Optimal job design in the presence of implicit contracts

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We characterize the optimal job design in a multitasking environment when the firms use implicit contracts (i.e., bonus payments). Two natural forms of job design are compared: (i) individual assignment, where each agent is assigned to a particular job and (ii) team assignment, where a group of agents share responsibility for a job and are jointly accountable for its outcome. Team assignment mitigates the multitasking problem but may weaken the implicit contracts. The optimal job design follows a cutoff rule where only the firms with high reputation concerns opt for team assignment. However, the cutoff rule need not hold if the firm can combine implicit incentives with explicit pay-per-performance contracts.

1. Introduction

Firms frequently give work assignments to a group of employees (or “team”) and hold all members of the group jointly accountable for the outcome of their assignment. In fact, firms often adopt such a strategy even when it is technologically feasible to give the same work assignment to a single worker and hold the worker solely accountable for his own performance (Bartol and Hagmann, 1992; Shaw and Schneier, 1995). Such a practice may seem counterintuitive because team performance can obscure individual contributions and blunt incentives. There is a vast literature on agency theory that studies the optimal incentive provisions in teams, but Corts (2007) is perhaps the first to explore how, in a multitasking environment, team assignment may arise endogenously when individual assignment is still a technologically feasible option. Corts argues that team assignment may optimally balance the trade-off between mitigating the multitasking problem and exposing the workers to a higher performance volatility.

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This article contributes to the recent literature on endogenous job design by analyzing the optimality of team assignment in a multitasking environment when firms use implicit contracts (i.e., informal promises of reward that are sustained through the threat of future retaliation of the workers should firms renege on their promise).\(^1\) Indeed, it is not hard to find multitasking environments where the firms offer implicit incentives. For example, consider the mutual fund industry. Managing a mutual fund may require a fund manager to exert effort to find new investment opportunities as well as to gather information about the risks associated with such investments. Even though both risks and returns significantly affect the overall profitability of the firm, a fund manager may be tempted to undertake high return investments without paying sufficient attention to the underlying risks.\(^2\) Implicit contracts, in the form of bonus payments, are ubiquitous in this industry. Interestingly, job design in this industry has evolved over time. Traditionally, each fund was individually managed by an assigned manager. However, there is a recent trend to move toward “comanaged” and/or “team-managed” funds. In a team-managed fund, a group of employees are jointly responsible for the performance of a set of funds, and often different managers are assigned to manage different classes of asset where the fund is invested (see, e.g., Binsbergen, Brandt, and Koijen, 2008; Prather and Middleton, 2002; Chen et al., 2004; Massa, Reuter, and Zitzewitz, 2008).\(^3\)

As is the case with the mutual fund industry, there are two natural job designs: (i) individual assignment, where each agent is assigned to a particular job. An agent performs all of the tasks associated with the job and he is solely responsible for the outcome of the job he has been assigned to; and (ii) Team assignment, where a group of agents share the responsibility for a job, each agent performs a subset of the tasks associated with the job, and all agents are jointly accountable for the job outcome.\(^4\)

In this article, we argue that although team assignment might mitigate the multitasking problem, it may make implicit contracts harder to sustain. Consider the following example that illustrates this trade-off: suppose that a firm needs to hire workers for two jobs, jobs 1 and 2, and each job involves two tasks: tasks 1 and 2 for job 1 and tasks 3 and 4 for job 2. Let \(e_i\) be the effort in task \(i\), and let the worker’s cost of effort be \(e_2i/2\) when performing that task. The efforts in the two jobs produce a value for the firm that is equal to \(e_1 + e_2 + e_3 + e_4\). That is, the firm cares about all tasks equally. Hence, the effort levels that maximize the joint surplus \(\sum(e_i - e_2i/2)\), that is, the “first-best” effort levels, are one unit of effort in each task (\(e_i = 1\) for all \(i\)). The resulting surplus level is $2. The firm cannot observe the effort levels but can observe an imperfect measure of job performance, say “success” or “failure,” where \(Pr(\text{“success” in job 1}) = 0.2e_1 + 0.4e_2\) (and similarly for job 2).

Suppose first that the firm adopts individual assignment. This means that the firm hires two workers and assigns one worker to each job: worker \(A\) is responsible for job 1 (and hence, for tasks 1 and 2) and worker \(B\) is responsible for job 2 (and hence, for tasks 3 and 4). The firm promises a bonus to each worker if he is “successful” in his assigned job. Given the bonus promise, each worker chooses his effort level to maximize his expected bonus reward net of the cost of effort. Consider the incentive effects of a bonus payment for job 1 (the case for job 2 is identical). Note that, given any bonus payment, the worker will exert twice as much effort in task 2 than in task 1. In other words, for any bonus payment, it must always be the case that in

\(^1\) Such contracts are also referred to as “relational” contracts (Levin, 2003; Rayo, 2007) or “self-enforcing implicit” contracts (Bull, 1987).

\(^2\) This might be especially true when the manager is primarily interested in increasing the fund’s returns in the short run. The most visible signal of the manager’s performance is perhaps the fund’s returns over the last few quarters rather than the details of where the fund has been invested and what type of underlying risks it bears.

\(^3\) In some cases, however, the difference between the two may not be one of job design but merely an issue of signalling the employees’ productivity (Massa et al., 2008) or exploiting the potential economies of scope in \(production\). In this article, we abstract away from this issue of the production economies in order to stay focused on the multitasking problem.

\(^4\) Some authors also denote these types of job design as “individual accountability” and “team accountability” (see, e.g., Corts, 2007).
equilibrium, $e_2 = 2e_1$, even though efficiency requires $e_1 = e_2$. The inefficiency arises from the fact that the marginal impact of $e_2$ on the probability of success is twice as much as that of $e_1$. Such a misalignment of the effort incentives is the source of the multitasking problem. Indeed, under individual assignment, it is not possible to ensure equal effort in the two tasks because the firm must provide a single bonus payment to provide incentives for both tasks. The best the firm can do is to offer a bonus payment that maximizes the total surplus subject to the multitasking problem. Indeed, in this case, the optimal bonus amount is $3$ in each job, or $6$ in total, which leads to effort levels $e_1 = e_3 = 0.6$ and $e_2 = e_4 = 1.2$. And it creates an aggregate surplus of $1.8$ from the two jobs. Note that the aggregate surplus is less than the first-best level of $2$. The loss of surplus stems from the misalignment of effort across tasks reflecting the underlying multitasking problem.

Now suppose that the firm resorts to team assignment and splits each job between two workers: worker $A$ is in charge of task 1 in job 1 and task 4 in job 2 and worker $B$ is in charge of task 2 (in job 1) and task 3 (in job 2). The firm can now resolve the multitasking problem by offering a $5$ bonus to $A$ and a $2.50$ bonus to $B$ if job 1 is successful and an additional $2.50$ bonus to $A$ and $5$ bonus to $B$ if job 2 is successful. Under these incentives, both $A$ and $B$ exert exactly one unit of effort in each of the two tasks they have been assigned to and, consequently, the first-best surplus is attained. The key issue to note is that, under team assignment, the firm can vary the power of incentives for the two workers, even if their pays are based on the same performance measure. Such flexibility in the incentive provision alleviates the multitasking problem. However, observe that the total bonus requirement, or the “bonus pool,” is now $15—much higher than the optimal (aggregate) bonus payment under individual assignment, which is $6$. In fact, the bonus pool required under team assignment to implement the exact same effort allocation that is implemented under individual assignment—that is, $e_1 = e_3 = 0.6$ and $e_2 = e_4 = 1.2$—is equal to $12$, which is also much larger than the bonus pool needed under individual assignment. Consequently, the firm’s gains from reneging on its promise are also higher and it is harder for the firm to credibly commit to such a bonus pool. This is the basic trade-off with team assignment that we will explore in this article.

The benefit of team assignment in solving the multitasking problem has been discussed by several authors (Dewatripont, Jewitt, and Tirole, 2000; Corts, 2007). The novel part of our analysis is to highlight the cost of team assignment in terms of weaker implicit incentives (or, equivalently, the need for a larger bonus pool), and to draw out the implications of the trade-off between mitigating the multitasking problem and weaker implicit incentives on a firm’s job design decision.

It is important to note that the necessity for a larger bonus pool under team assignment can be interpreted in terms of economies of scope in incentive provision. To see this point, consider the example discussed above. Under individual assignment, a single bonus payment (based on job performance) offers incentives for efforts in both tasks associated with the job. The resulting economies of scope in incentive provision reduces the size of the bonus pool the firm needs to commit to. In contrast, under team assignment, the job performance measure must be used twice to provide incentives for the efforts in each of the two tasks separately (hence two separate bonus payments are required). Therefore, the firm must commit to a larger bonus pool vis-à-vis the individual assignment case to provide the same incentive. One can also interpret the larger bonus requirement under teams as a cost of solving the underlying free-riding problem. The firm needs to offer separate bonus payments for each of the two tasks solely because a worker does not internalize the impact of his effort on the other agent’s payoff (we will further elaborate on this issue in Section 3).

We formalize this trade-off between mitigating the multitasking problem and weakening of the implicit incentives in a stylized model that is similar to the example discussed above. We consider an environment where an infinitely lived principal (firm) hires two infinitely lived agents.

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5 The derivation of the optimal bonus payment is discussed in detail in Section 3.
(workers) and characterize the optimal job design as a function of the firm’s discount factor ($\delta$) that parameterizes its reputation concerns.

We show that the optimal job design follows a cutoff rule: team assignment is strictly optimal only if the firm’s discount factor is sufficiently high. The intuition behind this result is simple but subtle. Recall that although team assignment allows the firm to overcome the multitasking problem, *ceteris paribus*, it requires the firm to credibly commit to a larger bonus pool vis-à-vis the case of individual assignment. If the firm’s discount factor ($\delta$) is sufficiently high, the threat of future punishment is significantly large for the firm, which, in turn, allows the firm to credibly commit to a larger bonus pool. Consequently, team assignment becomes optimal for firms with high $\delta$. However, for low $\delta$, the firm lacks credibility to offer large bonus payments, and the firm is better off by resorting to individual assignment. Under individual assignment, even a small bonus payment gives sharper incentives because it elicits effort in all tasks associated with a particular job. In other words, individual assignment allows the firm to exploit the economies of scope in incentive provision. A single bonus payment provides some work incentives in all tasks, and the stronger incentives can outweigh the inefficiencies originating from the multitasking problem.

Given this basic intuition on how job design may interact with the implicit incentive provisions, we consider a more general setting where the workers’ performances in a subset of jobs are indeed verifiable. In such an environment, the firm may opt to provide incentives to its workers through a combination of explicit and implicit incentives. It may offer explicit pay-per-performance contracts to the agents assigned to the jobs where contractible performance measures are available and promise implicit contracts to the others.

The scenario described above can be readily accommodated in our basic model. However, the cutoff result no longer holds in this setting. Instead, we find that the optimal job design must be one of the following: (i) only the firms with very high or very low $\delta$ opt for team assignment, but the firms with intermediate $\delta$ opt for individual assignment, or (ii) team assignment is optimal for all $\delta$. The former is the case when the extent of the multitasking problem is low; the latter is the case when the multitasking problem is severe.

The intuition behind this result is similar to the case of the cutoff result discussed above, except in the case of sufficiently low $\delta$. What drives the optimality of teams for low $\delta$? For $\delta$ small, the firm has low reputation concerns and, hence, the implicit incentives are infeasible under both types of job design. The firm’s profit under team assignment is higher because the explicit incentive can elicit effort more efficiently (for the explicitly contracted job) under a team setting by mitigating the multitasking problem. However, if the multitasking problem is sufficiently severe, then even for a moderate $\delta$, the stronger implicit incentives under individual assignment need not be enough to compensate for the associated multitasking problem. In this case, team assignment remains optimal for all values of $\delta$.

**Related literature.** This article relates to two broad strands of literature on agency theory— incentives in teams and implicit contracts—and highlights how team assignment may emerge endogenously as the optimal job design in the presence of implicit contracts.

Both explicit and implicit contracts in teams are well studied in the literature (Holmström, 1982; Che and Yoo, 2001; Kvaløy and Olsen, 2006; Bar-Isaac, 2007; Rayo, 2007). However, this literature generally assumes that team assignment arises exogenously and focuses solely on the incentive issues that may materialize in teams. Two important exceptions are Itoh (1991) and Bar-Isaac (2007), who argue that teamwork may indeed originate endogenously. But none of these authors considers the role of multitasking in team formation. Our article is perhaps more closely related to Corts (2007), who studies how team assignment may endogenously emerge as the optimal job design in a multitasking environment with explicit pay-per-performance contracts.

