have had little gains in productivity had they been assigned to job 2. Thus, in such a setting, the marginal gains from more efficient promotion that is brought about by stipulating a break-up fee is relatively small. However, such a break-up fee would hinder the efficient turnover of the promoted workers by lowering the raiders' bid and the marginal loss due to inefficient turnover is relatively large. (We later argue that there is no net change in the ex-ante turnover efficiency for the workers who were not promoted.) Consequently, it is optimal not to stipulated such a fee.

In contrast, when the difference in expected productivity in the two jobs is relatively small the firm would promote very few workers, only those with sufficiently high ability. Also, the marginal worker who is denied promotion would have been considerably more productive if he were promoted. Thus, the marginal gain from the improved job-assignment is high whereas the marginal loss from the reduced turnover is low. Therefore, it becomes optimal to stipulate a break-up fee as it eases the inefficiency in promotion but costs little in terms of the turnover inefficiency that it creates.

Our findings on the effects and the optimality of breakup fees have a few important implications. First, we offer a novel justification for the use of breakup fees where the oft-cited benefits of such fees—protection of investment or proprietary knowledge—are absent. Furthermore, as the asymmetric learning of the worker's quality is a key driver of the above findings, they suggest that break-up fees are more likely to be used where the information about the workers' quality remains private (to the initial employer) but the information about job-assignment becomes public. Indeed, breakup fees need not be used by the firms when the workers' promotions are not visible to the market or when the workers' productivity is perfectly observed by the market. This observation is consistent with the simultaneous

(2016), and Cassidy et al. (2016). Dato et al. (2016) also shows similar evidence in experimental labor markets. See also Baker et. al (1994a, 1994b) and McCue (1996) for empirical evidence that promotion is often associated with large wage increases.

rise in the use of restrictive covenants and visibility of workers' career progress within the firm in the US labor market as mentioned earlier.

Second, we highlight how the optimality of breakup fee is linked to the underlying production technologies in an organizational hierarchy. To the best of our knowledge, such a link has not been hitherto explored in the extant literature. An interesting implication of our result is that the breakup fees are less likely to be used when the nature of the tasks in the two jobs are significantly different as it may also lead to a large difference in the worker's productivity in the two jobs.

Finally, our findings also contribute to the debate on the enforcement of employment contracts that attempt to restrict turnover. For example, in US, Courts often refrain from enforcing a non-compete clause citing harm from restricted labor mobility (Lobel, 2013; Malsberger, 2004). However, in our setting the Court should always enforce a contract with breakup fee—if the fee is optimal for the firm it is also optimal for social welfare.

We also analyze the role of firm-specific human capital in the optimality of breakup fees. The extant literature suggests that the distortion in job-assignment would be smaller when the firm-specific human capital plays a bigger role in driving the worker's productivity (Waldman, 1984, 2013; Ghosh and Waldman, 2010). When firm-specific human capital is important, the worker is more likely to be a better match with his initial employer. Hence, the outside labor market is less likely to bid for him, and the firm has a stronger incentive to promote the worker. In light of this observation one may anticipate that the breakup fees are used less often when firm-specific human capital is important for production. However, we argue that the relationship between the two is more nuanced than what the above intuition may suggest.

In particular, we show that for a given breakup fee (that is not too large), the above intuition continues to hold: In equilibrium, the firm promotes more workers as the firm-specific human capital becomes more important. But it can be argued that in such a scenario the firm also finds it more profitable ex-ante to promote a worker. Consequently, the firm stipulates a higher breakup fee ex-ante to ensure that it has a stronger incentive for promoting a worker ex-post.

It is interesting to note, however, that the above result need not hold if, following the "Invisibility hypothesis" (Milgrom and Oster, 1987) we assume that the outside labor market can only bid for a promoted worker as the workers in the low-level job may not be visible to the market. In such an environment it is indeed the case that the firm is less likely to use breakup fees when firm-specific human capital becomes more critical. In other words, the impact of firm-specific human capital on the use of breakup fee critically hinges on the market visibility of the worker at the different tiers of the organizational hierarchy.

Related literature: The extant literature on breakup fees has studied its impact on various aspects of an employment relationship. There is a long literature on the role of deferred compensation in human capital investment (Backer, 1964), tenure (Lazear, 1979), and turnover (Salop and Salop, 1976). More recently, several authors have also shown how a non-compete clause may be used to protect the returns on investment in human capital (Rubin and Shedd, 1981; Posner et. al, 2004; Bishara, 2006) and to restrict the diffusion of proprietary knowledge (Franco and Filson, 2006), turnover (Kräkel and Sliwka, 2009; Garmaise, 2011; Mukherjee and Vasconcelos, 2012), and employee spinoffs (Franco and Mitchell, 2008; Rauch and Watson, 2015).

In contrast, this article highlights a different tradeoff that arises with the use of a breakup fee: it improves the efficiency in job-assignment but hinders efficient turnover. The environment where this trade-off appears has two salient features, both of which are well acknowledged in the current literature: (i) Asymmetric information among employers leads to inefficient turnover (Greenwald, 1986; Lazear, 1986; Gibbons and Katz, 1991; Laing, 1994; also see Gibbons and Waldman, 1999, for a survey). (ii) The initial employer's (publicly observable) decisions—e.g., promotions, outcome of a rank-order tournament, etc.,—may signal the outside labor market about a worker's quality (Waldman, 1984, 1990; Bernhardt and Scoones, 1993; Zábojník and Bernhardt, 2001; Golan, 2005; Mukherjee, 2008; Ghosh and Waldman, 2010; Koch and Peyrache, 2011).

As discussed earlier, our paper is closely related to Waldman (1984). In a framework similar to Waldman (1984), Bernhardt and Scoones (1993) considers a more general model of promotion and turnover in the presence of firm-specific matching gains. They assume that the raiders can invest to acquire information on the workers' quality and argue that in order to dissuade the raiders from doing so (as it increases turnover), the firm may promote the worker with a preemptively high wage. The wage signals a potentially good match between the worker and the firm and discourages the raiders to acquire information (as they anticipate a lower likelihood of successful raid). The assumption that the outside market can acquire the exact same information that the initial employer possesses is crucial for this finding. In our model such direct information acquisition is not feasible and the initial employer always enjoys some degree of information advantage.

Another article that is closely related to ours is Burguet et al. (2002). Burguet et al. study the link between the level of transparency about the worker's ability and the use of breakup fees. In their setting, such fees help the firm to extract the matching gains from the raider. They argue that the firm would stipulate a larger breakup fee when the worker's ability is public information as the raiders bid more aggressively when there is no adverse selection in turnover. This result is in sharp contrast to ours findings as in our case no breakup fee is necessary when the worker's ability is public information.

The role of breakup fee in our model is similar in spirit to that of restrictive covenants in the setup considered by Rauch and Watson (2015). In a model of employee spinoffs in client service firms, Rauch and Watson show that a restrictive covenant can create a favorable default option for the firm for future negotiations if the employee threatens to start a spinoff by stealing the firm's clients. But even though the covenant protects the firm from losing its clients, it could be socially inefficient as it thwarts the formation of efficient spinoffs. In our model the breakup fee also ensures a favorable default option for the firm when there is turnover and in the process, it protects the firm from the labor market competition. While such a fee distorts turnover and perpetuates poor firm-worker match, it improves efficiency in job assignment. Moreover, in our setting if breakup fee is profitable for the firm it is socially optimal as well.

Finally, it is also worth noting that our model is reminiscent of Laing (1994). Laing argues that asymmetric learning about the worker quality may distort a firm's layoff decision when the workers are risk-averse. As the laid-off workers are perceived as "inferior," the spot market competition creates a wedge between the laid-off and retained workers' wages leading to an inefficient risk sharing between the firm and the worker, which, in turn, distorts the firm's lay-off decision. However, Laing's model abstracts from the job-assignment issue as all workers are placed on the same job.

The rest of the paper is organized as follows. Section 2 presents our model and Section 3 characterizes the firm's equilibrium job assignment policy and worker's turnover for a given break-up fee. In Section 4 we elaborate on the trade-off between the inefficiencies in job-assignment and turnover. The optimal break-up fee is discussed in Section 5. Section 6 discusses some modeling extensions including the role of firm-specific human capital. A final section draws a conclusion. All proofs are given in the Appendix.

2. The Model

We consider a two-period principal-agent model that is described below in terms of its five key components: *players, technology, contracts and job assignment, raids and counteroffer,* and *payoffs.*

Players. A firm (or "principal"), F, hires a worker (or "agent"), A, at the beginning of period one. The worker works for the firm in the first period of his life, but he may get raided in period two by the outside labor market where two identical firms (or "raiders"), R_1 and R_2, bid competitively for the worker.

Technology. The technology specification of the firm is similar in spirit to that in Waldman (1984). The firm (F) has two types of jobs: job 1 and job 2. Job 1 is the entry level job where the worker (A) is assigned in period one. The worker's productivity in job 1 is assumed to be fixed at ψ_1 (> 0). However, in job 2 the worker's productivity depends on his ability, or "type", $a \in [0, 1]$: if assigned to job 2 (with F) a worker of type a produces $\psi_2 a$ (where $\psi_2 > 0$).

At the beginning of period one, the worker's ability (a) is unknown to all players (including the worker himself) and is assumed to follow a uniform distribution on $[0, 1]$. But at the end of period one, a is *privately* observed by the firm (but not by the raiders or the worker). Also, we assume that the information on a is non-verifiable and hence, the firm cannot credibly disclose it to a third party.

Job 1 is not available with the raiders, but they can employ the worker in job 2. However, the worker's productivity with the raiding firms depends not only on his ability but also on the firm-specific matching factor, m, where he produces $\psi_2 a\,(1 + m)$. The matching factor m is unknown to all players at the beginning of the game and it is assumed to be distributed on $[-1, 1]$ according to a piece-wise uniform probability density function $g(m)$ where:

$$g(m) = \begin{cases} \alpha & \text{if} \quad m \leq 0 \\ 1 - \alpha & \text{if} \quad m > 0 \end{cases}$$

and $\alpha \in [1/2, 1)$. Let the associated cumulative distribution function be $G\,(m)$. Note that $m \leq 0$—an event that occurs with probability α—implies that the worker is a better match with his initial employer than with the outside labor market. The parameter α can be interpreted as the measure for the importance of firm-specific human capital in job 2. The more critical is the role of the firm-specific human capital in job 2, the less likely it is that the worker would be a better match with the raiders. The value of m is revealed in period two and we will elaborate on this shortly.

We assume the following restriction on the parameters.

Assumption 1. $\frac{\psi_1}{\psi_2} G\left(2\frac{\psi_1}{\psi_2} - 1\right) \leq \alpha$.

The above assumption implies that the ratio ψ_1/ψ_2 cannot be too large and it simplifies our subsequent analysis by ruling out certain corner solutions in the firm's optimal contracting problem.

Contracts and job assignment. We assume that long-term contracts on wages are not feasible. Also, as A's ability (a) is neither observable nor verifiable to a third party, F cannot commit to a promotion policy that is contingent on a. Hence, we restrict attention to the following class of contracts: At the beginning of period one, F makes a take-it-or-leave-it offer (w_1, d) to A where w_1 is the period-one wage and d is a breakup fee that A must pay to F if A decides to leave for the raiders in period two.[3] At the end of period one, after observing a, F decides whether to assign (or "promote") A to job 2. Both the initial contract (w_1, d) at the beginning of period one and the subsequent job assignment at the end of period one are *publicly* observed.

Raids and counteroffer. At the beginning of period two, the raiding firms (R_1 and R_2) observe A's job assignment ($j \in \{1, 2\}$) as well as the matching factor m and make simultaneous wage bids b_i ($i = 1, 2$) for A.[4] We will maintain the convention that $b_i = 0$ when a raider refrains from bidding. Observing the bids, F may make a counteroffer to A: if F prefers to retain A (who has been assigned to job j), it offers a period-two wage of w_2^j; and if F prefers to let A leave for a raider, it may renegotiate the breakup fee d by offering a lower fee $d_R < d$ whenever it is efficient from them (as a coalition) to do so. That is, d is renegotiated whenever A receives an offer that is more than his productivity with F but less than the initially stipulated fee d. To streamline notations, we set $d_R = d$ if there is no renegotiation. The worker chooses the employer who offers the highest wage *net* of the (potentially renegotiated) breakup fee. In case of a tie, A stays with F.

Payoffs. All players are risk neutral and do not discount the future. Upon successfully hiring the worker, the firm's payoff in period one is $\pi_1 = \psi_1 - w_1$. But in period two, the payoff depends on the ability of the worker, whether the worker is promoted, and whether he is retained by the firm. So, the firm's payoff in period two from a worker with ability a is:

$$\pi_2 = \begin{cases} \psi_1 - w_2^1 & \textit{if } A \textit{ is not promoted but retained} \\ \psi_2 a - w_2^2 & \textit{if } A \textit{ is promoted to job 2 and retained} \\ d_R & \textit{if } A \textit{ is raided} \end{cases}.$$

[3] Even though we define the breakup fee d as a payment from the worker to the firm if he decides to quit, all our results remain unchanged if we model d as a deferred compensation, i.e., a part of the worker's period one wage that is paid at the end of period two, in addition to a *non-negative* period-two wage. In other words, in a contract with deferred payment d the firm is assumed to be contractually obligated to pay the worker *at least* d (as total compensation) in period two if he stays with the firm.

[4] That m is revealed to the raiders *after* the firm makes its promotion decision is assumed only for modeling convenience. For the purpose of our analysis, the key assumption is that m is *not* known to the firm when it makes the promotion decision. This is a natural assumption in many environments where the initial employer may not have complete information on the productivity of his worker in a competing firm (or even on job vacancies in the competing firms), and this information is revealed only after the worker generates offers from the potential raiders.

Similarly, the worker's payoff in period one is $u_1 = w_1$ but the period-two payoff, u_2, depends on his period-two job assignment and the offer/counteroffer that he subsequently receives. That is,

$$u_2 = \begin{cases} b_i - d_R & \textit{if } A \textit{ joins raider } i \\ w_2^j & \textit{if } A \textit{ stays with } F \textit{ in job } j \end{cases}.$$

Let $\Pi := \pi_1 + \pi_2$ and $U := u_1 + u_2$ be the aggregate payoffs of the firm and the worker respectively. Finally, the raider's payoff from a worker with ability a is:

$$\pi_{R_i} = \begin{cases} \psi_2 a (1+m) - b_i & \textit{if } R_i \textit{ successfully raids the worker} \\ 0 & \textit{otherwise} \end{cases}.$$

We assume that both the worker and the firm have a reservation payoff of 0.

Time Line. The following time line summarizes the game described above.

- *Period 1.0.* F publicly offers a contract (w_1, d) to A. If accepted, the game proceeds but ends otherwise.
- *End of Period 1.* Period-one output is realized and period-one wage (w_1) is paid. F privately observes A's ability (a) and decides on job assignment.
- *Period 2.0.* R_1 and R_2 observe job assignment as well as the matching factor m and simultaneously bid (b_1 and b_2) for A.
- *Period 2.1.* After observing the bids, F makes a counteroffer: To retain A in job j, F offers period-two wage w_2^j; if A is not retained, breakup fee may be renegotiated to d_R $(\leq d)$.
- *Period 2.2.* A chooses which employment contract to accept; pays d_R to F if he leaves for a raiding firm.
- *End of Period 2.* Period-two output is realized, period-two wage is paid and the game ends.

Strategies and equilibrium concept: The firm's strategy, σ_F, has three components: (i) at the beginning of period one, choose the initial contract offer (w_1, d), (ii) at the end of period one, decide on job assignment $j \in \{1,\ 2\}$ upon observing the worker's ability, and (iii) at the beginning of period two, upon observing the raiders' bids, decide on the counteroffer (period-two wage w_2^j if firm decides to retain the worker or renegotiated breakup fee $d_R \leq d$ otherwise). The worker's strategy, σ_A, has two components: (i) accept or reject the firm's initial contract, and (ii) choose period-two employer given the raiders' offer and the firm's counteroffer. Finally, raider i's strategy, σ_{R_i}, is to choose a wage bid b_i given the matching factor and the firm's job assignment decision (for $i = 1, 2$).

We use *perfect Bayesian Equilibrium (PBE)* as a solution concept (as defined in Fudenberg and Tirole, 2000; also see Watson, 2016, for a general definition of PBE that is applicable to a larger class of games).[5] Note that in a PBE if F deviates from its initial contract offer,

[5]Formally, in our framework, a PBE is defined as follows: Given the initial contract (w_1, d) and the subsequent job assignment $j \in \{1,\ 2\}$, let $\mu\left(a \mid (w_1, d), j\right)$ be the posterior belief of the raiders. A profile of strategies $\sigma^* = \left\langle \sigma_F^*, \sigma_A^*, \sigma_{R_1}^*, \sigma_{R_2}^* \right\rangle$ along with the raiders' belief μ^* constitute a PBE if (i) σ^* is sequentially rational given μ^*, (ii) on-equilibrium path μ^* is obtained through Bayes rule given the prior belief on ability and the strategies of the players, and (iii) off-equilibrium path μ^* satisfies the following restriction. If

the raiders' posterior belief in the continuation game is also obtained through Bayes rule. Thus, the equilibrium strategy profile and belief must induce a PBE in every continuation game following any initial offer (w_1, d). Hence, the optimal breakup fee is simply the one that induces the highest PBE payoff in the continuation game. In what follows, we analyze the optimal contracting problem accordingly.

3. Job assignment and turnover

In order to derive the optimal contract for the firm, we first need to analyze the players' equilibrium behavior in the continuation game following an initial contract (w_1, d). In what follows, we characterize the firm's equilibrium job assignment policy and worker turnover for any arbitrary value of d specified in period one. Notice that the wage in period one, w_1, has no impact on F's decision to promote the worker or raider's decisions in period two and hence, is ignored in the analysis below.

3.1. **An efficiency benchmark.** We begin our analysis by characterizing the promotion rule that maximizes the aggregate surplus assuming that following the job assignment decision, the turnover is always efficient (i.e., the worker leaves whenever he is a better match with the raider). Notice that the expected surplus generated by a worker with ability a (assuming efficient turnover) when he is promoted (S_P) and when he is not (S_N) are given as:

$$S_P(a) = \mathbb{E}_m[\max\{\psi_2 a, \psi_2 a(1+m)\}] \text{ and } S_N(a) = \mathbb{E}_m[\max\{\psi_1, \psi_2 a(1+m)\}].$$

Since the worker's productivity with the raiders is independent of his job assignment by the firm, the promotion rule that maximizes the aggregate surplus (when turnover is efficient) only needs to compare the worker's productivity in the two jobs when he stays with the firm. So, $S_P(a) \geq S_N(a)$ if and only if $a\psi_2 \geq \psi_1$. Thus, the efficient promotion rule is to promote a worker of ability a if and only if:

$$a \geq \frac{\psi_1}{\psi_2} \left(=: a^E\right). \tag{1}$$

In what follows, the threshold a^E serves as an benchmark for evaluating the extent of allocative inefficiency in equilibrium where private observability of ability leads to inefficiencies in turnover as well as in the firm's job assignment decision.

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the firm deviates in period one and offers an initial contract (w_1', d'), the posterior belief of the raiders $\mu^*(a \mid (w_1', d'), j)$ must also be obtained through Bayes rule defined as follows: Given an initial contract $(w_1', d') \in \mathbb{R}^2$ and the worker's type $a \in [0,1]$, denote $\sigma_F^{*J} : \mathbb{R}^2 \times [0,1] \to \{1, 2\}$ as the component of the firm's strategy σ_F^* that defines the firm's job assignment decision. We require,

$$\mu^*(a \mid (w_1', d'), j) = \frac{\Pr\left(j \mid a, (w_1', d'), \sigma_F^{*J}\right) \Pr(a)}{\Pr\left(j \mid (w_1', d'), \sigma_F^{*J}\right)}.$$

Also, the worker's belief on his ability remains unaffected by the firm's initial offer. Note that the restriction on the off-equilibrium belief invokes the "no signaling what you don't know" and "use of Bayes rule whenever possible" conditions suggested by Fudenberg and Tirole (2000). The initial contract (w_1, d) does not affect beliefs on ablility as it is offered before the ability is revealed. Also, in every continuation game following any initial contract offer by the firm, the raiders update their beliefs using Bayes rule given their (common) prior belief and the firm's job assignment decision (under his strategy σ_F^{*J} given the initial contract (w_1, d)).

3.2. **Equilibrium job assignment and turnover (given d).** We now analyze the equilibrium job assignment and turnover and explore how the extent of inefficiency is affected by the breakup fee.

As the firm cannot ex-ante commit to a promotion rule, at the end of period 1 the firm promotes a worker if and only if it is optimal to do so given the worker's ability and the offer-counteroffer game that follows in period 2. Also notice that similar to our benchmark analysis above, the firm's promotion decision in equilibrium continues to follow a cutoff rule. The argument is straightforward: Recall that the worker's productivity in job 2 is increasing in his ability (i.e., $a\psi_2$) but in job 1 it is constant (i.e., ψ_1). Now, as the worker's period-two wage is determined in the spot market and the raiders do not observe the worker's ability (a), the worker's wage *conditional* on job assignment is independent of his ability. So, the firm's payoff from offering promotion is increasing in a while denying promotion yields a constant payoff. Consequently, the firm promotes a worker if and only if his ability is greater than a cutoff value a^* (say).

In what follows, we solve for the equilibrium cutoff ability level a^* as a function of the breakup fee (d). Note that if a cutoff a^* constitutes an equilibrium promotion policy, the firm must be indifferent between promoting and not promoting the marginal worker with ability a^*—i.e., the firm's expected payoff from the marginal worker must be the same irrespective of the worker's job assignment. However, the derivation of the firm's payoff is somewhat involved as it depends on the raiders bid, which, in turn, depends on the firm's job assignment and counteroffer decisions. As the equilibrium strategies must be sequentially rational, we derive these payoffs through backward induction.

First, consider the firm's payoff from keeping a worker (including the marginal one) in job 1. We begin our derivation of the firm's payoff by first considering its counteroffer decision. Trivially, if there are no offers from the raiders (i.e., $b_i = 0$ for all i), the firm offers a wage $w_2^1 = 0$ to the worker to match his outside option and retains him in job 1.[6] But if the worker receives an external offer the firm's period-two wage offer needs a more careful study. Let b denote the highest bid that the worker receives; i.e., $b = \max\{b_1, b_2\}$. Throughout this article we refer to b as the market bid. Notice that upon receiving a market bid b, the worker (assigned in job 1) leaves the firm if and only if $b > \psi_1$. When $b \le \psi_1$, the firm offers $w_2^1 = 0$ to the worker if $b \le d$ and $w_2^1 = b - d$ if $b > d$, retaining him in both cases. But if $b > \psi_1$, the firm lets the worker go and collects $\min\{b, d\}$ from the worker as a breakup fee: if $b < d$ the firm and the worker renegotiate the breakup fee down to $d_R = b$; and if $b \ge d$ no renegotiation is called for and $d_R = d$.

Now, moving backwards in the game, consider the raiders' bidding strategy for a worker assigned to job 1 given a promotion theshold a^* (i.e., where the worker is promoted only if his type exceeds a cutoff a^*). Since the raiders compete for the worker, they make zero expected profit (in equilibrium) and bid the expected value of the worker whom they could successfully raid (given the firm's counteroffer decision). That is, the raiders successfully bid for a worker assigned in job 1 when:

[6] In our setting, the outside option of the worker in period two is the payoff he would get in case he leaves the firm but does not get hired by any raider. By modeling assumption, this payoff is zero. That is, we implicitly assume that the firm cannot contractually require a break-up fee from the worker should he leave the firm *irrespective* of his subsequent employment status. This is a natural assumption as in practice, any clause in a labor contract that imposes restrictions on the workers' mobility, such as a break-up fee, must be limited in scope in order to be enforced by courts; also, the court cannot force the worker to continue working for any specific employer.

$$\mathbb{E}_a\left[\psi_2 a(1+m) \mid a \in [0, a^*)\right] = \frac{1}{2}\psi_2 a^*(1+m) > \psi_1 \Leftrightarrow m > \frac{2\psi_1}{a^*\psi_2} - 1. \tag{2}$$

So, the raiders' equilibrium wage bids are $b_1 = b_2 = b_N^*$ where:

$$b_N^*(m; a^*) = \begin{cases} 0 & \text{if } m \leq \frac{2\psi_1}{a^*\psi_2} - 1 \\ \frac{1}{2}\psi_2 a^*(1+m) & \text{if } m > \frac{2\psi_1}{a^*\psi_2} - 1 \end{cases}. \tag{3}$$

Note that b_N^* is increasing in both m and a^*. A larger m implies a higher productivity and hence, leads to a higher bid. Also, a larger a^* implies that the firm is more selective in its promotion decision, and hence, the expected ability of the worker who misses promotion also increases. However, the equilibrium bid does not depend on d as the firm and the worker renegotiate the fee whenever it is efficient for the coalition to do so.

Using b_N^* we can derive the firm's payoff from keeping the marginal worker in job 1. From the firm's counteroffer strategy we know that when $b_N^*(m; a^*) = 0$, the worker stays with the firm and the firm earns ψ_1; but when $b_N^*(m; a^*) > 0$, the worker leaves the firm and the firm earns $\min\{b_N^*(m; a^*), d\}$. That is, the firm's payoff (as a function of m and d given a^*) is:

$$\pi_N(m, d; a^*) = \begin{cases} \psi_1 & \text{if } m \leq \frac{2\psi_1}{a^*\psi_2} - 1 \\ \min\{b_N^*(m; a^*), d\} & \text{otherwise} \end{cases}. \tag{4}$$

Next, consider the firm's payoff from promoting the marginal worker to job 2. As before, a promoted worker who does not receive any market offer gets $w_2^2 = 0$. If the worker receives a market offer of b, the firm makes a counteroffer and retains him if $b \leq a\psi_2$ but lets him go otherwise by (possibly) renegotiating the breakup fee down to $d_R = \min\{b, d\}$.