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6 Itoh’s argument relies on the need to foster collaborations among employees (also see Ramakrishnan and Thakor, 1991). Bar-Issac shows how a team formed with old and young workers can restore the reputation concerns of the old workers, who otherwise have nothing more to prove to the outside labor market.

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Similar to our article, Corts highlights the benefit of team in overcoming the multitasking problem. However, he considers a different cost of team: teams make agents’ income more noisy, for which the risk-averse agents must be offered a higher risk premium. Our article complements Corts’s article by highlighting a different trade-off associated with team assignment in the presence of implicit incentives. We abstract away from the risk-aversion issue by assuming that the agents are risk neutral, but introduce a new friction in terms of implicit contracts.

Another article that is closely related to ours is Levin (2002). Levin discusses the costs and benefits of multilateral contracting over bilateral contracting in employment relationships. In multilateral contracting, similar to team assignment, the firm makes commitments to a large group of employees, whereas under bilateral contracting, similar to individual assignment, the firm makes commitments to individuals or small groups. There is no multitasking issue in Levin’s model, and he highlights the trade-off that although multilateral contracting is difficult to adjust in response to exogenous shocks to the business environment, it facilitates implicit contracts. The latter effect is in sharp contrast with the effect highlighted in our article, where team assignment hinders implicit incentive provisions. We will revisit this issue in Section 3.

Finally, our article is also related to two “subbranches” of the agency theory literature: (i) multitasking (Holmström and Milgrom, 1991; Dewatripont et al., 2000; Besanko, Regibeau, and Rockett, 2005) and (ii) interaction between explicit and implicit incentives (Gibbons and Murphy, 1992; Baker, Gibbons, and Murphy, 1994). In contrast with our article, the existing literature on multitasking focuses primarily on explicit incentives. And the literature on the interaction between incentives primarily discusses the characteristics of the optimal contract (that emerges from such interplay between incentives) and is silent about its potential implications for job design in a multitasking environment. An important exception is Schöttner (2008). Schöttner investigates when to split three tasks between two agents, where both explicit and implicit contracts may be feasible. In her model, task splitting can elicit the first-best effort level in one of the three tasks, but implicit contracts are sharper under “no task splitting.” However, these benefits and costs of task splitting come as the result of very different reasons (compared to our model), namely, specific restrictions on the agents’ cost function and larger punishment threats when tasks are not split. Also, in contrast to our results, Schöttner finds that task splitting can be optimal only when implicit contracts are infeasible.

This article is organized as follows. Section 2 discusses the basic model and Section 3 characterizes the optimal job design. The role of explicit contracts is discussed in Section 4. Section 5 discusses the empirical implications of our main results and their robustness to alternative modelling assumptions. Finally, Section 6 concludes. Unless mentioned otherwise, all proofs are given in the Appendix.

2. The basic model

We use a model that formalizes the example discussed in the Introduction. The details of the model are as follows.

Players. A long-lived firm, \( F \), attempts to hire two long-lived agents, \( A \) and \( B \), to manage two funds.\(^7\)

Time is discrete and indexed by \( \tau \in \{0, 1, 2, \ldots \} \). In period 0, the firm decides on the job design. As we will see below, the choice of job design consists of choosing how to assign the two agents to the different tasks associated with the management of the two funds that the firm owns. In each subsequent period \( \tau \in \{1, 2, \ldots \} \), the firm and the agents play the following stage game.

\(^7\) The reference to the mutual fund industry is only to maintain a parallel with the example discussed in the Introduction. The model presented here does not purport to be a model of mutual funds, but a general model of multitasking with implicit incentives.
Stage game. At the beginning of the period, the firm offers contracts to the two agents and assigns them to different tasks associated with the management of the two funds. The agents decide whether to accept or reject their contracts. The game unfolds further if at least one agent accepts his contract but ends otherwise. Upon accepting the firm’s contract, an agent exerts effort in the tasks he has been assigned to. Subsequently, the performances of the two funds are realized and the firm pays the agents based on the funds’ performances and the promised contract.

We complete the description of the stage game by elaborating on its three key ingredients: technology and job design, contracts, and players’ payoffs.8

Technology and job design. The technology is modeled after the canonical task allocation model of Dewatripont et al. (2000). Denote the two funds owned by $F$ as 1 and 2, and assume that in each period the performance of fund $i$, $x_i$, can be either good ($x_i = 1$) or bad ($x_i = 0$). We define managing a fund as a “job” (thus there are two jobs in total). Each job consists of two tasks: (i) finding investment opportunities that yield higher returns and (ii) assessing the underlying risks associated with such an investment opportunity. We denote tasks 1 and 2 as the tasks required for job 1, and tasks 3 and 4 as those required for job 2. Task $j \in \{1, 2, 3, 4\}$ requires an effort level of $e_j \in [0, 1]$. Let the cost of effort in task $j$ be $c(e_j) = e_j^2/2$. The tasks affect the value, $V$, that $F$ receives from the two funds, where

$$V(e) = \phi \times (e_1 + e_2 + e_3 + e_4),$$

and $\phi > 0$. The value $V$ is not observable to the agents. One may interpret $V$ as the impact of the agent’s effort in various tasks on the bottom line of the firm’s profitability. This value accrues directly to the firm and may not be clearly observed by the rank and file of a hierarchical organization. Furthermore, efforts are also unobservable, but the funds’ performance measures, $x_1$ and $x_2$, are observable. The efforts in each of the two jobs determine their performance as follows:

$$\Pr(x_1 = 1 \mid e) = e_1 + \gamma e_2,$$

$$\Pr(x_2 = 1 \mid e) = e_3 + \gamma e_4,$$

where $\gamma > 1$. Although $x_1$ and $x_2$ are observable, neither of them is verifiable (we will relax this assumption later in Section 4). The parameter $\gamma$ measures the extent of the multitasking problem. An agent who is compensated on the basis of $x_1$ (or $x_2$) has incentives to substitute away from $e_1$ (or $e_3$) and concentrate more on $e_2$ (or $e_4$), even though all tasks have the same marginal impact on the firm’s value ($V$). We assume the following parametric restriction to ensure that the probabilities in equation (1) are well defined (i.e., probabilities lie between 0 and 1) in any equilibrium of this game:

Assumption 1. $\phi < 1/(1 + \gamma)$.

Assumption 1 requires $\phi$ to be small when $\gamma$ is large and vice versa. An important issue to note about this technology specification is that it rules out all interactions across efforts in different tasks. The costs ($c$) and value ($V$) are additively separable in efforts, ruling out any substitutability or complementarity across tasks. The assumption of additive separability streamlines the model and improves the exposition of the key trade-off between multitasking and implicit incentives.9

Each agent is responsible for exactly two tasks. The type of allocation of tasks between the two agents is referred to as the “job design.” The firm can choose between two designs: individual assignment and team assignment. Under individual assignment, one agent is responsible for all tasks in job 1 (i.e., sole responsibility of managing fund 1), whereas the other one is responsible for all tasks in job 2 (i.e., fund 2). Without loss of generality, we assume that under individual

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8 For the sake of clarity, while describing the stage game, we will suppress the time suffix $\tau$ associated with each variable.

9 This trade-off itself is not driven by this assumption, nor does this assumption drive our key results (see Mukherjee and Vasconcelos, 2011).
assignment, $A$ is responsible for job 1 (i.e., tasks 1 and 2), whereas $B$ is responsible for job 2 (i.e., tasks 3 and 4). In contrast, we assume that under team assignment, agent $A$ is assigned to tasks 1 and 4, whereas agent $B$ is assigned to tasks 2 and 3. This is to say that, in a team, both agents have responsibility for both jobs.

It is important to note that in order to form teams, one can also consider grouping tasks (1 and 3) and (2 and 4). This configuration has a natural interpretation that, under team assignment, each agent specializes in a task (say, ‘γ-task’ or ‘non-γ-task’) and performs this task for all jobs. However, we specify the grouping (tasks 1 and 4) and (tasks 2 and 3) to maintain a parallel with the case of individual assignment, where each agent is responsible for exactly one ‘γ-task’ (i.e., task 2 or 4). Such a specification is without loss of generality, because both configurations yield the same surplus to the firm (the total bonus requirement to implement a given effort profile $\mathbf{e}$ remains the same under the two configurations).

**Contracts.** The form of the contracts depends on the firm’s decision on the job design.

**Assumption 2.** The firm’s decision on the job design is irreversible.

Once a particular design is chosen at the beginning of the game, it is prohibitively costly to change it at a future date. This assumption simplifies the analysis of the firm’s punishment payoff and allows us to draw out the implications of the key trade-off between the multitasking problem and sustenance of the implicit contract more succinctly.\(^{10}\)

The contract offered under each of the two types of job design is as follows. Because the performance in both jobs is nonverifiable, irrespective of the job design, the firm can only offer an implicit contract promising (in each period) a bonus payment if the fund performance turns out to be good. So, under individual assignment (where agent $A$ is assigned to job 1, and $B$ is assigned to job 2), a contract offered to agent $A$ (in each period) is a tuple $(W_A, \beta_A)$, where $W_A$ is a lump-sum wage and $\beta_A$ is an implicitly contracted bonus payment that is offered only if $x_1 = 1$. The contract offered to agent $B$ is of similar form. In contrast, in a team setting, a contract offered to agent $k \in \{A, B\}$ (in each period) is a tuple $(W_k, \beta_{k1}, \beta_{k2})$, where $W_k$ is a lump-sum wage and $\beta_{k1}$ is an implicitly contracted bonus payment that is offered only if $x_i = 1$.\(^{11}\) We define the total bonus offered for a job as the “bonus pool” for that job. So, for example, the bonus pool for job 1 is $\beta_A$ under individual assignment and $\beta_{A1} + \beta_{B1}$ under team assignment.

**Payoffs.** We assume that both the firm and the two agents are risk neutral. The expected payoff of the firm is simply the overall value that the firm receives from the two funds net of its expected wage payments. The expected payoff of an agent is the expected wages he receives net of the cost of effort. The exact expressions for the expected payoffs will depend on the job design. Under individual assignment, the firm’s expected payoff in each period is

$$\pi^I = V(\mathbf{e}) - W_A - \beta_A \Pr(x_1 = 1 | \mathbf{e}) - W_B - \beta_B \Pr(x_2 = 1 | \mathbf{e})$$

and the expected payoffs of agents $A$ and $B$ are defined as

$$u^I_A = W_A + \beta_A \Pr(x_1 = 1 | \mathbf{e}) - c(e_1) - c(e_2),$$

$$u^I_B = W_B + \beta_B \Pr(x_2 = 1 | \mathbf{e}) - c(e_3) - c(e_4),$$

respectively. In contrast, under team assignment, the firm’s expected payoff is

$$\pi^T = V(\mathbf{e}) - \sum_{k \in \{A, B\}} [W_k + \beta_{k1} \Pr(x_1 = 1 | \mathbf{e}) + \beta_{k2} \Pr(x_2 = 1 | \mathbf{e})].$$

\(^{10}\) This assumption does not affect our key trade-off and, therefore, it does not affect the qualitative nature of our results. However, if one relaxes this assumption, on the punishment path, the firm will always reoptimize its job design. Consequently, the firm’s punishment payoff will increase and so will its temptation to cheat.

\(^{11}\) An important implication of such contract specification is that bonuses are additively separable across jobs. This assumption is not crucial for the qualitative nature of our findings, but we maintain this assumption because it considerably improves the analytical tractability of our model. We further discuss this issue in Section 5.
and the expected payoffs of agent $A$ (who is assigned to tasks 1 and 4) and agent $B$ (who is assigned to tasks 2 and 3) are

$$u^I_A = W_A + \beta_A \Pr(x_1 = 1 | e) + \beta_{x_2} \Pr(x_2 = 1 | e) - c(e_1) - c(e_4),$$

$$u^I_B = W_B + \beta_B \Pr(x_1 = 1 | e) + \beta_{x_2} \Pr(x_2 = 1 | e) - c(e_2) - c(e_3),$$

respectively. The outside option of both agents is assumed to be 0.

The discussion above completes our description of the stage game.

**Repeated game.** The repeated game is simply the above-mentioned stage game repeated in each period. Both the firm and the agents discount the future payoff at the rate of $\delta \in [0, 1)$ per period.

**History of the game.** We denote the public history of play in period $\tau$ (or in the $\tau$th stage game) by $h_{\tau}$. For $\tau = 0$, this history consists of $F$’s choice of job design $d \in \{\text{team assignment, individual assignment}\}$. Thus, $h_0 = d$. For $\tau \geq 1$, $h_{\tau}$ is a tuple that reports: (i) the contract offered by $F$ to the two agents ($W_{A\tau}, W_{B\tau}$) at the beginning of period $\tau$; (ii) A’s and B’s decisions on whether to accept the contract, $z_{i\tau} \in \{\text{accept, reject}\}, i \in \{A, B\}$; (iii) the outcomes in jobs 1 and 2, $x_1$, and $x_2$; and (iv) $F$’s actual bonus payments at the end of period $\tau$, $\tilde{\beta}_i$. That is, $h_{\tau} = ((W_{A\tau}, W_{B\tau}), (z_{A\tau}, z_{B\tau}), (x_{1\tau}, x_{2\tau}), \tilde{\beta}_i)$. We denote the public history of the game at the beginning of period $\tau$ by $h^\tau$ and the set of all possible public histories of the game at the beginning of period $\tau$ by $H^\tau$. For $\tau \geq 1$, $h^\tau = (h_1, h_2, \ldots, h_{\tau-1})$. With a slight abuse of notation, we define $h^0 = \emptyset$ and $H^0 = \emptyset$.

**Strategies, Implicit contracts, and equilibrium.** We focus on pure strategy equilibria due to their analytical tractability. A strategy of $F$, $\sigma_F$, consists of the following decisions: (i) the decision made in period zero on the job design $d$; (ii) the decisions made at the beginning of each period $\tau \geq 1$ on the contracts ($W_{A\tau}, W_{B\tau}$), given the history $h^\tau \in H^\tau$; and (iii) the decisions made at the end of each period $\tau \geq 1$ on the actual bonus payments $\tilde{\beta}_i$, given the history $h^\tau \in H^\tau$, contracts ($W_{A\tau}, W_{B\tau}$), and outcomes $(x_{1\tau}, x_{2\tau})$. A strategy of agent $i \in \{A, B\}, \sigma_i$, consists of the following decisions: (i) whether to accept the contract offered by $F$ at the beginning of each period $\tau \geq 1$, given the history $h^\tau \in H^\tau$; and (ii) the effort decisions in each period $\tau$ in which he accepts $F$’s contract offer, given the history $h^\tau \in H^\tau$ and contract ($W_{A\tau}, W_{B\tau}$).

For each period $\tau \geq 1$ and every history $h^\tau \in H^\tau$, an implicit contract between the firm and the two agents describes: (i) the compensation the firm should offer (and that should be paid) to both agents; (ii) whether the agents should accept or reject the firm’s offer; and, in the event of acceptance, (iii) the agents’ effort levels in the tasks they are assigned to. Because a part of the compensation (the bonuses) in this contract is only promised, the firm may have the incentive to renege once production occurs.

An implicit contract is self-enforcing if it constitutes a perfect public equilibrium (PPE) in trigger strategies of the repeated game.\footnote{See Fudenberg, Levine, and Maskin (1994) for a detailed discussion of the PPE solution concept.} A strategy profile ($\sigma_F, \sigma_A, \sigma_B$) constitutes a PPE in trigger strategies if: (i) given any public history, the strategy profile ($\sigma_F, \sigma_A, \sigma_B$) induces a Nash equilibrium in the continuation game, and (ii) both agents revert back to playing their static best responses forever if $F$ reneges on its promise to either of the two agents..\footnote{Note that we have assumed that regardless of the job design, both agents trigger punishment if the firm cheats on at least one of the two agents. This specification, however, is not essential for our main results.}

### 3. Optimal job design

Before we delve into the analysis of optimal job design, it is instructive to briefly discuss the first-best solution to the firm’s contracting problem, as the first-best solution serves as a benchmark for evaluating the efficacy of a given job design. The first-best solution is the one
that maximizes the joint surplus between the firm and the two agents. That is, the first-best effort levels, \( e^{E_B} \), solve \( \max_e V(e) - \sum c(e_j) \), or, equivalently, must satisfy the first-order conditions

\[
e^{E_B}_j = \phi \forall j.
\]  

Equation (2) suggests that the first-best effort in all four tasks should be the same and equal to \( \phi \), the marginal benefit of effort to the firm.

As the optimal job design is derived by comparing the firm’s payoff under individual and team assignment when the associated incentive contracts are optimally chosen, a characterization of the optimal job design first requires a characterization of the optimal contracts. The following lemma (à la Levin, 2003) simplifies the analysis by ensuring that without loss of generality, one can restrict attention to the class of stationary contracts; that is, we can characterize the optimal contract in the repeated game as a tuple \( (W_A, W_B, \beta_A, \beta_B) \) under individual assignment and as a tuple \( (W_A, W_B, \beta_{A1}, \beta_{B1}, \beta_{A2}, \beta_{B2}) \) under a team setting, where the optimal contract does not vary over time. (We omit the proof, as it directly follows from the proof of Theorem 2 in Levin’s article.)

**Lemma 1.** If an optimal contract exists, there exists a stationary contract that is also optimal (Levin, 2003).

Based on this observation we characterize below the optimal contracts under different job designs.

□ **Individual assignment.** The optimal contract must satisfy three constraints, as follows. (i) Individual rationality (IR), that is, the contract must offer both agents rents at least as large as their outside options. (ii) Incentive compatibility (IC), that is, given the incentives, both agents choose their effort levels to maximize their expected payoffs. And finally, (iii) dynamic enforcement (DE), that is, the firm’s promise of bonus payments must be credible. We elaborate on each of these constraints below.

As the outside options of both agents are equal to 0, given the contracts \( (W_A, \beta_A) \) for agent \( A \) and \( (W_B, \beta_B) \) for agent \( B \) and the prescribed effort levels \( e \), the (IR) constraints for agents \( A \) and \( B \) are

\[
W_A + \beta_A \Pr(x_1 = 1 | e) - c(e_1) - c(e_2) \geq 0, \quad (IR^A)
\]

\[
W_B + \beta_B \Pr(x_2 = 1 | e) - c(e_3) - c(e_4) \geq 0. \quad (IR^B)
\]

Next, consider the (IC) constraints. As we have discussed above, under individual assignment, agent \( A \) is responsible for tasks 1 and 2 (i.e., job 1), and agent \( B \) is responsible for tasks 3 and 4 (i.e., job 2). Given the implicitly contracted bonus payments \( \beta_A \) (offered if \( x_1 = 1 \)) and \( \beta_B \) (offered if \( x_2 = 1 \)), the optimization problems for the two agents are

\[
\max_{e_1, e_2} W_A + \beta_A \Pr(x_1 = 1 | e) - c(e_1) - c(e_2),
\]

\[
\max_{e_3, e_4} W_B + \beta_B \Pr(x_2 = 1 | e) - c(e_3) - c(e_4).
\]

Thus, for any credible promise of bonus amounts, the agents’ choice of effort levels must satisfy the following incentive compatibility conditions:

\[
\beta_A = e_1 = e_2 / \gamma. \quad (IC^{A})
\]

\[
\beta_B = e_3 = e_4 / \gamma. \quad (IC^{B})
\]

The (IC) constraints above highlight the multitasking problem. Consider the case of job 1 which is assigned to agent \( A \) (the case of job 2 is analogous). Given that both task 1 and task 2 are compensated based on the performance outcome in job 1, the effort levels exerted in these two tasks are linked by the relationship \( e_1 = e_2 / \gamma \). Because \( \gamma > 1 \), for any value of \( \beta_A \), agent \( A \)
will exert more effort in task 2 than in task 1 (similarly for $\beta_B$). And there cannot exist any values of $\beta_A$ and $\beta_B$ that can ensure the first-best allocation $e_1 = e_2 = \phi = e^{FB}_1 = e^{FB}_2$.

Finally, consider the dynamic enforcement (DE) constraint. If the bonus promises are to be credible, the discounted value of the firm’s payoff stream from offering such bonus payments (i.e., equilibrium payoff) must be greater than the bonus amount that the firm forfeits by reneging on its promise, plus the discounted value of the payoff stream the firm may earn if it reneges on its promise to one or both agents (i.e., punishment payoff). Now, by modelling specifications, if the firm reneges on its promise to at least one of the two agents, both agents revert back to their static best response, and do not exert any effort. Consequently, if the firm decides to reneg on its promise, it is optimal to renge on both agents, and its continuation payoff on the punishment path is zero. Hence, we must have the following constraint on $\beta_A$ and $\beta_B$ to ensure that reneging on bonus payments is not a profitable deviation for the firm:

$$\frac{\delta}{1-\delta}\pi^I \geq \beta_A + \beta_B.$$  \hfill (DE')

The optimal contracting problem for the firm can now be written as follows:

$$\max_{e, W_A, W_B, \beta_A, \beta_B} V(e) - W_A - W_B - \beta_A \Pr(x_1 = 1 | e) - \beta_B \Pr(x_2 = 1 | e)$$

s.t.  $(IR^I_A), (IR^I_B), (IC^I_A), (IC^I_B),$ and $(DE')$.

This problem can be simplified as follows by eliminating $W_A$, $W_B$, and $e$ by using the $(IR)$ and $(IC)$ constraints and using the fact that at the optimum, $\beta_A = \beta_B \equiv \beta$ (say), because $\pi^I(\beta_A, \beta_B)$ is concave and additively separable in $\beta_A$ and $\beta_B$:

$$\mathcal{P}_I : \begin{cases} 
\pi^I_e \equiv \max_{\beta} 2 \left[ \phi(1 + \gamma)\beta - \frac{1}{2}(1 + \gamma^2)\beta^2 \right] \\ 
s.t. \frac{\delta}{1-\delta} \left[ \phi(1 + \gamma)\beta - \frac{1}{2}(1 + \gamma^2)\beta^2 \right] \geq \beta. \end{cases} \hfill (DE'^I)$$

Let $r = (1 - \delta)/\delta$. Lemma 2 below characterizes the optimal profit of the firm associated with the above contracting problem.

**Lemma 2.** The optimal profit under individual assignment, $\pi^I_e(r)$, is a continuous and monotonically decreasing function in $r$.

For $r$ sufficiently large (i.e., $\delta$ sufficiently low), the optimal profit is simply equal to the punishment payoff of the firm, because no implicit incentives are feasible in equilibrium. As $r$ decreases (i.e., for larger values of $\delta$), the firm gains more credibility in promising implicit contracts. The resulting stronger implicit incentive induces greater effort and leads to an increase in the firm’s profit until the maximum profit under individual assignment is achieved. However, the maximum profit under individual assignment is less than the profit associated with the first best because the multitasking problem continues to prevail regardless of how strong an implicit incentive the firm can credibly offer. (See the proof of Lemma 2 in the Appendix for the complete analytical expression for $\pi^I_e(r)$.)

The basic argument for this lemma is as follows. Note that the solution to $\mathcal{P}_I$ is simply the highest $\beta$ feasible under $(DE')$ until the unconstrained maximum is reached. We can rewrite the $(DE'^I)$ constraint as

$$R^I(\beta) := \phi(1 + \gamma)\beta - \frac{1}{2}(1 + \gamma^2)\beta^2 \geq r\beta.$$  \hfill (3)

The function $R^I(\beta)$ reflects the credibility of the firm when it promises a bonus of the amount $\beta$, and this function can be interpreted as the “reputation capital” (or per-period reputational capital) of the firm, given the bonus $\beta$. Thus, for any given $r$, the optimal $\beta$ is simply the largest $\beta$ that the firm can credibly promise until the value of $\beta$ that solves the unconstrained
version of $\mathcal{P}_1$ becomes credible. One obtains $\pi^i(r)$ by plugging the value of the optimal $\beta$ into $\pi^i$.

Having characterized the profit function of the firm under individual assignment, we next analyze the case of team assignment.