Observe that the raiders face a winner's curse problem while bidding for a worker in job 2—a successful raid necessarily implies that the worker's ability is relatively low (i.e., $a \in [a^*, b/\psi_2)$) as a worker with a higher ability would be retained by the firm.[7] Therefore, the raiders' expected profit from bidding b (for a worker assigned in job 2) is:

$$\mathbb{E}_a\left[\psi_2 a(1+m) \mid a \in [a^*, b/\psi_2)\right] = \begin{cases} 0 & \text{if } b \leq \psi_2 a^* \\ \frac{1}{2}\psi_2(a^* + b/\psi_2)(1+m) & \text{if } \psi_2 a^* < b < \psi_2 \\ \frac{1}{2}\psi_2(a^*+1)(1+m) & \text{if } b > \psi_2 \end{cases}.$$

As discussed earlier, by virtue of competition between raiders, their equilibrium bids for a worker in job 2 must satisfy $b_1 = b_2 = b_P^*$ where b_P^* solves $b = \mathbb{E}_a\left[\psi_2 a(1+m) \mid a \in [a^*, b/\psi_2)\right]$. That is,

[7]The winner's curse problem with raiders' bid has also been highlighted in several others models of job assignment and raids, e.g., Golan, 2005; DeVaro and Waldman, 2012. In these models, the worker is always a better match with the initial employer and the winner's curse effect ensures that the raiders only bid for the least productive worker who could be promoted. In contrast, in our setting, the equilibrium bids could be higher than the productivity of the marginal type (i.e., $\psi_2 a^*(1+m)$) as we allow for the worker to be a better match with the raider.

$$b_P^*(m,d;a^*) = \begin{cases} 0 & \text{if } m \le 0 \\ \psi_2 a^* \frac{1+m}{1-m} & \text{if } 0 < m < \frac{1-a^*}{1+a^*} \\ \frac{1}{2}\psi_2(a^*+1)(1+m) & \text{if } m > \frac{1-a^*}{1+a^*} \end{cases} . \tag{5}$$

Notice that as is the case for b_N^*, b_P^* is also increasing in m and a^*. Also note that in response to the winner's curse problem, the raiders shade their bids; i.e., in equilibrium, the raiders bidding for a promoted worker correctly anticipate that given a bid b, they will successfully raid the worker only if his ability $a \in [a^*, b/\psi_2)$. Consequently, it dampens the period-two wages of the promoted worker.[8]

The firm's counteroffer strategy and the raiders' bidding strategies given above imply that if there are matching gains ($m > 0$) in equilibrium the marginal worker (i.e., the one with ability a^*) always receives a market offer and the firm always lets him leave. Otherwise, the worker stays with the firm at zero wage. Hence, the firm's payoff from promoting the marginal worker is:

$$\pi_P(m,d;a^*) = \begin{cases} \psi_2 a^* & \text{if } m \le 0 \\ \min\{b_P^*(m,d;a^*), d\} & \text{otherwise} \end{cases} . \tag{6}$$

If the cutoff a^* constitutes an equilibrium the firm must have the same (expected) payoff from the marginal worker irrespective of his job assignment. So, a^* solves:

$$\mathbb{E}_m \pi_N(m,d;a^*) = \mathbb{E}_m \pi_P(m,d;a^*). \tag{7}$$

The following proposition characterizes this solution.

Proposition 1. *Given a breakup fee d, there exists a unique cutoff level $a^*(d)$ such that the firm promotes a worker if and only if his ability $a \ge a^*(d)$. The cutoff $a^*(d)$ is strictly decreasing in d for $d < \hat{d}$ and independent of d for $d \ge \hat{d}$ where $\hat{d} \in (\psi_2, \psi_1 + \psi_2)$. Moreover, $a^*(0) > a^E$ $(= \psi_1/\psi_2)$ and $a^*(\psi_1) = a^E$.*

Proposition 1 has two key implications: First, the firm is more likely to promote a worker the larger is the associated breakup fee; i.e., the promotion cutoff a^* (weakly) decreases in d. The argument is as follows: An increase in d increases the firm's expected payoff from the marginal worker irrespective of his job assignment (i.e., both $\mathbb{E}_m \pi_N(m,d;a^*)$ and $\mathbb{E}_m \pi_P(m,d;a^*)$ increase with d); however the increase in profit is more pronounced when the worker is assigned in job 2 than when he is kept in job 1. That is, a breakup fee protects the firm from the labor market competition irrespective of its job assignment decision but this

[8] We implicitly assume that the raiders do not play weakly dominated strategies. Otherwise, there may exist other equilibria where the raiders bid more than the expected value of the worker (to the raiders) if the firm is expected to retain the worker with certainty by making a counteroffer (this can happen if $m < 0$). One may rule out such equilibria as they are not "trembling hand perfect"—if there is a small probability that the worker may mistakenly accept the raiders' bid, then the raider is strictly better off by not placing a bid that is higher than its valuation for the worker. Such equilibria in dominated strategies also do not survive the "market-Nash" refinement of Waldman (1984).

protection is more valuable when the worker is promoted than when he is not as a promoted worker attracts more intense competition.

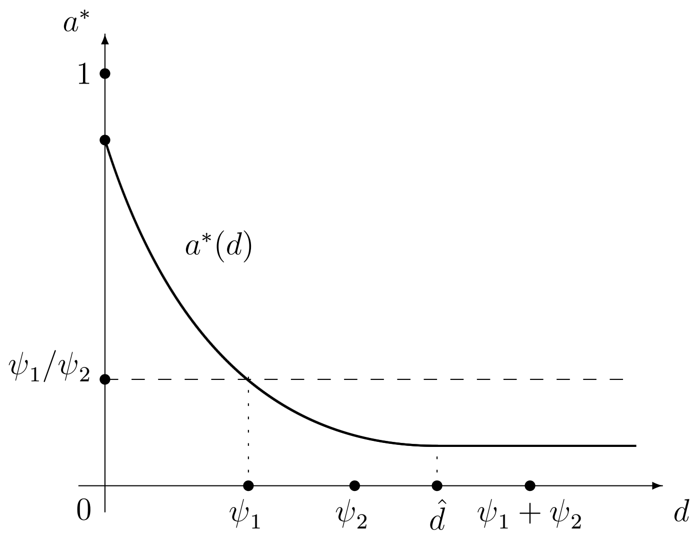

Figure 1. The equilibrium cut-off for promotion as a function of the break-up fee (d).

Observe that regardless of the firm's job assignment decision, the firm's payoff (i.e., both π_P and π_N) depends on d *only* when the market makes a relatively large bid for the worker (i.e., only when $b > d$ and b exceeds the productivity of the worker in the firm). In this case, the firm lets the worker go and receives d. Due to the signaling role of promotion, the market bids more often and more aggressively for a promoted worker than for a worker who is kept in job 1. Hence, the marginal worker's value to the firm increases more (with d) when he is promoted to job 2 than when he is kept in job 1—i.e., $\mathbb{E}_m\pi_P$ increases more than $\mathbb{E}_m\pi_N$. Consequently, the larger is the breakup fee the stronger is the firm's incentive to promote a worker. But when d is sufficiently large, it gets renegotiated down with certainty whenever the market makes an offer. Thus the firm's payoff, and hence, its promotion policy $a^*(d)$, no longer varies with d.

Second, in absence of any breakup fee, job assignment remains inefficient as too few workers are promoted ($a^*(0) > a^E$). This inefficiency is similar in spirit to the one discussed in Waldman (1984) and stems from the signaling role of job assignment that Waldman highlights. As a promoted worker is more likely to be of higher ability compared to the one who did not get the promotion, the market bids more aggressively for a promoted worker. Hence, it is costlier to retain a promoted worker vis-a-vis a worker who has not been promoted. As the firm's expected profit from promoting the worker decreases, in equilibrium, a worker is promoted only if he is significantly more productive in job 2 than in job 1 so that the resulting productivity gains could offset the wage premium that the firm must offer to a promoted worker.

We conclude this section with the following two remarks. First, it can be argued that in the absence of any firm-specific matching gains (i.e., if $m < 0$ with certainty), in our model the possibility of counteroffer can remedy the inefficiencies in job assignment as the raiders would refrain from bidding due to the winner's curse problem discussed above (Golan, 2005).

However, as Waldman and Zax (2016) points out, there is a distortion in job assignment à la Waldman (1984) whenever the signaling role of promotion leads to a wage premium for the promoted worker (also see DeVaro and Waldman, 2012). In our setting, the possibility that the worker could be a better match with the raiders (i.e., $m > 0$) gives rise to such a wage premium. Even though the counteroffer dampens the raiders' bid by creating a winners' curse problem, when $m > 0$ it is still profitable for the raiders to bid for the worker and they bid more aggressively for a promoted worker as promotion signals higher ability.

Second, even though the use of breakup fee may lessen the inefficiencies in job assignment (as a^* decreases in d), it may accentuate the inefficiencies in turnover through its influence on the raiders' bids. As the following sections elaborate, the *optimal* breakup fee trades off these two inefficiencies.

4. The nature of allocative inefficiencies

Before we characterize the optimal contract, it is instructive to illustrate the nature of the allocative inefficiencies that arise in our model given an arbitrary promotion policy and to highlight how these inefficiencies vary with a change in the promotion policy. Consider an arbitrary promotion policy where the firm assigns the worker to job 2 if and only if his ability $a \geq a_0$. Suppose that the cutoff $a_0 > a^E$, as is the case with the equilibrium promotion policy in absence of any breakup fee. Given this promotion policy, there are four sources of inefficiencies in the allocation of the worker. It is helpful to discuss them using Figure 2 (panel (i)) where these inefficiencies correspond to the areas labeled as A, B, C and D.

For $m < 0$, there is inefficiency in job assignment (reflected by area A): the firm assigns a worker with $a \in \left[a^E, a_0\right]$ in job 1 even though he is more productive in job 2. When $m > 0$, it is efficient for a worker of ability $a > \psi_1/\psi_2\left(1+m\right)$ to leave the firm. However, as discussed earlier, if the worker is assigned to job 1, he stays in the firm if $m \leq 2\psi_1/a_0\psi_2 - 1$ (see equation (3)) and if he is assigned to job 2, he is retained by the firm when $\psi_2 a \geq b_P^*$, or, equivalently, $m \leq \left(a - a_0\right)/\left(a + a_0\right)$ (see equation (5)). Thus, in the former case (which corresponds to area B), the worker remains with the firm in job 1 and in the latter case (which corresponds to area D), the worker stays with the firm in job 2 even though in both cases he is more productive with the raiders. Finally, when $m > 2\psi_1/a_0\psi_2 - 1$, a worker with ability $a \in \left[0, \psi_1/\psi_2\left(1+m\right)\right]$ is successfully raided by the market even though he is more productive with the firm in job 1 (shown by area C).

As mentioned earlier, the distortion captured by area D stems from the winner's curse effect. The raiders shade their bids as a successful raid may carry a negative signal about the worker's ability, i.e., the initial employer did not find the worker productive enough to warrant a matching wage offer. Thus, for small m, the bid b_P^* may be less than worker's productivity with the firm in job 2 ($\psi_2 a$) and the firm would find it profitable to match the bid even though the worker would have been more productive with the raiders.

Next, consider the marginal effects of the promotion threshold (a_0) on these inefficiencies and suppose that the threshold is lowered from a_0 to a_1 (see panel (ii)). Clearly, this change leads to more efficient allocation of a worker with ability $a \in [a_1, a_0]$: first, as the efficiency in job assignment would require, such a worker is now promoted to job 2 rather than kept in job 1; second, as turnover efficiency would require, such a worker would stay with the firm if $m < 0$ (gains shown by area A') and leave for the raiders if $m > 0$ (gains shown by area B').