**Team assignment.** Recall that without any loss of generality, we assumed that under team assignment, agent $A$ is responsible for tasks 1 and 4, whereas agent $B$ is responsible for tasks 2 and 3. As in the case with individual assignment, we first discuss the (IR), (IC), and (DE) constraints associated with the optimal contracting problem. The (IR) constraints are analogous to the case of individual assignment and are given as follows:

\[
W_A + \beta_{A1} \Pr(x_1 = 1 \mid e) + \beta_{A2} \Pr(x_2 = 1 \mid e) - c(e_1) - c(e_4) \geq 0, \quad (IR^T_A)
\]
\[
W_B + \beta_{B1} \Pr(x_1 = 1 \mid e) + \beta_{B2} \Pr(x_2 = 1 \mid e) - c(e_2) - c(e_3) \geq 0. \quad (IR^T_B)
\]

However, the nature of the (IC) and the (DE) constraints is significantly different compared to the previous case. Consider the (IC) constraints first. Given the bonus payments $\beta_{A1}$ and $\beta_{B1}$ (offered if $x_1 = 1$) and the bonus payments $\beta_{A2}$ and $\beta_{B2}$ (offered if $x_2 = 1$), the optimization problems for the two agents are

\[
\max_{e_1, e_4} W_A + \beta_{A1} \Pr(x_1 = 1 \mid e) + \beta_{A2} \Pr(x_2 = 1 \mid e) - c(e_1) - c(e_4),
\]
\[
\max_{e_2, e_3} W_B + \beta_{B1} \Pr(x_1 = 1 \mid e) + \beta_{B2} \Pr(x_2 = 1 \mid e) - c(e_2) - c(e_3).
\]

Thus, the (IC) constraints that the optimal contract must satisfy are

\[
\beta_{A1} = e_1, \ \beta_{A2} = e_4/\gamma, \quad (IC^T_A)
\]
\[
\beta_{B1} = e_2/\gamma, \ \beta_{B2} = e_3. \quad (IC^T_B)
\]

Note that the (IC) constraints above highlight both the benefit and the cost of team assignment. Because two different agents are performing the two tasks, they can be paid differently for the same performance outcome in job 1 (i.e., $\beta_{A1}$ need not be equal to $\beta_{B1}$). Consequently, the firm can alleviate the multitasking problem by better aligning the incentives of an agent with the tasks he has been assigned to. However, for any job, the firm now needs to make two separate bonus payments to elicit effort in the two tasks that are associated with that job (e.g., for job 1, a total bonus of $\beta_{A1} + \beta_{B1}$ is needed to elicit effort in tasks 1 and 2). Hence, the firm needs to be able to commit to a larger bonus pool to provide work incentives for all tasks.

Next, consider the dynamic enforcement (DE) constraint. Similar to the case of individual assignment, even under team assignment, both agents become aware of the firm’s deviation (and hence, trigger punishment), even if the firm reneges on its promise only with one of the two agents. Thus, if the firm decides to deviate, it is optimal to renege on both agents, and the punishment payoff of the firm would be 0. Consequently, the relevant (DE) constraint is

\[
\frac{\delta}{1 - \delta} \pi^T \geq \beta_{A1} + \beta_{B1} + \beta_{A2} + \beta_{B2}. \quad (DE^T)
\]

The optimal contracting problem can now be formulated as follows:

\[
\max_{e, W_A, W_B, \beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2}} V(e) - W_A - W_B - (\beta_{A1} + \beta_{B1}) \Pr(x_1 = 1 \mid e)
\]
\[
- (\beta_{A2} + \beta_{B2}) \Pr(x_2 = 1 \mid e)
\]
\[
s.t. \quad (IR^T_A), \ (IR^T_B), \ (IC^T_A), \ (IC^T_B), \ and \ (DE^T).
\]

Using the (IR) and (IC) constraints to eliminate $W_A, W_B$, and $e$, and using the fact that at the optimum, we must have $\beta_{A1} = \beta_{B2}$ and $\beta_{B1} = \beta_{A2}$ (because the agents are ex ante symmetric, and $\pi^T$ is concave and additively separable in $\beta$s), we can rewrite the firm’s problem as
Lemma 3. The optimal profit under team assignment, $\pi^T_\tau(r)$, is a continuous and monotonically decreasing function in $r$.

(The complete analytical expression of $\pi^T_\tau(r)$ is given in the proof of Lemma 3 in the Appendix.) A discussion of the basic argument behind Lemma 3 is useful in understanding the nature of optimal contracts under team assignment. We solve this problem in two steps. The first step is to solve an auxiliary problem of maximizing the firm’s profit by choosing $\beta_{d1}$ and $\beta_{b1}$ when the firm can commit to a fixed amount of total bonus $\beta$ ($= \beta_{d1} + \beta_{b1}$). Let the solutions to this problem (i.e., the optimal $\beta_{d1}$ and $\beta_{b1}$ as a function of total bonus $\beta$) be $\beta^T_{d1}$ and $\beta^T_{b1}$. In the second step, we write the firm’s profit function and, hence, the $(DE^T)$ constraint in terms of the total bonus pool $\beta$ by plugging the solution of the auxiliary problem $\beta^T_{d1}$ and $\beta^T_{b1}$. Now, similar to the case of individual assignment, the $(DE^T)$ constraint can be written as $R^T(\beta) \geq r\beta$, where $R^T(\beta)$ is the “reputation capital” of the firm when it promises a total bonus pool of $\beta$. Therefore, the firm’s optimization problem boils down to the problem of finding the largest value of $\beta$ subject to the $(DE^T)$ constraint: $R^T(\beta) \geq r\beta$ (until the unconstrained argmax value of $\beta$ is obtained). Clearly, when $r$ is small (i.e., $\delta$ is large), the firm can offer a higher bonus (i.e., $\beta$ is large) and the optimal $\beta$ decreases with $r$. Given the solution to this problem, one can compute the optimal values for $\beta^T_{d1}$ as a function of $r$. The functional form for $\pi^T_\tau(r)$ is obtained by plugging the optimal $\beta^T_{d1}$’s (as a function of $r$) into $\pi_T$.

In the context of Lemma 3, the following remarks are in order. First, the solutions to the auxiliary problem, $\beta^T_{d1}$ and $\beta^T_{b1}$, have the following characteristics: when the size of the available bonus pool ($\beta$) is small, the firm should optimally give incentives only for the ‘$\gamma$-task’ (i.e., task 2). However, as the amount of total available bonus increases, the firm starts offering bonuses for both tasks and eventually reaches the first-best effort levels. This suggests that for high $r$ (i.e., low $\delta$), the firm only offers incentives for the $\gamma$-tasks ($e_2$ and $e_4$). But as $r$ decreases (i.e., $\delta$ increases), the firm earns credibility in offering a larger bonus pool and offers bonuses for both tasks.

Second, in general, $\beta^T_{d1} \neq \beta^T_{b1}$. That is, under the optimal contract, different members of the team receive different bonus amounts based on the same performance measure, that is, the team performance ($x_{11}$). This finding is similar in spirit to the “team goal”-based incentive

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14 Note that in our model, the free-riding problem in teams does not restrain the firm from achieving the first best because the firm is the residual claimant and works as the “budget breaker” (see Holmström, 1982).
systems observed in several team-based production processes (Hoffman and Rogelberg, 1998). An oft-cited example of such an incentive system is the “Risk and Reward” program in Saturn automobile plants (a subsidiary of General Motors). Under this program, if a team meets its productivity and/or quality goal, all members of the team are offered a bonus, but different team members may get different bonus amounts based on their relative contributions to the team performance.

Finally, when the first best is attainable, the firm sets $\beta_{A1}^T = \phi$ and $\beta_{B1}^T = \phi/\gamma$. That is, agent $B$ in the $\gamma$-task gets weaker incentives compared to agent $A$, because the team performance measure is more sensitive to agent $B$’s effort than to agent $A$’s. This observation parallels the finding of Banker and Datar (1989) in the single-agent case, where they argue that the power of incentive offered to an agent should be inversely related to the sensitivity of the performance measure with respect to the agent’s effort.

Given the characterization of the firm’s profit under team and individual assignment, we can now address the issue of optimal job design.

**Optimal job design.** The optimal job design for a given value of $r$ (and hence, for a given $\delta$) is the one that yields the highest profit to the firm. A comparison between the optimal profits under individual and team assignment leads to the following characterization of the optimal job design.

*Proposition 1.* There exists a value of $r$, say $r^*$, such that team assignment is strictly optimal if $r < r^*$, individual assignment is strictly optimal for $r > r^*$, and both job designs yield the same expected payoff to the firm for $r = r^*$.

The intuition behind this result is simple. Recall that although team assignment allows the firm to overcome the multitasking problem, it also requires the firm to credibly commit to a larger bonus pool in order to elicit effort in all tasks. If the firm’s discount factor ($\delta$) is sufficiently high, the threat of future punishment is significantly large for the firm, which, in turn, allows the firm to credibly promise a high level of bonus payments. Thus, team assignment becomes optimal. However, for low $\delta$, the firm may not have the credibility to offer high bonus payments. In such a setting, the firm might be better off by resorting to individual assignment. Individual assignment offers economies of scope in incentive provisions where a single performance bonus payment based on the job outcome can simultaneously provide incentives for all tasks associated with the job. The sharper incentives under individual assignment may outweigh the inefficiencies that originate from the multitasking problem.

Proposition 1 suggests that the optimal job design follows a cutoff rule: team assignment is optimal for firms with sufficiently high reputation concerns, that is, sufficiently high $\delta$ (or, equivalently, sufficiently low $r$). And individual assignment is optimal otherwise. This is because the optimal profit functions $\pi^I$ and $\pi^T$ intersect at only one point, $r^*$, and the optimal profit from team assignment lies above (below) the optimal profit from individual assignment for all $r < r^*$ ($r > r^*$). Figure 1 depicts a generic example of the optimal profit functions $\pi^I$ and $\pi^T$ (plots are based on their analytically derived functional forms).

*Proposition 2.* The threshold $r^*$ is increasing in $\gamma$.

The proposition above suggests that as the multitasking problem becomes more severe (i.e., as $\gamma$ increases), team assignment is more likely to be the optimal job design. This finding is quite intuitive, because the key benefit of team assignment is that it mitigates the multitasking problem. Thus, the more acute the multitasking problem, the more likely it is that the firm will opt for team assignment.

4. Interaction between explicit and implicit incentives

The previous section is instructive in drawing out the key trade-off associated with team assignment. But it does so under a simple framework where implicit contracts are the only form
of incentives available. However, in many real-life scenarios, firms augment implicitly contracted bonus incentives with explicit pay-per-performance contracts. For example, in the commercial insurance industry, insurers rely on the agents (brokers) to perform two key jobs: (i) search for (commercial) clients who are willing to buy insurance coverage (i.e., search job) and (ii) elicit information about the clients’ risk and coverage requirement to ensure that the insurer’s offered coverage matches the clients’ needs (i.e., match job). A broker is compensated for the search job by an explicitly contracted commission rate, but the compensation for the match job often comes in the form of a bonus payment, or “contingency fee” (see Wilder, 2002). Both search and match jobs may involve multitasking problems. An effective search may require active solicitation of new business from existing clients as well as advertising the insurer’s products to a broader clientele. Similarly, effective matching may require the agent to not only advise the client on the appropriate coverage but also to elicit accurate information about the risks borne by the clients. How would the presence of explicit contracts affect the optimal job design? This section discusses this issue.

In order to accommodate explicit incentives in our model, we can simply “relabel” job 1 as the verifiable job and job 2 as the nonverifiable job. In other words, we assume that \( x_1 \) is a verifiable signal whereas \( x_2 \) continues to be nonverifiable. Let the piece rates associated with job 1 under individual assignment be \( b_A \) and under team assignment be \( b_{A1} \) and \( b_{B1} \). Thus, the firm now chooses the tuple \( (W_A, W_B, b_A, \beta_A) \) under individual assignment and the tuple \( (W_A, W_B, b_{A1}, b_{B1}, \beta_{A2}, \beta_{B2}) \) under team assignment.

Observe that for given values of \( bs \) and \( \beta s \), the presence of explicit contracts does not change the agents’ incentives in any substantive way (compared to the case where all incentives are implicit). It is merely a matter of relabeling \( \beta s \) as \( bs \). Thus, it does not affect the \( (IR) \) and \( (IC) \) constraints. However, the \( (DE) \) constraint changes substantially for two reasons. First, instead of both jobs, only the incentives associated with job 2 are now implicitly contracted upon. Second, the presence of explicit contracts changes the firm’s punishment payoff. This is due to the fact

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15 Commercial insurance coverage can often be a complex product, and it may be difficult for the client to assess his exact needs and the best suitable coverage. “Misselling” of products is indeed a major concern in the insurance industry and, in general, in the financial sector (see Inderst and Ottaviani, 2009).
that the firm can continue to rely on the explicit incentives to elicit some effort from the agents even on the punishment path (this issue is similar to the one discussed in Baker et al., 1994). Therefore, while discussing below the optimal contracts under individual and team assignment, we will primarily focus on the (DE) constraint.

Consider first the case of individual assignment. On the punishment path, agent $B$ reverts back to the static best response and does not exert any effort, that is, $e_3 = e_4 = 0$. Consequently, the optimal explicit contract on the punishment path simply solves the following program (after eliminating $W_A$, $W_B$, and $e$ using (IR) and (IC)):

$$\hat{\pi}^I = \max_{b_A} b_A \phi(1 + \gamma) - \frac{1}{2} b_A^2 (1 + \gamma^2).$$

The optimal bonus thus obtained is $b_A^I = \phi(1 + \gamma)/(1 + \gamma^2)$ and the associated punishment payoff is $\hat{\pi}^I = \frac{1}{2} \phi^2 (1 + \gamma)^2 / (1 + \gamma^2)$.

Now, analogous to the program $P_I$ (i.e., the optimization problem of the firm when both jobs are implicitly contracted, as discussed in the previous section), the firm’s optimization problem can be written as (again, after eliminating $W_A$, $W_B$, and $e$ using (IC) and (IR)):

$$\hat{P}_I : \left\{ \begin{array}{l}
\hat{\pi}^I = \max_{b_A, b_B} \hat{\pi}^I (b_A, b_B) = \phi(1 + \gamma)(b_A + b_B) - \frac{1}{2} (1 + \gamma^2)(b_A^2 + b_B^2) \\
s.t. \quad \frac{\delta}{1 - \delta} [\hat{\pi}^I (b_A, b_B) - \hat{\pi}^I] \geq b_B. \quad (DE')
\end{array} \right.$$ 

It is important to note the following about $\hat{P}_I$. As before, $\hat{\pi}^I (b_A, b_B)$ is additively separable in $b_A$ and $b_B$. Thus, fixing $b_B$, the optimal $b_A$, say $b_A^*$, is independent of $b_B$ and is exactly equal to $b_A^I$. In other words, the firm continues to offer the same explicit contract on both the equilibrium and the punishment paths. Plugging $b_A = b_A^I$ into $\hat{P}_I$, this program becomes identical to $P_I$, except for the fact that the objective function in $\hat{P}_I$, $\hat{\pi}^I (b_A^*, b_B)$, is a linear transformation of the objective function in $P_I$. Thus, one readily obtains the following relationship between the profits associated with the optimal contracts in the two scenarios:

$$\hat{\pi}_*^I = \frac{1}{2} \pi_*^I + \hat{\pi}^I. \quad (4)$$

The same logic holds in the case of team assignment. But, of course, the punishment payoff under team assignment is different from its individual assignment counterpart. On the punishment path under team assignment, both agents exert effort only in response to the explicit incentives. Thus, $e_3 = e_4 = 0$, as job 2 is compensated only through implicit contracts. For efforts associated with job 1, the (IC) constraints for the agents imply that $e_1 = b_{a1}$ and $e_2 = \gamma b_{b1}$. Therefore, analogous to the case of individual assignment, the optimal explicit contract on the punishment path simply solves the following program (after eliminating $W_A$, $W_B$, and $e$ using (IR) and (IC)):

$$\max_{b_{a1}, b_{b1}} \hat{\pi}^T = \phi(b_{a1} + \gamma b_{b1}) - \frac{1}{2} (b_{a1}^2 + \gamma^2 b_{b1}^2).$$

The optimal punishment payoff thus obtained is $\hat{\pi}^T = \phi^2$. Now, analogous to the case of individual assignment (i.e., as given in equation (4)), the equilibrium payoff under team assignment, say, $\hat{\pi}_*^T$, is given as follows:

$$\hat{\pi}_*^T = \frac{1}{2} \pi_*^T + \hat{\pi}^T. \quad (5)$$

Equations (4) and (5) offer a simple characterization of the firm’s equilibrium payoff under team and individual assignment when explicit contracts are combined with implicit incentives. Using these relationships, the following proposition shows that the optimal job design no longer follows a cutoff rule in the presence of explicit incentives.

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FIGURE 2
THE MAXIMAL PROFITS UNDER TEAM AND INDIVIDUAL ASSIGNMENT IN THE PRESENCE OF EXPLICIT INCENTIVES: THE CASE WHERE $\gamma$ IS SMALL (TEAM IS OPTIMAL FOR $r < r_1$ AND $r > r_2$)

Proposition 3. If $\gamma$ is sufficiently large, team assignment is optimal for all values of $r$. Else, there exist two values of $r$, say $r_1$ and $r_2$, such that individual assignment is optimal for all $r \in [r_1, r_2]$, and team assignment is strictly optimal otherwise.

The intuition behind this result is similar to the cutoff result discussed in Proposition 1, particularly when $r$ is not too large. For $r$ sufficiently small (i.e., $\delta$ sufficiently large), the firm’s reputational capital is sufficiently large. Thus, the firm can offer strong implicit incentives even under team assignment. Consequently, team assignment becomes optimal because it overcomes the multitasking problem. In contrast, for moderate values of $r$, individual assignment dominates. This is due to the fact that for a moderate $r$, the firm has some reputational capital that allows it to offer implicit incentives. In such a scenario, the implicit incentives are sharper under individual assignment. This is because the firm needs to promise a bonus payment to only one of the two agents and, hence, can credibly promise a larger bonus amount than it could if it were to promise a bonus payment to each of the two agents (as is the case with team assignment). When the multitasking problem is not too large (i.e., moderate $\gamma$), this incentive effect outweighs the multitasking problem associated with individual assignment. But what drives the optimality of teams for sufficiently large $r$? For $r$ sufficiently large (i.e., $\delta$ sufficiently small), the firm has few reputation concerns and, hence, the implicit incentives are infeasible under both types of job design. The firm’s profit under team assignment is higher because the explicit incentive can elicit more search effort under a team setting by mitigating the multitasking problem.

However, if the multitasking problem is sufficiently large, then even for a moderate $r$, the stronger implicit incentives under individual assignment need not be enough to compensate for the associated multitasking problem. In this case, team assignment remains optimal for all values of $r$. Figure 2 depicts a generic example of the case where individual assignment is optimal for intermediate values of $r$ (plots are based on their analytically derived functional forms).

The following proposition presents comparative statics with respect to $\gamma$.

Proposition 4. Both $r_1$ and $r_2$ are increasing in $\gamma$. Moreover, there exist two threshold values of $\gamma$, say $\gamma$ and $\overline{\gamma}$, such that (i) for $\gamma < \gamma$, $r_2 - r_1$ (the size of the interval for which individual
assignment is optimal) may increase or decrease in \( \gamma \), (ii) for \( \gamma < \gamma_1 < \gamma \), \( r_2 - r_1 \) decreases in \( \gamma \), and (iii) for all \( \gamma > \gamma_1 \), team assignment is always optimal.

The proposition above suggests that if \( \gamma \) is not too small to begin with, team assignment becomes more likely to be the optimal job design as the extent of the multitasking problem (\( \gamma \)) increases. Indeed, when \( \gamma \) is sufficiently large, individual assignment is never optimal. This result is similar in spirit to Proposition 2, but with one caveat: for sufficiently low \( \gamma \), an increase in the extent of the multitasking problem may favor individual assignment. This happens for the following reason. An increase in \( \gamma \) has two effects on the firm’s payoff: (i) Incentive effect: It increases the effort level in the \( \gamma \)-tasks as the marginal benefit of task 2 and task 4 increases with \( \gamma \). This effect favors both individual and team assignment.\(^{16}\) However, it is a priori unclear under which job design this effect is more pronounced. (ii) Multitasking effect: It accentuates the multitasking problem and, therefore, increases the loss of efficiency due to the misallocation of effort across tasks. This effect works in the favor of team assignment. When \( \gamma \) is sufficiently small, the loss of surplus due to the multitasking problem is small. Thus, if the underlying parameters are such that the incentive effect is significantly stronger under individual assignment, it may dwarf the multitasking effect. Therefore, when \( \gamma \) is sufficiently small to begin with, an increase in \( \gamma \) may favor individual assignment.

We conclude this section with a brief discussion on the role of Assumption 2 (irreversibility of job design) in this context. Note that on the punishment path the firm’s payoff is limited to its profit from job 1 (where only explicit contracts are used to provide work incentives). So, if feasible, on the punishment path the firm would always prefer to adopt team assignment in job 1 so as to maximize its payoff by alleviating the multitasking problem. Thus, this assumption has no effect on the optimal profit of the firm under team assignment but does affect the optimal profit of the firm under individual assignment. In other words, if Assumption 2 is relaxed, \( \hat{\pi}_t \) remains unchanged but \( \hat{\pi}_i \) does change. Under individual assignment, the punishment payoff of a firm will increase because the firm will now switch to team assignment (for job 1) once it reneges on its promise. As the punishment payoff increases, the firm’s credibility in promising bonuses decreases, leading to weaker incentives and lower profits.

In other words, the firm’s profit function under individual assignment “shifts to the left” if the assumption of irreversibility of the job design is relaxed. Regarding the characterization of the optimal job design in this case, we obtain that Proposition 3 continues to hold. Of course, the cutoff points \( r_1 \) and \( r_2 \) will be different from the ones derived under Assumption 2.\(^{17}\) Because the payoff from individual assignment (weakly) decreases, team assignment becomes more likely to be optimal. Thus, as discussed in the context of Proposition 3, for high \( \gamma \), team assignment remains optimum for all values of \( r \), and for low \( \gamma \), team assignment is optimum only for sufficiently high or sufficiently low values of \( r \).

5. Extensions and discussion

The results discussed in the previous sections (Propositions 1 and 3) offer a sharp characterization of the optimal job design. But to what extent can one generalize our result in a model with an arbitrary number of jobs, tasks, and agents? Also, does the main result continue to hold if a more general class of contracts becomes feasible? This section discusses the robustness of our main results to each of these issues. It also explores some of the salient empirical implications of our results.

\(^{16}\) It is straightforward to see this effect from the agents’ (IC) constraints. For example, for agent \( A \), under individual assignment, \((IC)'_I \) implies \( e_2 = \gamma \beta_a \) and \((IC)'_T \) implies \( e_4 = \gamma \beta_a \). Thus, in equilibrium, an increase in \( \gamma \) increases both \( e_2 \) and \( e_4 \).

\(^{17}\) See Mukherjee and Vasconcelos (2011) for the proof of Proposition 3 when Assumption 2 is relaxed.
Suppose there are $N$ agents, $N$ jobs, and each job involves $N$ tasks. Let $e_{it}$ denote the effort exerted on task $k$ in job $j$. Let $V(e) = \phi \sum_{i,j} e_{ij}$ and $Pr(x_j = 1 \mid e) = \sum_k \gamma_k e_{jk}$, and $\gamma_k \neq \gamma_{k'}$ for some tasks $k$ and $k'$ in job $j$. We continue to maintain the assumption that the total cost of effort to an agent is $\sum e_{jk}^2 / 2$ (where the sum is taken over all the tasks that have been assigned to the agent). Observe that the key trade-off between multitasking and the sustenance of implicit contracts continues to prevail. Effort in each task has the same marginal impact on the firm’s value, but an agent assigned to job $j$ will continue to exert more effort on the tasks that have higher impact on the performance measure $x_j$ (i.e., that have higher $\gamma$ coefficient in the $Pr(x_j = 1 \mid e)$ function). As before, under team assignment, the firm can assign each of the $K$ tasks to $k$ different agents and fine-tune its incentives to overcome the multitasking problem. But, the firm now has to commit to a larger bonus pool, because it has to make separate payments to each of the $k$ different agents to elicit effort in all of the $K$ tasks.

In fact, the cutoff result discussed in Proposition 1 continues to hold in a general setting. For analytical tractability, suppose that there are $K_1$ jobs that are “non-$\gamma$ jobs” (i.e., $\gamma_k = 1$ for $K = 1, 2, \ldots, K_1$) and $K_2$ “$\gamma$-jobs” (i.e., $\gamma_k = \gamma$ for $K = K_1 + 1, \ldots, K$), where $K_1 + K_2 = N$. Proposition 5 shows that team assignment is still optimal only for firms with sufficiently high reputation concerns (low $r$).

**Proposition 5.** There exists a value of $r$, say $r^*$, such that team assignment is strictly optimal if $r < r^*$, individual assignment is optimal if $r > r^*$, and both job designs yield the same expected payoff to the firm if $r = r^*$.

Two issues are worth mentioning in this regard. First, one may ask the following: if there are arbitrary numbers of jobs, tasks, and agents (i.e., the number of jobs, tasks, and agents need not be equal to each other), under what condition does team assignment necessarily solve the multitasking problem (i.e., elicit the first-best effort in all tasks in all jobs)? The answer to this question follows directly from Corts (2007). Corts shows that team assignment can elicit first-best effort allocation as long as there are enough agents so that in order to elicit effort in all tasks in all jobs, the firm does not have to assign any agent to more tasks than there are jobs.\(^{18}\) The intuition is that, in order to overcome the multitasking problem, the team must generate a sufficiently rich set of performance measures for all agents. If an agent is assigned to more tasks than there are jobs, then it must be the case that his performance in at least two tasks is rewarded based on the outcome of a single job. Consequently, the multitasking problem prevails.