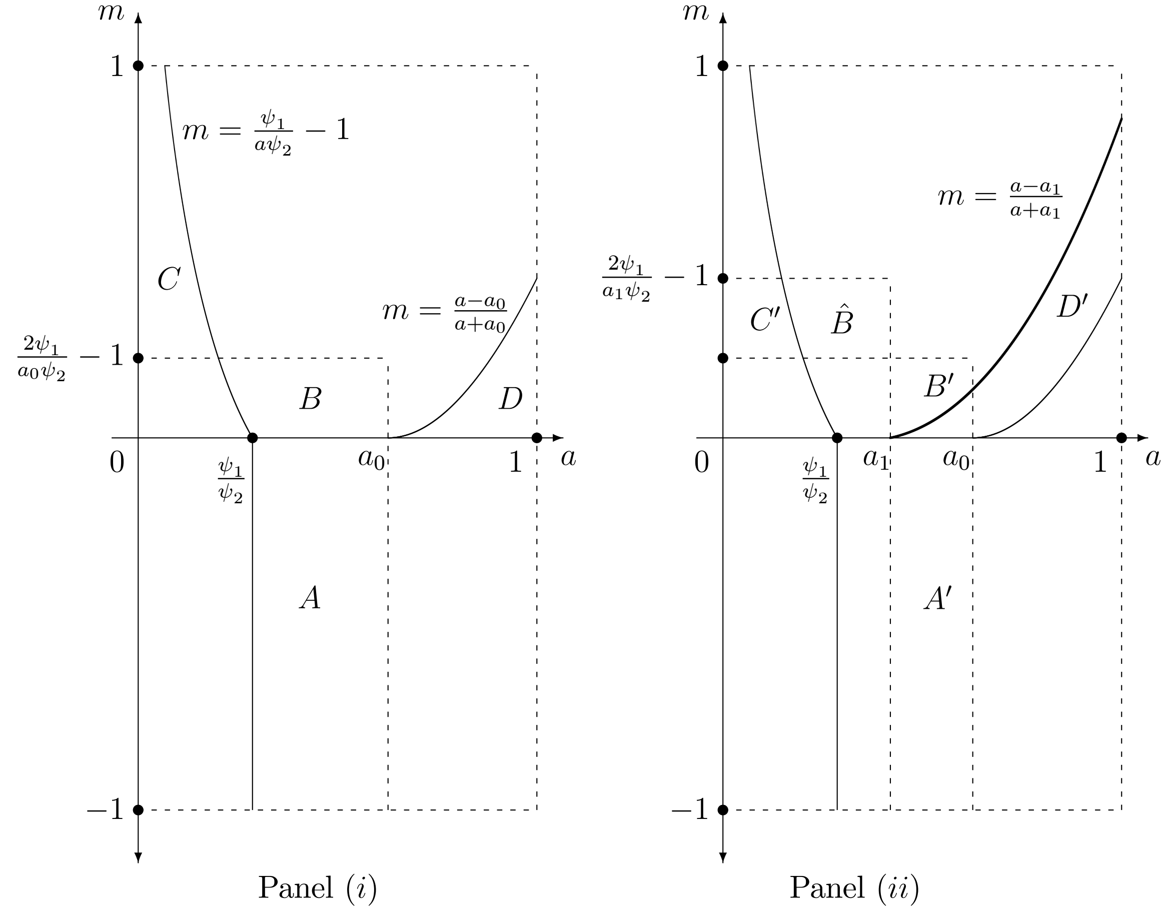

Figure 2. The allocative inefficiencies associated with a given promotion policy.

But the improved allocation of these worker types comes at a cost of distorted worker turnover. First, note that a promoted worker is now more likely to be retained by the firm due to an aggravated winner's curse problem even if he is more productive with the raiders. As the promotion threshold is lowered, the expected productivity of a promoted worker decreases and so does the equilibrium bid. Thus, the firm will retain a higher share of the workers: now a worker of ability a is successfully raided only if $m > (a - a_1) / (a + a_1) > (a - a_0) / (a + a_0)$ (the increased turnover inefficiency is shown by area D').

Second, turnover is reduced even for the worker types who are not promoted. As the ability threshold for promotion is reduced, the expected quality of a worker who is kept is job 1 is also lowered. Hence, the worker leaves from the raider only if the matching factor is sufficiently high, i.e., $m > 2\psi_1/a_1\psi_2 - 1 > 2\psi_1/a_0\psi_2 - 1$. Such reduction in turnover decreases surplus when the worker is of ability $a \in [\psi_1/\psi_2 (1 + m), a_1]$ as he would have been more productive with the raiders (loss shown by area $\hat{B}$) but increases surplus otherwise as such a worker is more productive with the firm but would have left for the raider when promotion threshold was higher (gain shown by area C'). But notice that the matching factor cutoff, $2\psi_1/a_0\psi_2 - 1$, is the one for which the worker has the same expected productivity with the firm (in job 1) and raiders (see equation (2)). Hence, the aforementioned gains and losses (areas $\hat{B}$ and C') exactly offset each other.

Therefore, the promotion policy that maximizes the expected aggregate surplus ex-ante must balance the trade-off between improved worker-job matching (areas A' and B') and worsened worker-firm matching for the promoted workers (area D'). As we discuss below, this is also the trade-off that drives the firm's choice of optimal breakup fee.

5. The optimal breakup fee

As the raiders make zero profit due to competition and the firm extracts all rents from the worker by sufficiently lowering the first-period wage (w_1), the firm appropriates the entire surplus that is generated by the coalition of the firm, worker and the raiders. Consequently, the problem of choosing the optimal breakup fee can be conceived as the problem of choosing d such that the equilibrium promotion rule $a^*(d)$ maximizes the aggregate surplus over the two periods. Thus, the firm's optimal contracting problem boils down to:

$$\max_{d} \ \Pi(d) := \psi_1 + S(a^*(d)),$$

where $S(a_0)$ represents the expected aggregate surplus in period two under an arbitrary promotion threshold a_0, i.e.,

$$\begin{aligned} S(a_0) := \ & \psi_1 \Pr[\textit{no turnover, no promotion} \mid a_0] \\ & + \ \mathbb{E}_{a,m}[\psi_2 a \mid \textit{no turnover, promotion, } a_0] \Pr[\textit{no turnover, promotion} \mid a_0] \\ & + \ \mathbb{E}_{a,m}[\psi_2 a(1+m) \mid \textit{turnover, } a_0] \Pr[\textit{turnover} \mid a_0]. \end{aligned} \tag{8}$$

The following proposition characterizes the optimal breakup fee.

Proposition 2. *There exists a strictly positive cutoff $\underline{\psi}_1$ (given α and ψ_2) such that the optimal breakup fee is zero if $\psi_1 \leq \underline{\psi}_1$ but is strictly positive otherwise. Moreover, for $\psi_1 > \underline{\psi}_1$ (i) the optimal breakup fee is increasing in ψ_1 and (ii) the use of breakup fee in the optimal contract enhances welfare as it increases the aggregate surplus.*

Notice that the firm need not use any breakup fee if the worker's ability is public or if the promotion decision is private—in both of these cases promotion does not play any signaling role and hence, there is no distortion in job-assignments. Thus, a key implication of the above proposition is that breakup fees are more likely to be observed when the information on the workers' ability is private (to the initial employer) but information on job-assignment is public.

This prediction of the model is consistent with the recent surge in the use of noncompete clauses in employment contracts that could be conceived as contracts with steep breakup fees (Lobel, 2013). One may assume that the recent growth in the recruiting networks (e.g., LinkedIn) has made a worker's career progress within a firm clearly visible to the outsiders while his actual quality is still his employer's private information. It is often argued that noncompete clauses protect the firm's investment in human capital. But Lobel also finds proliferation of noncompete clauses even in the industries where human capital investments hardly play a role and the former argument fails to explain this observation.

Another salient implication of the above finding is that the optimality of a breakup fee is driven by the relative productivity of the worker in the two jobs: it is never optimal to stipulate a breakup fee if the worker's productivity in job 1 (i.e., ψ_1) is too low compared to his expected productivity in job 2 (as reflected by ψ_2). Otherwise, it is always optimal to specify a breakup fee in the employment contract and the size of the fee increases as the difference between the worker's expected productive in the two jobs gets smaller. In other words, breakup fees are more likely to be used when the production technologies in the pre- and post-promotion jobs become similar (e.g., they involve similar sets of tasks).[9]

The intuition behind this finding is as follows. As discussed above, the firm's promotion threshold $a^*(d)$ is decreasing in d. Also recall that such a reduction in promotion threshold leads to a trade-off between the gains from improved efficiency in job assignment and the loss from more inefficient turnover for the promoted workers. When ψ_1 is small, the marginal gain from the former effect is lower than the marginal loss from the latter one. To see this, note that for low ψ_1, the equilibrium promotion rule a^* is also low even in the absence of any breakup fee—as the worker is hardly productive in job 1, the firm has a strong incentive to assign him to job 2. As most workers are promoted (when ψ_1 is small), the marginal worker who remains in job 1 is of relatively low ability and assigning him to job 2 (as efficiency in job assignment dictates) has only a small impact on his productivity. Thus, while the introduction of a breakup fee does improve job assignment, its marginal benefit is rather small. In contrast, its marginal cost stemming from inefficient turnover of the promoted workers is still significant as most types of the worker are assigned to job 2 at the first place. Hence, when ψ_1 is small, the marginal benefit of the breakup fee (in terms of efficient promotion) is more than offset by its marginal cost (in terms of reduced turnover of the promoted workers) and it is optimal not to use such a fee in the employment contract.

But when ψ_1 is high, the opposite happens—the marginal benefit from efficiency in job assignment dominates the marginal cost of inefficiency in turnover. When ψ_1 is large, in absence of any breakup fee very few types of the worker are promoted in equilibrium. Thus, the marginal worker who misses promotion is of relatively high ability and the gains in productivity from (efficiently) promoting him are relatively large. In contrast, the loss from the inefficiencies in turnover are small as very few types of the workers are promoted in absence of any breakup fees. Hence, when ψ_1 is large, the firm can increase its profit by stipulating a breakup fee that ensures a more efficient promotion policy.

Finally, consider the optimality of breakup fees from the social welfare perspective. Since the firm extracts the entire surplus generated by the worker, if the inclusion of a breakup fee is profit-enhancing for the firm, it is also socially optimal—it increases the aggregate social surplus generated by the coalition of the firm, worker and the outside labor market.

6. Discussion and extensions

In this section, we highlight the implications of the firm-specific human capital in our model and also explore the robustness of our key findings to a set of alternative modeling assumptions.

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[9]Unfortunately, empirical findings on this issue are rather scant as task variations across jobs in the organizational hierarchies may be difficult to measure. An empirical test of our prediction can potentially follow the approach suggested in DeVaro et al. (2012). In an analysis of discrimination in labor markets, they construct a measure of task variability from information on factors such as knowledge required, supervision received, guidelines, etc., that describe the nature of a given job within the organizational hierarchy of a given firm.

6.1. **Role of firm-specific human capital.** The the key role of a breakup fee in our setting is to shield the firm from the competitive pressure on wages that stems from the promotion signaling. And in the process, breakup fee improves the efficiency in job-assignment at the cost of inefficiencies in turnover. But such a competitive pressure on wages is less likely to arise when the firm-specific human capital becomes more important in driving the worker's productivity. Indeed, the canonical models on the signaling role of job-assignment (Waldman, 1984; Ghosh and Waldman, 2010) suggest that the extent of inefficiency in job-assignment becomes smaller the larger is the role of the firm-specific human capital. Thus, one may conjecture that the firm is less likely to use a breakup fee when firm-specific human capital is more critical for the production process.

Recall that in our model, one may interpret α as the measure for importance of firm-specific human capital: the larger is α the less likely it is that the worker would be a better match with the outside labor market. Unfortunately, an analytical derivation of the comparative statics of the optimal breakup fee (d^*) with respect to α appears to be algebraically intractable. Nevertheless, as the following proposition indicates, the impact of firm-specific human capital is more nuanced than what the above conjecture suggests.

Proposition 3. *(i) The promotion cutoff $a^*(d)$ decreases with α when $d < \psi_1$ and increases with α otherwise. (ii) The threshold $\underline{\psi}_1$ (i.e., the value of ψ_1 above which it is optimal to specify a breakup fee) is decreasing in α.*

Proposition 3 has two important implications: First, if the breakup fee is not too high ($d < \psi_1$), the larger is the role of firm-specific human capital the stronger is the firm's incentive to promote a worker. But otherwise ($d \geq \psi_1$), an increased importance of firm-specific human capital leads to fewer promotions. When $d < \psi_1$, the argument behind this finding is exactly the same as the one discussed above: as the market is less likely to compete for the worker, it mutes the upward pressure on wages following promotion, and hence, the firm is more likely to promote a worker. But when the breakup fee is sufficiently large, the promotion threshold becomes too low: for $d > \psi_1$, $a^*(d) < \psi_1/\psi_2$; that is the marginal worker is now more productive in job 1 than in job 2 and the firm gains more if the worker receives an external offer and leaves rather than if he stays back in job 2 (since $\psi_2 a^* < \psi_1 < d$). The firm promotes such a worker since the probability of an external offer is higher when he is promoted than when he is not. But as α increases the worker is less likely to receive an external offer irrespective of his job assignment. Hence, it becomes more profitable for the firm to retain the worker in job 1 and raise the promotion threshold.