The second issue concerns the comparative statics predictions. In the basic model, if one increases the multitasking problem by increasing $\gamma$, team assignment becomes more likely to be the optimal job design. In the general model, one can also increase the extent of the multitasking problem by increasing the number of tasks in each job (i.e., increase $K$). Does an increase in $K$ also favor team assignment? It may not. The reason is that an increase in the number of tasks cuts both ways: it aggravates the multitasking problem under individual assignment and favors team assignment, but it also requires more agents to be assigned to the same job (albeit to different tasks) to overcome the aggravated multitasking problem. This calls for an increased bonus pool (equivalently, a more severe free-riding problem) in teams that the firm may not be able to commit to.

\[ \square \textbf{General class of contracts.} \] In the model, we have considered a class of contracts that is additively separable across jobs. Although nonseparable compensation schemes may have their own disadvantages (such as incentives for sabotage or collusion among coworkers), additively separable contracts rule out certain interesting compensation schemes such as relative performance evaluations. For example, under individual assignment, agent $A$’s bonus following a

\(^{18}\) Note that this condition is trivially satisfied in our basic model as well as in this general discussion. In the general case, the number of jobs, tasks, and agents are all equal to $N$. The basic model is a special case of the general model where $N = 2$.
success in job 1 may depend on whether agent \( B \) is also successful (Levin, 2002 shows that such contracts can be optimal in some settings).

The qualitative nature of our results does not change even if we allow for such relative performance evaluations. To see this, consider a general contract under individual assignment that is defined as follows: the contract offered to agent \( i \in \{A, B\} \) consists of a fixed wage \( W_i \) and bonus payments \( \beta_i \) given the performance outcome in jobs 1 and 2, \( x = (x_1, x_2) \).\(^{19}\) Now, the agents’ optimization problems are given by

\[
\max_{e_1, e_2} W_A + \sum_x \beta_{iA} \Pr(x | e) - c(e_1) - c(e_2)
\]

and

\[
\max_{e_3, e_4} W_B + \sum_x \beta_{iB} \Pr(x | e) - c(e_3) - c(e_4),
\]

where \( \Pr(x | e) \) is the probability of outcome \( x \in \{(0, 0), (0, 1), (1, 0), (1, 1)\} \) given \( e = (e_1, e_2, e_3, e_4) \). Solving these problems, one finds that the optimal effort choice of agent \( A \) is\(^{20}\)

\[
e_1 = \frac{\beta_{A10} + (1 + \gamma^2)\beta_{B01}(\beta_{A11} - \beta_{A10})}{1 - (1 + \gamma^2)(\beta_{A11} - \beta_{A10})(\beta_{B11} - \beta_{B01})} \quad \text{and} \quad e_2 = \gamma e_1,
\]

and that of agent \( B \) is

\[
e_3 = \frac{\beta_{B01} + (1 + \gamma^2)\beta_{A10}(\beta_{B11} - \beta_{B01})}{1 - (1 + \gamma^2)(\beta_{A11} - \beta_{A10})(\beta_{B11} - \beta_{B01})} \quad \text{and} \quad e_4 = \gamma e_3.
\]

In the solutions above, two issues are important to note. First, the multitasking problem continues to hold under individual assignment. For any arbitrary contract, \( e_1 \) and \( e_2 \) cannot be perfectly aligned (similarly for \( e_3 \) and \( e_4 \)). Second, relative performance evaluation might be optimal when the firm has few reputation concerns. For example, from (6) one finds \( \partial e_1 / \partial \beta_{A10} = 1 - (1 + \gamma^2)\beta_{B01} \) when \( \beta_{A10} = \beta_{A11} \) (i.e., when the initial contract does not consider relative performances). But \( \beta_{B01} \) must be less than the total bonus, say \( \beta \), that the firm can credibly commit to. Hence, when \( \beta \) is sufficiently small and \( \beta_{A10} = \beta_{A11}, \partial e_1 / \partial \beta_{A10} > 0 \). In other words, when the firm does not have the credibility to promise a large bonus, it is more efficient to pay a higher bonus to a successful agent \( A \) when agent \( B \) fails than when agent \( B \) succeeds. Analogous argument holds for agent \( B \).

One can also consider a more general contract under team assignment where the firm promises an additional bonus when both jobs are successful. For example, the firm may promise a bonus, say \( \beta_A \) (\( >0 \)), to agent \( A \) in addition to the bonuses \( \beta_{A1} \) (paid when job 1 is successful) and \( \beta_{A2} \) (paid when job 2 is successful). Similarly, the firm can promise an additional bonus \( \beta_B \) to agent \( B \).

However, such a contract may not be optimal for a firm with low reputation concerns, however. Note that when a firm has low reputation concerns, it is optimal to set both \( \beta_A \) and \( \beta_B \) to zero because such incentives are less efficient in eliciting effort than the more direct bonus incentives \( \beta_{A1}, \beta_{A2}, \beta_{B1}, \text{and} \beta_{B2} \). Because \( \beta_A \) and \( \beta_B \) are offered only if both jobs are successful, the agent will collect them only with probability \( \Pr(x_1 | e) \times \Pr(x_2 | e) \). When the firm cannot promise a strong implicit contract, efforts will be small and the marginal impact of effort on the product \( \Pr(x_1 | e) \times \Pr(x_2 | e) \) would be significantly smaller than the marginal impact of effort on each of these individual probabilities. Hence, the firm gets more effort per dollar of bonus when the bonus is offered as a direct incentive (i.e., \( \beta_{A1}, \beta_{A2}, \beta_{B1}, \text{and} \beta_{B2} \)).

\(^{19}\) Thus, because there are four possible outcomes of the performance measures in jobs 1 and 2, there are four possible bonuses for agent \( A: \beta_{A11} \) (if \( x = (1, 1) \)), \( \beta_{A10} \) (if \( x = (1, 0) \)), \( \beta_{A01} \) (if \( x = (0, 1) \)), and \( \beta_{A00} \) (if \( x = (0, 0) \)). The same holds for agent \( B \).

\(^{20}\) The associated first-order conditions reveal that the optimal effort of an agent, say agent \( A \), is driven only by the differences in the bonus payments \( \beta_{A10} - \beta_{A00} \) and \( \beta_{A11} - \beta_{A01} \) (similarly for agent \( B \)). Thus, in the optimal contract, we will normalize \( \beta_{A01} = \beta_{A00} = \beta_{B10} = \beta_{B00} = 0 \).
How would the availability of such general contracts affect the optimal job design? Although a complete characterization of the optimal job design under the general class of contracts appears intractable in our model, the above discussion suggests that the qualitative nature of our main finding continues to hold even if general contracts are in place: team assignment is optimal for firms with high reputation concerns, and individual assignment is optimal for firms with low reputation concerns.

The argument is as follows: first consider the case when \( \delta \) is low. If relative performance contracts are feasible under individual assignment, the firm can (weakly) enhance its payoff. But for low \( \delta \), the payoff under team assignment may not change even if general contracts become feasible. To see this, note that under team assignment, the firm might offer an additional bonus when both jobs are successful (e.g., worker \( A \) earns \( \beta A_i \) if job \( i \) is successful, but earns \( \beta A_1 + \beta A_2 + \beta \) where \( \beta > 0 \) if both jobs are successful). But as argued above, such a bonus payment can be profitable only when \( \delta \) is sufficiently large. More generally, one can also assume that when both jobs are successful, the firm offers a bonus payment \( \beta_{12} \) that is completely independent of the bonus offered for success in a particular job. That is, \( \beta_{12} \) is independent of \( \beta_{A1} \) and \( \beta_{A2} \), and, more importantly, the firm can set \( \beta_{12} < \beta_{A1} + \beta_{A2} \). But even if such contracts are used, it is still the case that for low \( \delta \), implicit contracts are easier to sustain under individual assignment. Thus, when \( \delta \) is low, general contracts increase the payoff under individual assignment but have little impact under team assignment. Because individual assignment outperforms team assignment for low \( \delta \) when only additively separable contracts are feasible, individual assignment remains optimal even when general contracts are used. What happens when \( \delta \) is large? Note that even when one allows for relative performance contracts under individual assignment, the multitasking problem prevails. But for sufficiently high \( \delta \), team assignment completely resolves the multitasking problem and ensures first-best outcome. Thus, even under general contracts, team assignment is optimal when \( \delta \) is sufficiently large. Consequently, our initial finding that team assignment is optimal for high \( \delta \) and individual assignment is optimal for low \( \delta \) continues to hold even when additively nonseparable contracts are used.

It is important to note, however, that the above argument does not posit that a cutoff result as that given by Proposition 1 continues to hold under the general class of contracts. Although we argue that team assignment remains optimal for high \( \delta \) and individual assignment remains optimal for low \( \delta \), theoretically, we cannot rule out the case where the payoff functions under team and individual assignment intersect each other multiple times over a range of intermediate values of \( \delta \).

\[ \square \]

**Empirical implications.** Propositions 1–4 have important empirical implications. They highlight the fact that when bonus incentives are used in a multitasking environment, a key parameter that affects the firm’s job design decision (and hence, its performance) is the firm’s discount factor, \( \delta \). Furthermore, these results suggest that the relationship between \( \delta \) and the optimal job design crucially depends on the type(s) of incentives that is in place (i.e., explicit and/or implicit). When only implicit incentives are feasible, team assignment is more likely for firms with high \( \delta \). In contrast, when both implicit and explicit contracts are in place, team assignment becomes more likely for firms with either sufficiently low or sufficiently high \( \delta \). The extent of the multitasking problem also plays an important role. However, the comparative statics with respect to \( \gamma \) is relatively straightforward: for a given \( \delta \), team assignment is more likely when the multitasking problem is more severe (i.e., when \( \gamma \) is large).

A potential challenge in testing these predictions is that the appropriate empirical measures of \( \gamma \) and \( \delta \) in a given industry might be difficult to obtain. Also, in many cases, a firm’s job design
decision may be an artifact of the underlying production technology rather than a strategic choice made by the firm. However, the results of our model can be put to the test in the context of certain industries where these measures are perhaps easier to obtain. The mutual funds industry may be one such candidate. One can use a fund’s liquidation probability as a measure of $\delta$ (Getmansky, Lo, and Wei, 2004). But finding an empirical measure of $\gamma$ is more challenging. Information on the types of assets a fund is invested in may be indicative of the extent of the multitasking the fund’s managers are exposed to. For example, a fund that is primarily invested in government bonds and treasury bills faces lower risks compared to a fund that is entirely invested in the stock market. One may argue that the manager of the latter fund faces a higher multitasking problem because she not only is responsible for increasing the returns of the investment but also has to pay close attention to the underlying risks that the fund is bearing. As discussed above, mutual funds often classify themselves as ‘comanaged’ or ‘team-managed’ funds, where several employees are jointly responsible for the performance of a set of funds. In many cases, the different managers are assigned to the task of managing different classes of assets where the fund is invested. Moreover, the compensation of each of the managers is often specified as a percentage of the net asset value of the fund she manages (Coles, Suay, and Woodbury, 2000). Such a compensation practice bears suggestive evidence that the total volume of the variable pay (i.e., in terms of our model, the amount of bonus pool) is likely to be higher under a team-managed fund than under an individually managed fund. In a team-managed fund, the firm needs to pay a percentage of the fund’s asset value to each member of its managerial team, whereas under individually managed funds, such a payment needs to be made to only one manager.

If one can interpret individually managed funds as individual assignment and team-managed funds as team assignment, data on mutual funds may be used to test the predictions discussed above. For example, our result suggests that, if implicit contracts are the only incentive device used by the funds, one may expect to find that funds with low liquidation probabilities (high $\delta$) and a relatively risky portfolio (high $\gamma$) are team managed, whereas funds with high liquidation probabilities (low $\delta$) and a relatively safe portfolio (low $\gamma$) are individually managed.

Empirical studies on the role of the firms’ discount factor on its job design decision are scarce. However, there is some anecdotal evidence that seems to corroborate our findings. For example, Bär, Kempf, and Ruenzi (2005) find that compared to individually managed funds, team-managed funds are usually larger and have less turnover at the managerial level. If one interprets the higher stability of the management as an environment where the players have a higher discount factor $\delta$ (i.e., the firm-manager relationship is more likely to continue into the next period), Bär et al.’s finding is consistent with our prediction that team management is more likely to be the optimal job design when $\delta$ is high.

6. Conclusion

In many industries, firms often adopt team assignment, even when individual assignment remains a technologically viable option. This article highlights how team assignment may emerge endogenously in a multitasking environment, where the firm relies on implicit contracts, that is, bonus payments. In the presence of implicit contracts, team assignment involves an interesting trade-off: it alleviates the multitasking problem but weakens the implicit incentives. The contribution of this article is to formalize this trade-off and to draw out its implications for the firms’ optimal job design policy.

The key result is that the optimal job design follows a cutoff rule. Only the firms with high enough reputation concerns (i.e., discount factor) opt for team assignment. The more acute the multitasking problem is, the more likely it is that the firm would opt for team assignment. However, the cutoff result need not hold when, in addition to implicit contracts, explicit pay-per-performance contracts are also feasible.

However, one must be wary of a potential endogeneity problem stemming from the fact that the type of assets in which the fund is invested is also a choice variable for the firm.
The multitasking problem need not be the only driver of a firm’s job design decision. For example, team assignment may emerge to manipulate career concerns of the agents by obscuring their individual contributions to the project’s overall performance (Massa et al., 2008). Teams may also arise from the need to facilitate cooperation within organizations (Shaw and Schnieer, 1995). But the main contribution of this article is in extending our understanding of how job design interacts with implicit contracts and how firms may profit from adopting team assignment in a multitasking environment when they must rely on implicit incentives.

Appendix

Proofs.

Proof of Lemma 2. The optimal profit under individual assignment is obtained by solving program $P_i$. Let $\bar{\pi}(\beta) = 2(\phi(1+\gamma)\beta - 1/2 (1+\gamma^2) \beta^2)$, which is the function being maximized in $P_i$; and note that constraint $DE^i$ in $P_i$ can be written as $R^i(\beta):= \phi(1+\gamma)\beta - 1/2 (1+\gamma^2)\beta^2 \geq r \beta$. The remainder of the proof is given by the following steps.

Step 1. Suppose that $r \leq \phi(1+\gamma)/2$. In this case, the $DE^i$ constraint is satisfied when $\beta = \phi(1+\gamma)/(1+\gamma^2)$, which is the (unconstrained) argmax $\bar{\pi}(\beta)$. Thus, in this case, $\beta = \phi(1+\gamma)/(1+\gamma^2)$ must be the solution to $P_i$.

Step 2. Suppose that $r \geq \phi(1+\gamma)$. In this case, $R^i(0) \leq r$. Because $R^i(0) = 0$ and $R^i$ is concave, this implies that $R^i(\beta) < r \beta$ for all $\beta > 0$, and the only value of $\beta$ that satisfies the $DE^i$ constraint is $\beta = 0$. Thus, in this case, $\beta = 0$ is the solution to $P_i$.

Step 3. Suppose that $\phi(1+\gamma)/2 < r < \phi(1+\gamma)$. Because $r > \phi(1+\gamma)/2$, the $DE^i$ constraint is violated for all $\beta \geq \phi(1+\gamma)/(1+\gamma^2)$. Thus, from direct inspection of the $DE^i$ constraint and $\bar{\pi}(\beta)$, it immediately follows that the solution to $P_i$ is the highest value of $\beta$ that satisfies the $DE^i$ constraint. Because $r < \phi(1+\gamma)$, $R^i(0) > r$. This, together with the fact that $R^i$ is concave and $R^i(\beta) < r \beta$ for all $\beta \geq \phi(1+\gamma)/(1+\gamma^2)$, implies that the highest value of $\beta$ that satisfies the $DE^i$ constraint is given by the condition $R^i(\beta) = r \beta$. Thus, in this case, $\beta = (2\phi(1+\gamma) - 2r)/(1 + \gamma^2)$ is the solution to $P_i$.

To obtain $\pi^*_i(r)$, we plug the solutions obtained for $\beta$ in Steps 1–3 into $\bar{\pi}(\beta)$, and obtain

$$\pi^*_i(r) = \begin{cases} 
\phi^2(1+\gamma)/(1+\gamma^2) & \text{if } r \leq \frac{1}{2} \phi(1+\gamma) \\
4(\phi(1+\gamma)r - r^2)/(1+\gamma^2) & \text{if } \frac{1}{2} \phi(1+\gamma) < r < \phi(1+\gamma) \\
0 & \text{if } r \geq \phi(1+\gamma)
\end{cases}$$

It is routine to check that $\pi^*_i(r)$ is continuous and decreasing in $r$.

Proof of Lemma 3. The optimal profit under team assignment is obtained by solving problem $P_i$. We solve this problem in two steps. First, for a given value of total bonus payments $\beta = \beta_{a1} + \beta_{b1}$, we characterize the optimal individual bonus payments $\beta_{a1}$ and $\beta_{b1}$. Second, given the optimal $\beta_{a1}$ and $\beta_{b1}$ as a function of $\beta$, we find the optimal $\beta$ that the firm can sustain.

The optimal individual bonus payments $\beta_{a1}$ and $\beta_{b1}$ given total bonus $\beta$ solve the following problem:

$$R^i(\beta) := \max_{\beta_{a1}, \beta_{b1}} \phi(\beta_{a1} + 2\gamma \beta_{b1}) - \frac{1}{2} (\beta_{a1}^2 + \gamma^2 \beta_{b1}^2)$$

s.t. $\beta_{a1} + \beta_{b1} \leq \beta$, $\beta_{a1} \geq 0$, and $\beta_{b1} \geq 0$.

Note that $R^i(\beta)$ denotes the reputational capital that is achievable given total bonus $\beta$. Using the Kuhn-Tucker optimization method, we obtain that the solution to this problem is

$$\beta^*_{a1} = 0, \quad \beta^*_{b1} = \beta \quad \text{if } \beta < \frac{\gamma - 1}{\gamma^2}$$

$$\beta^*_{a1} = \frac{\gamma \beta - \phi(\gamma - 1)}{1 + \gamma^2}, \quad \beta^*_{b1} = \frac{\beta + \phi(\gamma - 1)}{1 + \gamma^2} \quad \text{if } \frac{\gamma - 1}{\gamma^2} \leq \beta < \frac{\gamma + 1}{\gamma}.$$ 

$$\beta^*_{a1} = \phi, \quad \beta^*_{b1} = \frac{\phi}{\gamma} \quad \text{if } \beta = \frac{\gamma + 1}{\gamma}.$$ 

This solution implies that

$$R^i(\beta) = \begin{cases}
\phi \gamma \beta - \frac{1}{2} \gamma^2 \beta^2 & \text{if } \beta < \frac{\gamma - 1}{\gamma^2} \\
(2\phi \gamma (1+\gamma) + \phi^2 (1-\gamma)^2 - \gamma^2 \beta^2) / 2(1+\gamma^2) & \text{if } \frac{\gamma - 1}{\gamma^2} \leq \beta < \frac{\gamma + 1}{\gamma} \\
\phi^2 & \text{if } \beta \geq \frac{\gamma + 1}{\gamma}
\end{cases}$$

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Now the firm’s optimization problem reduces to \( \pi^*_r \equiv \max \ 2R^I(\beta) \) s.t. \( R^I(\beta) \geq r\beta \). Using the same procedure that is used to solve program \( \mathcal{P}_I \) (see the proof of Lemma 2), we obtain that the solution to this problem is

\[
\beta^*_r = \begin{cases} 
\frac{1}{\gamma} \phi(y + 1) & \text{if } r \leq \frac{\phi y}{y + 1} \\
\frac{1}{\gamma^2} K(r) & \text{if } \frac{\phi y}{y + 1} < r \leq \frac{1}{2} \phi(y + 1) \\
\frac{2}{\gamma^2}(\phi y - r) & \text{if } \frac{1}{2} \phi(y + 1) < r < \phi y \\
0 & \text{if } r \geq \phi y 
\end{cases}
\]

where the function \( K(r) = \phi y(1 + \gamma) - r(1 + y^2 + ((\phi y)(1 + \gamma) - r(1 + y^2)))^2 + y^2(1 - y^2)\phi^3(y^3)^{1/2} \). To obtain \( \pi^*_r \), we plug this solution into \( 2R^I(\beta) \) and obtain

\[
\pi^*_r = \begin{cases} 
2\phi \gamma & \text{if } r \leq \frac{\phi y}{y + 1} \\
2\gamma^2 K(r)/\gamma^2 & \text{if } \frac{\phi y}{y + 1} < r \leq \frac{1}{2} \phi(y + 1) \\
4\gamma^2 (\phi y - r)/\gamma^2 & \text{if } \frac{1}{2} \phi(y + 1) < r < \phi y \\
0 & \text{if } r \geq \phi y 
\end{cases}
\]

It is routine to check that \( \pi^*_r \) is continuous and decreasing in \( r \).

**Proof of Proposition 1.** The proof is given by the following steps.

**Step 1.** We first show that for \( r \in [\phi(1 + y)/2, \phi y] \), \( \pi^*_r \) and \( \pi^*_r \) cannot intersect. To prove this fact, we simply note the following. First, \(|\partial \pi^*_r / \partial r| < |\partial \pi^*_r / \partial r| \) for \( r \in [\phi(1 + y)/2, \phi y] \). Second, \( \pi^*_r (\phi(1 + y)/2) = \phi^2 (1 - y^2) < \phi^2 (1 + y^2)/(1 + y^2) = \pi^*_r (\phi(1 + y)/2) \).

**Step 2.** For \( r \in [\phi y/(1 + y), \phi y + 1/2] \), \( \pi^*_r \) and \( \pi^*_r \) must intersect at a unique point. To see this, note that for all \( r \in [\phi y/(1 + y), \phi y + 1/2] \), \( \pi^*_r (r) = \phi^2 (1 + y^2)/(1 + y^2) < \phi^2 (1 + y^2)/(1 + y^2) \). So, by the mean value theorem, there must exist a value of \( r \in [\phi y/(1 + y), \phi y + 1/2] \), say \( r^* \), such that \( \pi^*_r (r^*) = \pi^*_r (r^*) = \phi^2 (1 + y^2)/(1 + y^2) \). Finally, \( r^* \) is unique, as \( \pi^*_r \) is monotone for \( r \in [\phi y/(1 + y), \phi y + 1/2] \).

**Proof of Proposition 2.** As shown in the proof of Proposition 1, \( r^* \) solves \( 2r^* K(r)/\gamma^2 = \phi^2 (1 + y^2)/(1 + y^2) \). This equation has two solutions for \( r \). Using the fact that \( r^* < \phi y/(1 + y) \), the only admissible solution is \( r^* = \gamma y(1 + y^2)/4(1 + y^2) \). Observe that \( |\partial r^*/\partial y| = \phi(1 + (2 + 2y^2)/4(1 + y^2)) > 0 \).

**Proof of Proposition 3.** The proof is similar in spirit to the proof of Proposition 1, and it is given by the following steps.

**Step 1.** We first show that for \( \forall \gamma \in [\phi(1 + y)/2, \phi y] \), \( \pi^*_r \) and \( \pi^*_r \) can intersect at most once. To prove this fact, we proceed as follows. For \( r \in [\phi(1 + y)/2, \phi y] \), denote \( \Delta(r) = \pi^*_r - \pi^*_r \). Thus, if \( \pi^*_r \) and \( \pi^*_r \) intersect for any value of \( r \), say \( r, \Delta(r) = 0 \). Now, \( \Delta(r) = -2(\phi y(y - 1))/(y + y^2) < 0 \), thus, there cannot exist more than one value of \( r \) such that \( \Delta(r) = 0 \).

**Step 2.** Because \( \Delta(r) > 0 \) and \( \phi y > \phi(1 + y)/2 \), we need to consider three cases: (i) \( \Delta(\phi y) > 0 \) (\( \Delta(\phi(1 + y)/2) > 0 \)), (ii) \( \Delta(\phi y) < 0 \) and \( \Delta(\phi(1 + y)/2) > 0 \), and (iii) \( \Delta(\phi(1 + y)/2) < 0 \) (\( \Delta(\phi y) < 0 \)). The rest of the proof characterizes the nature of the intersection of \( \pi^*_r \) and \( \pi^*_r \) in each of these three cases.

**Step 3.** If \( \Delta(\phi y) = 0 \) (which is the case if \( 1 + y \) is sufficiently large), \( \Delta(r) > 0 \) for all \( r \in [\phi(1 + y)/2, \phi y] \). In this case, \( \pi^*_r > \pi^*_r \). To see this, note that \( \forall \gamma \in [\phi(1 + y)/2, \phi y] \), \( \pi^*_r = \phi^2 (1 + y^2)/(1 + y^2) < \pi^*_r (\phi(1 + y)/2) \) (where the last inequality follows from the fact that \( \pi^*_r (r) \) is a (weakly) decreasing function in \( r \), and \( \Delta(\phi(1 + y)/2) > 0 \) \( \Rightarrow \pi^*_r (\phi(1 + y)/2) > \pi^*_r (\phi(1 + y)/2) \)). Similarly, \( \forall \gamma \in (\phi y, \infty) \), \( \pi^*_r (\phi y) > \pi^*_r (\phi(1 + y)/2) \) (where the last inequality follows from the fact that \( \pi^*_r (r) \) is a (weakly) decreasing function in \( r \), and \( \Delta(\phi y) > 0 \) \( \Rightarrow \pi^*_r (\phi y) > \pi^*_r (\phi y) \)).