Second, in our setting the above conjecture on the negative relationship between the use of the breakup fee and the importance of firm-specific human capital need not hold. In particular, as the firm-specific human capital becomes more essential, the firm is more likely to stipulate a breakup fee. To see the intuition, recall that the use of a breakup fee trades off the marginal gains for more efficient job assignment with the marginal loss from inefficiencies in turnover. Moreover, the optimal fee is one that induces a promotion rule a^* that maximizes the expected aggregate surplus $S(a_0)$.

Now, as the firm-specific human capital becomes more critical (i.e., the α increases), there are two opposing effects on the optimal breakup fee: As discussed earlier, breakup fee mutes the competitive pressure on wages (following job assignment) and incentivizes the firm to promote more workers; i.e., $a^*(0)$ decreases with α. Clearly, this effect reduces the need for breakup fee in the optimal contract when the firm tries to implement a specific promotion cutoff.

But there is also a countervailing effect: When α increases the promotion cutoff that maximizes the expected aggregate surplus also decreases. As the worker is likely to be more productive with the firm than with the raiders, turnover is less likely to be efficient at the first place. So, if the promotion threshold is lowered, the associated marginal (expected) loss from an inefficient turnover gets reduced whereas the marginal (expected) gain from a more efficient job-assignment increases (as the worker is now more likely to stay with the firm, assigning him in the right job becomes more important). Thus, even though $a^*(0)$ decreases with α, the firm may now want to implement an even lower cutoff for promotion and use a breakup fee in order to achieve the same.

However, it is important to note that this finding critically hinges on the modeling assumption that the market could raid a worker irrespective of his job assignment. The result is overturned if one assumes—in the spirit of the "Invisibility hypothesis" à la Milgrom and Oster (1987)—that the raiders can bid for the worker only if he is promoted whereas a worker who remains in job 1 is insulated from the outside labor market (i.e., a worker becomes visible to the market only after he is assigned to a high-level job). In such a setting, the firm is indeed less likely to use a breakup fee when α increases as the countervailing effect mentioned above is weaker.

The argument for this observation is somewhat involved and a working paper version of this article (Mukherjee and Vasconcelos, 2016) presents a complete analysis of this setting. But a brief intuition is as follows. In our model, a worker who is assigned to job 1 in spite of being more productive in job 2 is likely to leave when he is a better match with the raiders (i.e., $m > 0$). Thus, the inefficiency in job assignment affects the firm's payoff only when the worker stays with the firm; i.e., the firm benefits from a more efficient job assignment primarily when $m < 0$. An increase in α makes this event more likely, and hence, the firm's expected marginal benefit (from a more efficient job assignment) increases as well.

Now consider the case where the raiders can only bid for a worker if he has been assigned to job 2. In such a setting, a worker in job 1 always stays with the firm irrespective of his match quality, i.e., as the job assignment becomes more efficient the firm gains not only when $m < 0$ but also when $m \geq 0$. But notice that as α increases, it increases the weight on the firm's gains when $m < 0$ but reduces the weight on the same when $m \geq 0$. Hence, the firm's marginal expected gains from improving the efficiency in job-assignment become smaller (compared to the setting considered in our main model where the gains are mostly accrued when $m < 0$). As a result, when α increases, the promotion threshold that maximizes the aggregate expected surplus need not decrease as much as it does in our main model and the resulting decrease in $a^*(0)$ may be sufficiently large compared to the promotion cutoff that the firm prefers to implement. Hence, the firm is less likely to use a breakup fee as α increases.

The discussion above highlights that the relationship between the use of breakup fee and the importance of firm-specific human capital is more subtle than what the extant literature suggests and the market visibility of the workers who remain in the low level jobs plays a key role in governing this relationship.

6.2. **Breakup fee based on ability.** In some settings the firm may choose the breakup fee *after* observing the workers' type. That is, the firm may simultaneously decide on the promotion of the worker and on the breakup fee. How would the optimal contract change in such a setting? While a complete characterization of the equilibrium appears intractable, two salient observations can be made: first, in equilibrium, the breakup fee may vary with ability, and hence, the optimal contract also serves as a signal (in addition to signal implied by job-assignment) on the worker's quality.[10] Second, the breakup fee is used regardless of the difference of the workers' productivity between the two jobs. The latter observation is somewhat nontrivial and the argument is as follows.

Note that in our baseline model, the issue of allocational efficiencies and surplus extraction can be decoupled: surplus extraction is done using the period 1 wage w_1 and d is chosen so as to implement the promotion policy that maximizes the aggregate surplus. When the difference in a worker's productivity between jobs is high, the gain in worker-job allocation from using d does not compensate the loss in worker-firm allocation and the firm optimally sets $d = 0$. But if d is specified along with the promotion decision at the end of period 1, the choice of d also affects surplus extraction, i.e., it protects the firms' profit in case it decides to promote the worker. As long as there is a chance that a worker would receive an offer from the raiders, it will be optimal to set a break up fee—with a breakup fee it is always cheaper to retain a worker and the firm obtains a compensation in case the worker leaves. This argument holds even if the difference in the worker's productivity between jobs is large. So, in this case, d is used more as a tool to appropriate surplus than as a tool to achieve allocational efficiency. Of course, even in this case, the use of d still has the trade-off we highlight earlier: it leads to more efficient promotion but compromises turnover efficiencies. But this trade-off never precludes the use of breakup fees in the optimal contract.

6.3. **Renegotiation of breakup fee.** Our model allows the firm and worker to renegotiate the breakup fee (d) whenever it is efficient for them (as a coalition) to do so. It turns out that our key results continue to hold even if we assume that the breakup fee is not renegotiable. However, in such an environment breakup fee leads to a new effect: it may aggravate inefficiencies in turnover by directly foreclosing raiders from bidding for the worker. A detailed analysis of this case is available in Mukherjee and Vasconcelos (2016) but it is relatively straightforward to see why the foreclosure effect may arise. When the breakup fee is not renegotiable, a raider successfully bids away the worker only if his bid exceeds the fee. Therefore, when the fee is sufficiently large (in particular, if $d > \psi_1$), the raider may refrain from bidding (even when the worker is more productive with the raider) as in order to successfully bid away the worker, he must pay more than the worker's expected productivity.

As this effect arises only when the fee is sufficiently large, it does not affect our findings on when such a fee should be used. When $d = 0$, the effects of raising d on the margin is still driven by the same trade-off between efficiencies in job-assignment and turnover that we have discussed earlier. Hence, Proposition 2 remains largely unaffected: breakup fee is optimal only when ψ_1 is sufficiently large and in this case the use of breakup fee is also welfare enhancing. Similarly, the characterization of the equilibrium promotion threshold as given in Proposition 1—$a^*(d)$ decreases with d—continues to hold as long as d is not too large. However, if d is sufficiently large, $a^*(d)$ starts to increase with d. When the fee is sufficiently large and cannot be renegotiated down it may be more profitable for the firm to let the worker leave than to retain him. So, the firm may find it optimal to be more selective in its promotion policy—as promotion becomes a stronger signal of ability, it elicits a more aggressive bidding from the raider and raises the likelihood of turnover. Of course, when d

[10] A similar issue is highlighted in Bernhardt and Scoones (1993).

becomes even larger, the raiders are completely foreclosed and the promotion policy is no longer affected by d.

It is interesting to note that the discussion above is reminiscent of the model of bilateral trade with potential entrants à la Aghion and Bolton (1987) where the seller may stipulate a breakup fee in his contract offer to the buyer in order to foreclose a more efficient entrant from the market. However, as is the case in our model, such a foreclosure effect disappears if the buyer and the seller can renegotiate the breakup fee up on entry (Spier and Whinston, 1995).[11]

6.4. **Severance payments and long-term wage contracts.** Our analysis assumes that long-term wage contracts are infeasible and wages in period two are set in the spot market. While this is a common assumption in the literature (see, for example, Zabojnik and Bernhardt, 2001; DeVaro and Waldman, 2012) it is interesting to note the implications of long-term contracts in our setting. Instead of relying on breakup fees, the firm can use long-term contracts that commit to severance pays or period-two wages to alleviate the inefficiencies in job assignment.

Consider the use of severance pay where the firm commits to make lump-sum payments to the worker (depending on his job assignment) when the employment relation terminates, *irrespective* of whether the worker stays with the firm in period two (and leaves at the end of period) or leaves at the beginning of the period to join the raider. The firm can always implement the promotion rule that maximizes the aggregate surplus $S(a^*)$ by choosing the payments appropriately to mitigate the wage differentials between jobs that stems from a job assignment signal. As in our model, the optimal promotion rule trades off efficiencies in job-assignment and turnover and, in equilibrium, both inefficiencies persist. Also, the equilibrium, severance pay is larger in job 1 compared to job 2, as it must generate a stronger incentive for the firm to promote the worker.

However, such a contract is profitable provided that the firm can ex-ante recover the severance payments by lowering the period-one wage of the worker. As these payments are made to all workers irrespective of their ability and job assignments, it would require the firm to significantly lower the worker's period-one wage to extract all rents. So, if the worker has liquidity constraints, such a low period-one wage may not be feasible and the optimal contract may still fall short of achieving the promotion cutoff that maximizes the aggregate surplus $S(a_0)$.[12]

The implications of the long-term wage contracts are also similar. Waldman (1984) shows that long-term wage contracts that commit to period-two wages of the worker (along with the period-one wage) can ensure efficient job-assignment by making the period-two wage contingent on the job assignment. As one would expect, the same holds in our setting as well even though, in equilibrium, job-assignment may remain inefficientas it trades off

[11]Spier and Whinston (1995) also note that even with renegotiation, the market foreclosure effect reappears if the seller needs to make relationship specific investments and the entrant has some market power. In the context of our model, this finding suggests that if the initial employer invests in its worker for firm-specific human capital accumulation and if the raider can make take-it-or-leave-it offer, then contract renegotiation need not rule out the possibility of market foreclosure. A complete analysis of this issue is beyond the scope of this article and remains an interesting topic for future research.

[12]Liquidity constraints can be less binding under contracts with breakup fee as the worker may have lower rents in period two (hence, period-one wage need not have to be lowered as much to ensure complete rent extraction).

efficieny in turnover. Similar to the case of severance payments, such a contract improves efficiency in job assignment by making it more costly for the firm to keep a worker in job 1—the firm implements the promotion rule that maximizes the aggregate surplus $S(a_0)$ by committing to a period-two wage that is larger when the worker stays in job 1 rather than in job 2.

It is important to note, however, that the use of such long-term wage contract is seldom observed in practice as the firm may lack the necessary commitment power. Also, the key feature of the optimal contract mentioned above—i.e., committing a higher wage to the workers who *fails* to get promoted—is rather unrealistic. As promotion tournaments are often used to provide work incentives, such a wage schedule may undermine the incentive role of promotions. Also, similar to the case of severance payments, if the workers are liquidity constrained, such a contract may not be feasible and the optimal contract may fail to ensure efficient promotion.

7. Conclusion

Breakup fees are contracting tools that firms frequently use to restrict turnover. Several authors have argued that such a restriction could be beneficial to the firm as it increases the firm's incentives for investment in its workers' human capital, guards against diffusion of proprietory knowledge, and protects the firm from potential losses associated with employee spinoffs. This article highlights a novel trade-off associated with the use of such fees in an environment with asymmetric learning about the worker's productivity and firm-specific matching gains. The use of breakup fees reduces inefficiencies in job-assignment à la Waldman (1984) that stems from its signaling value but creates inefficiencies in turnover.

Our key finding is that the optimality of the breakup fee depends on the relative size of the worker's expected productivity across jobs. If there are substantial (expected) productivity gains from promotion, then it is never optimal to specify any breakup fee in the employment contract. Moreover, when the use of a breakup fee is optimal for the firm it is also socially optimal as it increases the aggregate social welfare. Our analysis also suggests a subtle link between the optimality of breakup fee and the importance of firm-specific human capital as it critically depends on the market visibility of the workers at different levels of the organizational hierarchy.

It is important to note in the presence of asymmetric information on workers' quality any personnel decision by the initial employer, including, but not limited to job assignment, that releases information on the workers' quality to the outside labor market makes the firm vulnerable to raids. This leads to higher wages for the retained workers and the threat of such competition distorts the firm's personnel decisions at the first place.[13] Thus, the value of breakup fee that we highlight here is not limited to improving the efficiency in job assignments, per se. The fee may be used to mitigate inefficiencies in any personnel decision that may be distorted due to its signaling role in the outside labor market.

[13] For example, Loveman and O'Connell (1996) offer a case study on an IT firm where the firm must decide whether to send its software programmers to the clients'premises or to require them to work in-house (and ship out the final product to the client). The firm experiences a high rate of turnover amongst the workers who work at the clients'site as the client firms learn more about the quality of the workers and bid away the better ones. The turnover risk distorts the firm's job design policy as it becomes biased towards in-house projects.

There are several other economic effects that are interesting and relevant in our environment albeit beyond the scope of our model. One may assume that to be productive in the "post-promotion" job, it is necessary that the worker (and/or the firm) invests in human capital. How would the presence of breakup fees affect the incentives for investment? The answer to this question depends on whether the human capital is general or firm-specific and who undertakes the investments.[14] It would also be interesting to consider the case where the market can screen the promoted workers (see Ricart i Costa (1988) for a related model on managerial job assignment). Here, the firm's promotion policy continues to play an important role as it can affect that information rent that the worker earns from the market (which, in turn, can be extracted by the initial employer). Finally, if there is a moral hazard problem in the production process, the use of breakup fees may create an additional cost: it mutes work incentives by dampening the raiders' bid, and therefore, lowering the prospect to future wage increments (see, Kräkel and Sliwka (2009) for a similar discussion).

The issues raised above offer useful directions for future research and may offer additional insights into the firm's job assignment policies. However, the key trade-off between the job-assignment and turnover that we highlight in this article continues to play a critical role in all these setting and we expect our findings to be informative in analyzing such complex environments.

Appendix

This appendix contains the proofs omitted in the text.

Proof of Proposition 1. To simplify the exposition, let $\Pi_N(d;a^*) := \mathbb{E}_m \pi_N(m,d;a^*)$ and $\Pi_P(d;a^*) := \mathbb{E}_m \pi_P(m,d;a^*)$. Using (3) and (4), we obtain that:

$$\Pi_N(d;a^*) = \begin{cases} \psi_1 G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right) + d\left[1 - G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\right] & if \ d \le \psi_1 \\ \psi_1 G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right) + \int_{\frac{2\psi_1}{\psi_2 a^*}-1}^{\frac{2d}{\psi_2 a^*}-1} \frac{1}{2}\psi_2 a^*(1+m)dG(m) & \\ \qquad + d\left[1 - G\left(\frac{2d}{\psi_2 a^*} - 1\right)\right] & if \ d > \psi_1 \end{cases}; \tag{9}$$

and using (5) and (6), we obtain that

[14] Golan (2005) addresses these issues in a related environment but does not consider breakup fees or matching gains with the outside labor market. Also see Bernhardt and Scoones (1998) for a related discussion on the incentives to invest on human capital.

$$
\Pi_P(d;a^*) = \begin{cases} \psi_2 a^* G(0) + d\left[1-G(0)\right] & if\ d \le \psi_2 a^* \\ \psi_2 a^* G\left(0\right) + \int_0^{\frac{d-\psi_2 a^*}{d+\psi_2 a^*}} \frac{1}{2}\psi_2 a^*(1+m)dG(m) \\ \qquad +d\left[1-G\left(\frac{d-\psi_2 a^*}{d+\psi_2 a^*}\right)\right] & if\ \psi_2 a^* < d \le \psi_2 \\ \psi_2 a^* G\left(0\right) + \int_0^{\frac{1-a^*}{1+a^*}} \psi_2 a^* \frac{1+m}{1-m} dG(m) + \\ \quad \int_{\frac{1-a^*}{1+a^*}}^{\frac{2d}{\psi_2(1+a^*)}-1} \frac{a^*+1}{2}\psi_2(1+m)dG(m) + \\ \qquad +d\left[1-G\left(\frac{2d}{\psi_2(a^*+1)}-1\right)\right] & if\ d > \psi_2 \end{cases} \tag{10}
$$

The remainder of the proof is given in three steps, each characterizing the equilibrium promotion rule for a given range of values of the breakup fee d.

Step 1: *Equilibrium promotion rule when $d \le \psi_1$.* In this case, $\Pi_N(d;a^*) = \psi_1$ for all $a^* \le \psi_1/\psi_2$ (since $2\psi_1/\psi_2 a^* - 1 > 1$ and $G\left(2\psi_1/(\psi_2 a^*) - 1\right) = 1$), and $\Pi_N(d;a^*)$ decreases with a^* for $a^* > \psi_1/\psi_2$. Hence, $\Pi_N(d;a^*)$ is non-increasing in a^*. In contrast, $\Pi_P(d;a^*)$ is increasing in $a^* \in [0,1]$: clearly, $\Pi_P(d;a^*)$ increases with a^* when $d \le \psi_2 a^*$; and when $\psi_2 a^* < d$,

$$
\frac{\partial}{\partial a^*}\Pi_P(d;a^*) = \psi_2 G\left(0\right) + \int_0^{\frac{d-\psi_2 a^*}{d+\psi_2 a^*}} \frac{1}{2}\psi_2(1+m)dG(m) > 0.
$$

Thus, given d, the equation $\Pi_N(d;a^*) = \Pi_P(d;a^*)$, which defines $a^*(d)$, has at most one solution. To see that it has a solution, and that such solution $a^*(d) \in [\psi_1/\psi_2, 1)$, observe that Π_N and Π_P are continuous in a^*,

$$
\Pi_P\left(d;\frac{\psi_1}{\psi_2}\right) = \psi_1 G(0) + d\left[1-G(0)\right] \le \psi_1 = \Pi_N\left(d;\frac{\psi_1}{\psi_2}\right) \tag{11}
$$

and

$$
\begin{aligned} \Pi_P(d;1) &= \psi_2 G(0) + d\left[1-G(0)\right] \\ &\ge \psi_1 G\left(\frac{2\psi_1}{\psi_2}-1\right) + d\left[1-G\left(\frac{2\psi_1}{\psi_2}-1\right)\right] = \Pi_N\ (d;1), \end{aligned} \tag{12}
$$

where this inequality follows from Assumption 1, the fact that $\psi_1 < \psi_2$ (which is implied by Assumption 1), and $d \le \psi_1$.

We next show that $a^*(d)$ decreases with d. Since Π_P increases with a^* and Π_N decreases with a^*, it is sufficient to show that $\partial\Pi_P/\partial d > \partial\Pi_N/\partial d$ when $a^* = a^*(d)$. Observe that

$$
\frac{\partial \Pi_P}{\partial d} - \frac{\partial \Pi_N}{\partial d} = G\left(\frac{2\psi_1}{\psi_2 a^*}-1\right) - G(0).
$$

This is always positive when $a^* = a^*(d)$, since $a^*(d) < 2\psi_1/\psi_2$. To see that $a^*(d) < 2\psi_1/\psi_2$, note that

$$
\Pi_P\left(d;\frac{2\psi_1}{\psi_2}\right) = 2\psi_1 G(0) + d\left[1-G(0)\right] > \psi_1 G\left(0\right) + d\left[1-G\left(0\right)\right] = \Pi_N\left(d;\frac{2\psi_1}{\psi_2}\right)
$$

and recall that Π_P increases with a^* while Π_{NP} decreases with a^*.

Finally, we obtain $a^*(\psi_1) = \psi_1/\psi_2$ as $\Pi_P(\psi_1; \psi_1/\psi_2) = \Pi_N(\psi_1; \psi_1/\psi_2) = \psi_1$. And $a^*(0) > \psi_1/\psi_2$ as $a^*(\psi_1) = \psi_1/\psi_2$ and $a^*(d)$ decreases in d.

Step 2: *Equilibrium promotion rule when $\psi_1 < d \leq \psi_2$.* As before, $\Pi_N(d; a^*) = \psi_1$ for all $a^* \leq \psi_1/\psi_2$. Observe that $\Pi_P(d; a^*)$ is continuous and increasing in a^*, $\Pi_P(d; 0) = 0$, and

$$\begin{aligned}
(13) \qquad \Pi_P\left(d; \frac{2\psi_1}{\psi_2}\right) &= \psi_1 G(0) + \int_0^{\frac{d-\psi_1}{d+\psi_1}} \psi_1 \frac{1+m}{1-m} dG(m) + d\left[1 - G\left(\frac{d-\psi_1}{d+\psi_1}\right)\right] \\
&> \psi_1 G(0) + \psi_1 \int_0^{\frac{d-\psi_1}{d+\psi_1}} dG(m) + \psi_1\left[1 - G\left(\frac{d-\psi_1}{d+\psi_1}\right)\right] = \psi_1,
\end{aligned}$$

where the inequality follows from the fact that $(1+m)/(1-m) > 1$ and $d > \psi_1$. Thus, given d, the equation $\Pi_N(d; a^*) = \Pi_P(d; a^*)$ has a unique solution in the interval $(0, \psi_1/\psi_2)$. We next show it has no solution in $[\psi_1/\psi_2, 1]$ by showing that $\partial\Pi_P/\partial a^* > \partial\Pi_N/\partial a^*$ for all $a^* > \psi_1/\psi_2$ when $d \in (\psi_1, \psi_2]$. Regardless of the value of d,

$$\frac{\partial}{\partial a^*}\Pi_P(d; a^*) \geq \psi_2 G(0) = \alpha\psi_2.$$

Also, for $d > \psi_1$,

$$\begin{aligned}
(14) \qquad \frac{\partial}{\partial a^*}\Pi_N(d; a^*) &= \int_{\frac{2\psi_1}{\psi_2 a^*}-1}^{\frac{2d}{\psi_2 a^*}-1} \frac{1}{2}\psi_2(1+m)dG(m) \leq \frac{1}{2}\alpha\psi_2 \int_{\frac{2\psi_1}{\psi_2 a^*}-1}^{\max\{\frac{2d}{\psi_2 a^*}-1, 1\}} (1+m)dm \\
&\leq \frac{1}{2}\alpha\psi_2 \int_{\frac{2\psi_1}{\psi_2 a^*}-1}^{1} (1+m)dm \leq \alpha\psi_2
\end{aligned}$$

for all $a^* > \psi_1/\psi_2$. Thus, when $d \in (\psi_1, \psi_2]$, $a^*(d)$ is unique and $a^*(d) < \psi_1/\psi_2$. Finally, because (i) $\Pi_N(d; a^*) = \psi_1$ for all $a^* \leq \psi_1/\psi_2$, (ii) $\Pi_P(d; a^*)$ is increasing in a^* and (iii)

$$\frac{\partial}{\partial d}\Pi_P(d; a^*) = 1 - G\left(\frac{d - \psi_2 a^*}{d + \psi_2 a^*}\right) > 0$$

when $d > \psi_1$and $\psi_2 a^* \leq d$, we obtain that $a^*(d)$ is decreasing in $a^*(d)$.

Step 3: *Equilibrium promotion rule when $d > \psi_2$.* The analysis of this case follows closely the analysis in the previous step. Once again, $\Pi_N(d; a^*) = \psi_1$ for all $a^* \leq \psi_1/\psi_2$. Moreover, $\Pi_P(d; a^*)$ is continuous and increasing in a^* and $\Pi_P(d; 0) = 0$. Now, observe that Π_P is non-decreasing in d, which jointly with (13) in Step 2 implies that $\Pi_P(d; \psi_1/\psi_2) > \psi_1$. Hence, given d, $\Pi_N(d; a^*) = \Pi_P(d; a^*)$ has a unique solution in the interval $(0, \psi_1/\psi_2)$. We next show it has no solution in $[\psi_1/\psi_2, 1]$. It suffices to show that $\partial\Pi_P/\partial a^* > \partial\Pi_N/\partial a^*$ for all $a^* > \psi_1/\psi_2$ when $d > \psi_2$. Clearly, (14) holds when $d > \psi_2$ and, therefore, $\partial\Pi_N/\partial a^* \leq \alpha\psi_2$ for all $a^* > \psi_1/\psi_2$. Next, observe that when $d > \psi_2$,

$$\begin{aligned}
\frac{\partial}{\partial a^*}\Pi_P(d; a^*) &= \psi_2 G(0) + \int_0^{\frac{1-a^*}{1+a^*}} \psi_2 \frac{1+m}{1-m} dG(m) + \int_{\frac{1-a^*}{1+a^*}}^{\frac{2d}{\psi_2(1+a^*)}-1} \frac{1}{2}\psi_2(1+m)dG(m) \\
&> \psi_2 G(0) = \alpha\psi_2.
\end{aligned}$$

Thus, for each $d > \psi_2$, the promotion cut-off $a^*(d)$ is unique and satisfies $a^*(d) \leq a^*(\psi_2) < \psi_1/\psi_2$. The remaining question is whether $a^*(d)$ decreases with d. When $d > \psi_2$,

$$\frac{\partial}{\partial d}\Pi_P(d; a^*) = 1 - G\left(\frac{2d}{\psi_2(a^* + 1)} - 1\right).$$

Therefore, $\partial\Pi_P(d;a^*)/\partial d > 0$ if and only if

$$\frac{2d}{\psi_2(a^*+1)} - 1 < 1 \text{ or, equivalently, if } d < \psi_2(a^*+1).$$

Observe first that when $d = \psi_2$ this condition is satisfied for all a^*. Therefore, $a^*(d)$ decreases with d, at $d = \psi_2$. When $d = \psi_2 + \psi_1$, this condition requires that $a^* > \psi_1/\psi_2$. Since $(\psi_2 + \psi_1)a^* < \psi_1/\psi_2$, we obtain that $a^*(d)$ does not change with d when $d = \psi_2 + \psi_1$. Hence, the cut-off $\hat{d}$ is defined as $\hat{d} = \psi_2(a^*(\hat{d}) + 1)$ and $\hat{d} \in (\psi_2, \psi_2 + \psi_1)$. ■

Proof of Proposition 2. The proof is given in the following steps.

Step 1: *Characterization of S.* Given a promotion cut-off a^*, the expected total surplus in period two can be written as

$$\begin{aligned} S(a^*) = \; & \int_0^{a^*}\left[\int_{-\infty}^{\frac{2\psi_1}{\psi_2 a^*}-1}\psi_1 dG(m) + \int_{\frac{2\psi_1}{\psi_2 a^*}-1}^{\infty}\psi_2 a(1+m)dG(m)\right]da + \\ & \int_{a^*}^1\left[\int_{-\infty}^{\frac{a-a^*}{a+a^*}}\psi_2 a dG(m) + \int_{\frac{a-a^*}{a+a^*}}^{\infty}\psi_2 a(1+m)dG(m)\right]da. \end{aligned} \tag{15}$$

It is useful for the analysis that follows to characterize $S'(a^*)$ and $S''(a^*)$. We do so for all $a^* < 2\psi_1/\psi_2$. Since, $a^*(d) < 2\psi_1/\psi_2$ for all d (see the proof of Proposition **1**), only this range of values of a^* is relevant. Let $h_1(a,a^*)$ denote the function inside the first square brackets in the expression of $S(a^*)$ and $h_2(a,a^*)$ the function inside the second square brackets. Then,

$$S'(a^*) = h_1(a^*,a^*) + \int_0^{a^*}\frac{\partial h_1(a,a^*)}{\partial a^*}da - h_2(a^*,a^*) + \int_{a^*}^1\frac{\partial h_2(a,a^*)}{\partial a^*}da.$$

The second term of this expression is always zero. Simplifying the other terms we obtain that

$$\begin{aligned} & S'(a^*) = \\ & (\psi_1 - a^*\psi_2)G(0) + \int_0^{\frac{2\psi_1}{\psi_2 a^*}-1}[\psi_1 - \psi_2 a^*(1+m)]dG(m) + \int_{a^*}^1 2\psi_2 g\left(\tfrac{a-a^*}{a+a^*}\right)\tfrac{a^2(a-a^*)}{(a+a^*)^3}da. \end{aligned} \tag{16}$$

Differentiating this expression, using the fact g is piecewise uniform with support in $[-1,1]$ and simplifying, we obtain that for $a^* \le \psi_1/\psi_2$ (which implies that $2\psi_1/(\psi_2 a^*) - 1 \ge 1$),

$$S''(a^*) = \frac{1}{2}\psi_2(\alpha - 3) + \int_{a^*}^1 2a^2\psi_2(1-\alpha)\frac{\partial}{\partial a^*}\left(\frac{a-a^*}{(a+a^*)^3}\right)da, \tag{17}$$

and for $a^* > \psi_1/\psi_2$ (which implies $2\psi_1/(\psi_2 a^*) - 1 < 1$),

$$S''(a^*) = \frac{1}{2}\psi_2(1 - 3\alpha) + \int_{a^*}^1 2a^2\psi_2(1-\alpha)\frac{\partial}{\partial a^*}\left(\frac{a-a^*}{(a+a^*)^3}\right)da. \tag{18}$$

Both are strictly negative since by assumption $\alpha \in [1/2, 1)$ and in both the second term is clearly negative. Thus, S is concave in the interval $[0, 2\psi_1/\psi_2]$.

Step 2: *Optimality of a breakup fee.* Given that S is concave and $a^*(d)$ is decreasing in d, setting $d > 0$ in the contract is optimal (i.e., a breakup fee is optimal) if and only if $S'(a^*(0)) < 0$. In what follows, we show that $S'(a^*(0)) < 0$ if and only if ψ_1 is sufficiently high.

We begin by showing that $S'(a^*(0))$ decreases with ψ_1. Observe that ψ_1 affects both S' (directly) and $a^*(0)$. Therefore,

$$\frac{d}{d\psi_1}S'(a^*(0)) = \frac{\partial}{\partial\psi_1}S'(a^*(0)) + S''(a^*(0))\frac{\partial}{\partial\psi_1}a^*(0), \tag{19}$$

Let us analyze each term separately. Differentiating S' with respect to ψ_1 and simplifying (use (16) and focus on the case where $a^* \in (\psi_1/\psi_2, 2\psi_1/\psi_2)$, since $a^*(0)$ is always in this interval), we obtain

$$\frac{\partial}{\partial\psi_1}S'(a^*) = 2\alpha - 1. \tag{20}$$

To obtain $\partial(a^*(0))/\partial\psi_1$, we use the condition that defines $a^*(0)$. Specifically, $a^*(0)$ is the value of a^* that satisfies $\Pi_N(0; a^*) = \Pi_P(0; a^*)$. Using (9) and (10) in the proof of Proposition **1**, this condition is given by

$$\psi_1 G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right) = \psi_2 a^* G(0). \tag{21}$$

Using the Implicit Function Theorem, we obtain that

$$\frac{\partial a^*}{\partial\psi_1} = \frac{G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right) + g\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\frac{2\psi_1}{\psi_2 a^*}}{g\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\frac{2\psi_1^2}{\psi_2 a^{*2}} + \psi_2 G(0)} = \frac{a^*}{\psi_1},$$

where the second equality follows from using (21) to replace $G\left(2\psi_1/(a^*\psi_2) - 1\right)$ with $a^*\psi_2 G(0)/\psi_1$. Thus, $\partial a^*(0)/\partial\psi_1 = a^*(0)/\psi_1$. We can now sign $\partial S'(a^*(0))/\partial\psi_1$. From (19) and the analysis above, we obtain that:

$$\frac{\partial}{\partial\psi_1}S'(a^*(0)) =$$
$$2\alpha - 1 + \left\{\frac{1}{2}\psi_2(1-3\alpha) + \int_{a^*(0)}^{1} 2a^2\psi_2(1-\alpha)\frac{\partial}{\partial a^*(0)}\left(\frac{a-a^*(0)}{(a+a^*(0))^3}\right)da\right\}\frac{a^*(0)}{\psi_1}.$$

Now, observe that the second term inside curly brackets is negative. Moreover, since $a^*(0) > \psi_1/\psi_2$,

$$2\alpha - 1 + \frac{1}{2}\psi_2(1-3\alpha)\frac{a^*(0)}{\psi_1} < 0.$$

Hence, $\partial S'(a^*(0))/\partial\psi_1 < 0$ and, therefore, $S'(a^*(0))$ decreases with ψ_1.

Next, we show that $S'(a^*(0)) > 0$ for sufficiently low values of ψ_1 and $S'(a^*(0)) < 0$ for sufficiently high values of ψ_1. From (21), it follows that $\lim_{\psi_1\to 0} a^*(0) = 0$. From this and (16), it follows that

$$\lim_{\psi_1\to 0} S'(a^*(0)) = \int_0^1 2\psi_2 g(1)da = 2\psi_2(1-\alpha) > 0.$$

Let $\hat{\psi}_1$ denote the highest value of ψ_1 that satisfies Assumption 1. Observe that $a^*(0) \to 1$ as $\psi_1 \to \hat{\psi}_1$. From this, the fact $\hat{\psi}_1 < \psi_2$ and (16), it follows that

$$\lim_{\psi_1\to\hat{\psi}_1} S'(a^*(0)) = (\hat{\psi}_1 - \psi_2)G(0) + \int_0^{\frac{2\overline{\psi}_1}{\psi_2}-1}[\hat{\psi}_1 - \psi_2(1+m)]dG(m) < 0.$$

Since $S'(a^*(0))$ decreases with ψ_1, $\lim_{\psi_1\to 0} S'(a^*(0)) > 0$ and $\lim_{\psi_1\to\hat{\psi}_1} S'(a^*(0)) < 0$, there exists $\underline{\psi}_1$ such that $S'(a^*(0)) < 0$ (and a breakup fee is optimal) if and only if $\psi_1 > \underline{\psi}_1$.

Step 3: *The value of the optimal breakup fee increases with* ψ_1 for $\psi_1 > \underline{\psi}_1$. Suppose $\psi_1 > \underline{\psi}_1$. Let d^* denote the optimal breakup fee. Also, let $\hat{a} := \max_x S(x)$. Since S is differentiable, concave, $S'(0) > 0$, and $S'(a^*(0)) < 0$ (when $\psi_1 > \overline{\psi_1}$), we know that $S'(\hat{a}) = 0$. Moreover, observe that when $\psi_1 > \underline{\psi_1}$, then $\hat{a} < a^*(0) < 2\psi_1/\psi_2$. We consider separately two cases regarding the value of $\hat{a}$.

Step 3.1: *Suppose* $\hat{a} > \psi_1/\psi_2$. The optimal breakup fee d^* satisfies $a^*(d^*) = \hat{a}$ and by Proposition 1, $d^* < \psi_1$. Hence, d^* satisfies $S'(a^*(d^*)) = 0$. This condition is used to characterize how d^* changes with ψ_1. In particular, since S is concave and $a^*(d)$ is decreasing in d, then d^* increases with ψ_1 if $S'(a^*(d^*))$ decreases with ψ_1. We next show that indeed $S'(a^*(d^*))$ decreases with ψ_1. Since ψ_1 affects S' directly and $a^*(d^*)$, then

$$\frac{\partial}{\partial\psi_1}S'(a^*(d^*)) = \frac{\partial}{\partial\psi_1}S'(a^*(d^*)) + S''(a^*(d^*))\frac{\partial}{\partial\psi_1}a^*(d^*). \tag{22}$$

The terms $\frac{\partial S'}{\partial\psi_1}$ and S'' are given by (20) and (18), respectively. We next characterize $\partial a^*(d^*)/\partial\psi_1$. The cutoff $a^*(d)$ is defined as the value of a^* that satisfies $\Pi_N(0;a^*) = \Pi_P(0;a^*)$. Using (9) and (10) in the proof of Proposition **1** it is easy to obtain that when $d < \psi_1$ this condition is given by

$$\psi_1 G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right) + d\left[1 - G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\right] = a^*\psi_2 G(0) + d\left[1 - G(0)\right]. \tag{23}$$

By the Implicit Function Theorem, we obtain that:

$$\frac{\partial a^*(d)}{\partial\psi_1} = \frac{a^*}{\psi_1} \times \frac{G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right) + g\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\frac{2\psi_1}{\psi_2 a^*} - g\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\frac{2d}{\psi_2 a^*}}{g\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\frac{2\psi_1}{\psi_2 a^*} - g\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\frac{2d}{\psi_2 a^*} + a^*\frac{\psi_2}{\psi_1}G(0)} > \frac{a^*}{\psi_1},$$

where the inequality follows from the fact that the term inside the curly brackets is greater than one. To see this, note that the only difference between the numerator and the denominator of that expression is the first term of the former and the last term of the latter, and that by (23)

$$G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right) = a^*\frac{\psi_2}{\psi_1}G(0) + \frac{d}{\psi_1}\left[G\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right) - G(0)\right] > a^*\frac{\psi_2}{\psi_1}G(0)$$

where the last inequality follows from the fact that $2\psi_1/(\psi_2 a^*) - 1 > 0$ since we know that $a^*(d) < 2\psi_1/\psi_2$ (see proof of Proposition 1). Given the above, we can write

$$\begin{aligned}\frac{\partial}{\partial\psi_1}S'(a^*(d^*)) &= 2\alpha - 1 + \left\{\tfrac{1}{2}\psi_2\left(1 - 3\alpha\right) + \int_{a^*}^{1} 2a^2\psi_2(1-\alpha)\tfrac{\partial}{\partial a^*}\left(\tfrac{a - a^*}{(a+a^*)^3}\right)da\right\}\tfrac{\partial a^*(d^*)}{\partial\psi_1} \\ &\leq 2\alpha - 1 + \left\{\tfrac{1}{2}\psi_2\left(1 - 3\alpha\right) + \int_{a^*}^{1} 2a^2\psi_2(1-\alpha)\tfrac{\partial}{\partial a^*}\left(\tfrac{a - a^*}{(a+a^*)^3}\right)da\right\}\tfrac{a^*}{\psi_1} \\ &< 2\alpha - 1 + \left\{\tfrac{1}{2}\psi_2\left(1 - 3\alpha\right)\right\}\tfrac{a^*}{\psi_1} \\ &< 2\alpha - 1 + \left\{\tfrac{1}{2}\psi_2\left(1 - 3\alpha\right)\right\}\tfrac{1}{\psi_2} < 0,\end{aligned} \tag{24}$$

where the first inequality follows from the fact that the term inside curly brackets is negative (since $S'' < 0$) and $\partial a^*(d^*)/\partial\psi_1 > a^*/\psi_1$while positive , the second from the fact that the second term inside curly brackets is negative, and the third from the fact that $a^*(d^*) = \hat{a} > \psi_1/\psi_2$.

Step 3.2: *Suppose* $\hat{a} < \psi_1/\psi_2$. The proof is similar to that of the case when $\hat{a} > \psi_1/\psi_2$ analyzed in the previous step. Again, we show that $\partial S'(a^*(d^*))/\partial\psi_1 < 0$. The difference relative to that case is that now $d^* > \psi_1$, which implies that the terms in (22) are (quantitatively) different. Specifically, since we are analyzing cases where $a^* < \psi_1/\psi_2$ (which implies that $2\psi_1/(a^*\psi_2) - 1 > 1$), we have

$$\frac{\partial}{\partial\psi_1} S'(a^*(d^*)) = G(0) + \int_0^1 dG(m) = 1$$

and S'' is given by (17). Regarding $\partial a^*(d^*)/\partial\psi_1$, following the same procedure as in the previous step, we obtain that it is greater than $a^*(d^*)/\psi_1$ while positive. Given this, for all $d^* > \psi_1$,

$$\begin{aligned} \tfrac{\partial}{\partial\psi_1} S'(a^*(d)) &= 1 + \left\{ \tfrac{1}{2}\psi_2\,(\alpha - 3) + \int_{a^*}^1 2a^2\psi_2(1-\alpha) \tfrac{\partial}{\partial a^*} \left(\tfrac{a-a^*}{(a+a^*)^3} \right) da \right\} \tfrac{\partial a^*(d)}{\partial\psi_1} \\ &\le 1 + \left\{ \tfrac{1}{2}\psi_2\,(\alpha - 3) + \int_{a^*}^1 2a^2\psi_2(1-\alpha) \tfrac{\partial}{\partial a^*} \left(\tfrac{a-a^*}{(a+a^*)^3} \right) da \right\} \tfrac{a^*}{\psi_1} \\ &= 1 + \left\{ \tfrac{1}{2}\psi_2\,(\alpha - 3) - \psi_2(1-\alpha) \int_{a^*}^1 4a^2 \tfrac{2a-a^*}{(a+a^*)^4} da \right\} \tfrac{a^*}{\psi_1}. \end{aligned}$$

We next show this is negative. We begin by simplifying the second term inside the curly brackets. Define, $H(a) := (1-\alpha)2a^2\psi_2(a-a^*)/(a+a^*)^3$ and let $h(a) = H'(a)$. Observe that

$$-\psi_2(1-\alpha)\int_{a^*}^1 4a^2 \frac{2a-a^*}{(a+a^*)^4} da = -\frac{1}{a^*}\int_{a^*}^1 a h(a) da.$$

Using the rule of integration by parts (which implies that $\int_{a^*}^1 H(a)da = [aH(a)]_{a^*}^1 - \int_{a^*}^1 ah(a)da$) and the fact that $[aH(a)]_{a^*}^1 = 2(1-\alpha)\psi_2(1-a^*)/(1+a^*)^3$, we obtain

$$-\psi_2(1-\alpha)\int_{a^*}^1 4a^2 \frac{2a-a^*}{(a+a^*)^4} da = \frac{1}{a^*}\left\{ \int_{a^*}^1 H(a)da - \frac{2(1-\alpha)\psi_2(1-a^*)}{(1+a^*)^3} \right\}.$$

Now, observe that $\int_{a^*}^1 H(a)da$ is identical to last term of (16). Thus, from the fact that $S'(a^*(d^*)) = 0$, it follows that when $a^* = a^*(d^*)$,

$$\begin{aligned} &\int_{a^*}^1 2\psi_2(1-\alpha)a^2 \tfrac{\partial}{\partial a^*} \left(\tfrac{a-a^*}{(a+a^*)^3} \right) da = \\ &-\tfrac{1}{a^*}\left\{ \tfrac{1}{2}\left(2\psi_1 - 3\psi_2 a^* + \alpha\psi_2 a^*\right) + 2(1-\alpha)\psi_2 \tfrac{1-a^*}{(1+a^*)^3} \right\}. \end{aligned}$$

Using this in (24) and simplifying, we obtain that the expression in (24) is strictly negative if and only if

$$-2(1-\alpha)\psi_2 \frac{1-a^*}{(1+a^*)^3} < 0,$$

which is indeed the case. ■

Proof of Proposition 3. The proof is given in two steps. In the first we prove part (i) of the Proposition and in the second part (ii).

Step 1: *The promotion cutoff* $a^*(d)$ *decreases with* α *when* $d < \psi_1$ *and increases with* α *otherwise.* The promotion cutoff $a^*(d)$ is the value a^* such that $\Pi_N(d; a^*) = \Pi_P(d; a^*)$. When $d < \psi_1$, this condition is given by (use (9) and (10) in the proof of Proposition 1)

$$(\psi_1 - d)\left(\alpha + (1-\alpha)\left(\frac{2\psi_1}{\psi_2 a^*} - 1\right)\right) = (\psi_2 a^* - d)\alpha. \tag{25}$$

Using the Implicit Function Theorem and using (25) again to simplify the expression obtained, we get

$$\frac{\partial a^*(d)}{\partial \alpha} = \frac{a^*}{1-\alpha} \times \frac{\psi_1 - \psi_2 a^*}{\psi_1 - 2\alpha\left(\psi_1 - \psi_2 a^*\right) - d(1-\alpha)}, \tag{26}$$

which is negative, since the numerator in the second fraction is negative and the denominator is positive because $d < \psi_1$ and because by Proposition 1, $a^*(d) > \psi_1/\psi_2$ when $d < \psi_1$.

Consider now the case where $\psi_1 < d < \psi_2$. In this case, $\Pi_N(d; a^*) = \Pi_P(d; a^*)$ is given by

$$\psi_1 = \alpha\psi_2 a^* + (1-\alpha)\int_0^{\frac{d-\psi_2 a^*}{d+\psi_2 a^*}} \psi_2 a^* \frac{1+m}{1-m} dm + (1-\alpha)d\left(1 - \frac{d-\psi_2 a^*}{d+\psi_2 a^*}\right).$$

Using the same procedure as above, we obtain that

$$\frac{\partial}{\partial \alpha} a^*(d) = \left(\psi_1 - \psi_2 a^*\right) / \left[\alpha\left(1-\alpha\right)\psi_2 a^* + \int_0^{\frac{d-\psi_2 a^*}{d+\psi_2 a^*}} (1-\alpha)^2 \psi_2 a^* \frac{1+m}{1-m} dm\right],$$

Finally, when $d > \psi_2$, $\Pi_N(d; a^*) = \Pi_P(d; a^*)$ is given by

$$\begin{aligned} \psi_1 = \; & \alpha\psi_2 a^* + (1-\alpha)\int_0^{\frac{1-a^*}{1+a^*}} \psi_2 a^* \tfrac{1+m}{1-m} dm \\ & + (1-\alpha)\int_{\frac{1-a^*}{1+a^*}}^{\frac{2d}{\psi_2(1+a^*)}-1} \tfrac{a^*+1}{2} \psi_2 (1+m) dm + 2(1-\alpha)d\left(1 - \tfrac{d}{\psi_2(1+a^*)}\right). \end{aligned}$$

Following again the same procedure as above,

$$\begin{aligned} & \tfrac{\partial}{\partial \alpha} a^*(d) = \\ & \left(\psi_1 - \psi_2 a^*\right) / \left[\alpha\left(1-\alpha\right)\psi_2 + \int_0^{\frac{1-a^*}{1+a^*}} (1-\alpha)^2 \psi_2 \tfrac{1+m}{1-m} dm + \int_{\frac{1-a^*}{1+a^*}}^{\frac{2d}{(1+a^*)\psi_2}-1} \tfrac{1}{2} (1-\alpha)^2 \psi_2 (1+m) dm\right], \end{aligned}$$

Note that in both cases above, the derivative is positive, since the denominators are positive and by Proposition 1, $a^*(d) < \psi_1/\psi_2$ when $d > \psi_1$.

Step 2: *The threshold $\underline{\psi}_1$ is decreasing in α.* The threshold $\underline{\psi}_1$ is the value of ψ_1 for which $S'(a^*(0)) = 0$. Since $S'(a^*(0))$ decreases with ψ_1 (see Step 2 of the proof of Proposition 2), to prove that $\underline{\psi}_1$ is decreasing in α, it suffices to show that $\partial S'(a^*(0))/\partial \alpha < 0$ when $\psi_1 = \underline{\psi}_1$.

Since α affects S' directly and $a^*(d^*)$, then

$$\frac{d}{d\alpha} S'(a^*(0)) = \frac{\partial}{\partial \alpha} S'(a^*(0)) + S''(a^*(0)) \frac{\partial a^*(0)}{\partial \alpha}, \tag{27}$$

Next, we characterize each of the components of this expression. From (16) in the proof of Proposition 2, we obtain

$$\begin{aligned} S'(a^*) \; = \; & (\psi_1 - \psi_2 a^*)\alpha + (1-\alpha)\int_0^{\frac{2\psi_1}{\psi_2 a^*}-1} [\psi_1 - \psi_2 a^*(1+m)] dm \\ & + \int_{a^*}^1 2(1-\alpha)\psi_2 a^2 \frac{a-a^*}{(a+a^*)^3} da. \end{aligned} \tag{28}$$

Differentiating this expression with respect to α and using condition $S'(a^*(0)) = 0$, we obtain that when $a^* = a^*(0)$,

$$\frac{\partial}{\partial \alpha} S'(a^*) = \frac{\psi_1 - \psi_2 a^*}{1-\alpha}. \tag{29}$$

From (26) in Step 1, we obtain that

$$\frac{\partial a^*(0)}{\partial \alpha} = \frac{a^*(0)}{1-\alpha} \times \frac{\psi_1 - \psi_2 a^*(0)}{\psi_1 - 2\alpha\left(\psi_1 - \psi_2 a^*(0)\right)} \tag{30}$$

and is negative.

From (27), (29), (18) in the proof of Proposition 2 and (30), we obtain that:

$$\begin{aligned} &\tfrac{\partial}{\partial \alpha} S'(a^*(0)) = \\ &\tfrac{\psi_1 - \psi_2 a^*}{1-\alpha}\left[1 + \left\{\tfrac{1}{2}\psi_2\left(1-3\alpha\right) + \textstyle\int_{a^*}^{1} 2\psi_2(1-\alpha)a^2 \tfrac{\partial}{\partial a^*}\left(\tfrac{a-a^*}{(a+a^*)^3}\right) da\right\} \tfrac{a^*}{\psi_1 - 2\alpha(\psi_1 - \psi_2 a^*)}\right]. \end{aligned} \tag{31}$$

Observe that $(\psi_1 - a^*\psi_2) < 0$ since $a^*(0) > \psi_1/\psi_2$. Hence, to show that $\partial S'(a^*(0))/\partial\alpha$, we only need to show that term inside the square brackets is positive. Since $\psi_1 - 2\alpha\left(\psi_1 - a^*\psi_2\right) > 0$ (recall that $a^*(0) > \psi_1/\psi_2$), this is equivalent to showing that

$$\psi_1 - 2\alpha\left(\psi_1 - \psi_2 a^*\right) + \left\{\frac{1}{2}\left(1-3\alpha\right)\psi_2 - (1-\alpha)\psi_2 \textstyle\int_{a^*}^{1} 4a^2 \dfrac{2a - a^*}{\left(a+a^*\right)^4} da\right\} a^* > 0 \tag{32}$$

Using a procedure identical to that used in Step 3.2 of the proof of Proposition 2 (integration by parts combined with condition $S'(a^*(0)) = 0$), we obtain that

$$-(1-\alpha)\psi_2 \textstyle\int_{a^*}^{1} 4a^2 \dfrac{2a-a^*}{\left(a+a^*\right)^4} da = \dfrac{1}{2a^*}\left(\left(2-4\alpha\right)\psi_1 - \left(1-3\alpha\right)\psi_2 a^*\right) - \dfrac{2(1-\alpha)\psi_2\left(1-a^*\right)}{a^*(1+a^*)^3}.$$

Using this in (32) and simplifying, we obtain that condition is equivalent to

$$-\left(2\alpha - 1\right)\psi_1 + \alpha\psi_2 a^* - \frac{\left(1-a^*\right)\left(1-\alpha\right)}{\left(a^*+1\right)^3}\psi_2 > 0. \tag{33}$$

Using again condition $S'(a^*) = 0$ to eliminate the term with ψ_1, we obtain that (33) is equivalent to

$$\frac{1}{2}a^* + \textstyle\int_{a^*}^{1} 2a^2 \dfrac{a-a^*}{(a+a^*)^3} da - \dfrac{1-a^*}{\left(a^*+1\right)^3} > 0,$$

which holds for all $a^* \in [0,1]$ and therefore for $a^* = a^*(0)$. Hence the proof. ■

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