**Step 4.** If \( \Delta(\phi y) < 0 \) and \( \Delta(\phi(1 + y)/2) > 0 \), then \( \pi^*_r \) and \( \pi^*_r \) intersect at exactly two points. By the mean value theorem, there exists a value of \( r \), say \( r_1 \in [\phi(1 + y)/2, \phi y] \), such that \( \Delta(\phi y) = 0 \). Also, for the argument discussed in Step 3 above, there cannot exist any value of \( r \) such that \( \phi y > \phi(1 + y)/2, \phi y \) such that \( \Delta(r) = 0 \). However, there must exist another value of \( r \in (\phi y, \infty) \), say \( r_2 \), such that \( \Delta(r_2) = 0 \). The argument is as follows: \( \forall \gamma \in (\phi y, \infty) \), \( \pi^*_r (\phi y) > \pi^*_r (\phi y) \) (because we start with the premise that \( \Delta(\phi y) < 0 \) and \( \pi^*_r (\phi(1 + y)/2) = \phi^2 (1 + y^2)/(1 + y^2) < \phi^2 (\phi(1 + y)/2) \). As \( \pi^*_r \) is continuous and monotone, by the mean value theorem, there must exist a value of \( r \), say \( r \), such that \( \pi^*_r (r_2) = \phi^2 (r_2) \).

**Step 5.** Finally, consider the case where \( \Delta(\phi(1 + y)/2) < 0 \). Because \( \Delta(r) < 0 \) \( \forall \gamma \in (\phi y, \infty) \), there cannot exist any value of \( r \) such that \( \Delta(\phi(1 + y)/2) = 0 \). However, there must exist a value of \( r \) such that \( \Delta(\phi(1 + y)/2) = 0 \). The argument is as follows: \( r \leq \phi y/(1 + y) \), \( \pi^*_r (r) = 2\phi^2 > \phi^2 (1 + y^2)/(1 + y^2) = \pi^*_r (r) \) and, by assumption, \( \pi^*_r (\phi(1 + y)/2) = \pi^*_r (\phi(1 + y)/2) = \phi^2 (1 + y^2)/(1 + y^2) \). As \( \pi^*_r \) is continuous and...
monotonically decreasing, by the mean value theorem, there exists a unique value of \( r \), say \( r_1 \), such that \( \Delta(r_2) = 0 \). Also, by the argument discussed in Step 4, there must exist a unique value of \( r \in (\phi, \ell / \gamma) \), say \( r_2 \), such that \( \Delta(r_2) = 0 \). This observation completes the proof.

**Proof of Proposition 4.** The proof is given in the following steps.

**Step 1** (Comparative statics for \( r_1 \)). Recall from the proof of Proposition 3, that if \( r_1 < \phi(1 + \gamma)/2, r_1 \) solves
\[
\phi'(1 + \gamma^2)/(1 + \gamma^2) = \phi^2 + r, K(r_1)/\gamma^2, \tag{A1}
\]
whereas if \( r_1 \geq \phi(1 + \gamma)/2, r_1 \) solves
\[
\phi^2 + 2\phi(r\gamma - r) - \gamma^2 = (\phi^2(1 + \gamma^2) + 4\phi(1 + \gamma)r - 4r^2)/2(1 + \gamma^2). \tag{A2}
\]
If \( r_1 < \phi(1 + \gamma)/2 \), then \( \pi^*_1(\phi(1 + \gamma)/2) < \pi^*_1(\phi(1 + \gamma)/2) \). This holds only if \( \gamma < 4.0154 \). From equation (A1) one obtains \( r_1 = 2\phi \times (\gamma^2/(\gamma^2 + 1)) \times ((\sqrt{2} + 1)/3 + 2\sqrt{2} - \gamma) \) (this is the only root of equation (A1) in the relevant interval of values of \( r \), that is, in \([\phi(1 + \gamma)/2, \phi(1 + \gamma)/2] \). Now, from direct inspection of \( r_1 \), it follows that \( r_1 \) is an increasing function of \( r \) in the interval \((1, 4.0154]\).

If \( r_1 > \phi(1 + \gamma)/2 \), from equation (A2) one obtains \( r_1 = \frac{1}{2}(\sqrt{2} - 1)\phi(\gamma - 1) \) (this is the only positive root of equation (A2)). Now, \( \partial r_1/\partial \gamma = \frac{1}{2}(\sqrt{2} - 1)\phi(2\gamma - 1) > 0 \).

**Step 2** (Comparative statics for \( r_2 \)). Recall from the proof of Proposition 3, that \( r_2 \) solves
\[
\phi^2 = (\phi^2(1 + \gamma^2) + 4\phi(1 + \gamma)r - 4r^2)/2(1 + \gamma^2). \tag{A3}
\]
The only positive root of equation (A3) is \( r_2 = \frac{1}{2}\phi(1 + \sqrt{\gamma^2}) \). Thus, \( \partial r_2/\partial \gamma = \frac{1}{2}\phi(1 + 1/\sqrt{\gamma^2}) > 0 \).

**Step 3** (Upper threshold for \( \gamma \)). Define \( \gamma^* \) as the value of \( \gamma \) for which \( r_1 = \phi \gamma^* \). Now, for all \( r \neq r_1, \pi^*_1 > \pi^*_1 \). To see this, note the following: for \( r > r_1, \pi^*_1 > \pi^*_1 \) as \( \pi^*_1 \) is decreasing in \( r \) for \( r \in (\phi(1 + \gamma)/2, \phi(1 + \gamma)/2) \), but, by definition of \( r_1 \), for all \( r > r_1 = \phi \gamma^* \), \( \pi^*_1(r) = \pi^*_1(r_1) \). Thus, this implies \( r_1 = \phi \gamma^* \). Also, for \( r < r_1, \pi^*_1 > \pi^*_1 \). The argument is as follows: Because \( \pi^*_1(\phi \gamma^*) > \pi^*_1(\phi \gamma^*) \), and for all \( r \in (\phi(1 + \gamma)/2, \phi \gamma^* \), \( \pi^*_1(r) > \pi^*_1(\phi \gamma^*) \) (see Step 1 of the previous proof), \( \pi^*_1(\phi(1 + \gamma)/2) > \pi^*_1(\phi(1 + \gamma)/2) \). Now, for all \( r < \phi(1 + \gamma)/2, \pi^*_1(r) > \pi^*_1(\phi(1 + \gamma)/2) \), \( \pi^*_1(\phi(1 + \gamma)/2) = \pi^*_1(\phi \gamma^*) \). Now, because \( r_1 > \phi(1 + \gamma)/2 \), for \( \gamma \in (\gamma^*, \gamma^*), r_1 \) must solve (A2). But such an \( r_1 \) is increasing in \( \gamma \) by Step 2. So, for all \( \gamma > \gamma^*, r_1 > \phi \gamma^* \), which is inadmissible by the definition of \( r_1 \). Thus, \( \pi^*_1 > \pi^*_1 \) for all \( r \).

**Step 4** (Lower threshold for \( \gamma \)). Define \( \gamma^* \) as the value of \( \gamma \) for which \( r_1 = \phi(1 + \gamma)/2, \) or \( \gamma = 4.0154 \). For \( \gamma > \gamma^* \) (and \( \gamma < \gamma^* \)), \( r_1 \) must solve equation (A2) (see Step 1 of this proof). Now, for \( \gamma \in (\gamma^*, \gamma^*) \), \( r_1 \) is given by the solution to equation (A2), and \( r_2 \) is given by the solution to equation (A3). Therefore, \( r_2 - r_1 = \frac{1}{2}(\sqrt{2} - 1)\phi(\gamma - 1) - \frac{1}{2}\phi(1 + \sqrt{\gamma^2}) \).

**Step 5.** Finally, compute \( \partial (r_2 - r_1)/\partial \gamma = \frac{1}{2}(1/\sqrt{\gamma^2} + 2\gamma - \sqrt{2}(2\gamma - 1)) \). Observe that \( \partial (r_2 - r_1)/\partial \gamma < 0 \) for \( \gamma = \gamma^* = 4.0154 \), and \( \partial^2 (r_2 - r_1)/\partial \gamma^2 < 0 \). Therefore, for all \( \gamma > \gamma^* \), \( \partial (r_2 - r_1)/\partial \gamma < 0 \).

**Proof of Proposition 5.** Following the steps analogous to the derivation in the main model, one can derive the optimal contract and profit functions under individual and team assignment. Under individual assignment, the optimal contract and the associated profit function are
\[
\beta_i^r(r) = \begin{cases} \phi(k_1 + \gamma k_2)/(k_1 + \gamma k_2) & \text{if } r \leq \frac{1}{2}\phi(k_1 + \gamma k_2) \\ (2\phi(k_1 + \gamma k_2) - 2r)/(k_1 + \gamma k_2) & \text{if } \frac{1}{2}\phi(k_1 + \gamma k_2) < r < \phi(k_1 + \gamma k_2) \\ 0 & \text{if } r \geq \phi(k_1 + \gamma k_2) \end{cases}
\]
and
\[
\pi_i^r(r) = \begin{cases} \pi_1^r & \text{if } r \leq \frac{1}{2}\phi(k_1 + \gamma k_2) \\ 2\phi(k_1 + \gamma k_2)r - r^2/(k_1 + \gamma k_2) & \text{if } \frac{1}{2}\phi(k_1 + \gamma k_2) < r < \phi(k_1 + \gamma k_2) \\ 0 & \text{if } r \geq \phi(k_1 + \gamma k_2) \end{cases}
\]
Under team assignment, the optimal contract and the associated profit function are
\[
\beta_i^{r*} = 0, \quad \beta_i^{r*} = \frac{\beta r}{k_2} \quad \text{if } \beta < \phi k_2 \frac{1}{r^2} \gamma \leq \phi \frac{1}{r^2} (\gamma k_1 + k_2) 
\]
\[
\beta_i^{r*} = \frac{\gamma^2 \beta - \phi(\gamma - 1)k_2}{k_2 + \gamma^2 k_1}, \quad \beta_i^{r*} = \frac{\beta + \phi(\gamma - 1)k_1}{k_2 + \gamma^2 k_1} \quad \text{if } \phi k_2 \frac{1}{r^2} (\gamma k_1 + k_2) \leq \phi \frac{1}{r^2} (\gamma k_1 + k_2) 
\]
\[
\beta_i^{r*} = \phi, \quad \beta_i^{r*} = \frac{\phi}{\gamma} \quad \text{if } \beta \geq \phi \frac{1}{r^2} (\gamma k_1 + k_2) / \gamma.
\]
and
\[
\pi_t^r(r) = \begin{cases} 
J\phi_2 \left( \frac{k_1 + k_2}{2} \right) & \text{if } r \leq \frac{1}{2} \gamma (k_1 + k_2), \\
Jr K(r) / \gamma^2 & \text{if } \frac{1}{2} \gamma (k_1 + k_2) < r \leq \frac{1}{2} (\gamma + 1)\phi, \\
Jr \frac{2}{\gamma^2} k_1 (\gamma \phi - r) & \text{if } \frac{1}{2} (\gamma + 1)\phi < r < \phi \gamma, \\
0 & \text{if } r \geq \phi \gamma, 
\end{cases}
\]

where \( K(r) = A + \left[ A^2 + \gamma^2 (1 - \gamma) \phi^2 (k_1 + k_2) \right]^{1/2} \) and \( A = \phi (k_2 + y k_1) - (r - k_2 + \gamma k_1) \). Note that \( \frac{1}{2} \phi (k_1 + y k_1) > \frac{1}{2} (\gamma + 1), \) as \( k_1, k_2 \geq 1 \). The remainder of the proof is given by the following steps.

**Step 1.** We first show that for \( r \in \left( \frac{1}{2} \gamma \phi (k_1 + k_2) / (k_1 + y k_1), \phi \gamma \right] \), \( |\partial \pi_t^r / \partial r| < |\partial \pi_t^r / \partial r| \) for \( r \in \left( \frac{1}{2} \gamma \phi (k_1 + k_2) / (k_1 + y k_1), \phi \gamma \right) \). As \( \partial \pi_t^r / \partial r < 0 \) and \( \partial \pi_t^r / \partial r > 0 \), we have that \( |\partial \pi_t^r / \partial r| = 2 \left( \phi (k_1 + k_2) / (k_1 + y k_1) \right) \). As \( \partial \pi_t^r / \partial r < 0 \) and \( \partial \pi_t^r / \partial r > 0 \), we have that \( |\partial \pi_t^r / \partial r| = 2 \left( \phi (k_1 + k_2) / (k_1 + y k_1) \right) \). Moreover, as \( |\partial \pi_t^r / \partial r| < |\partial \pi_t^r / \partial r| \) for \( r \in \left( \frac{1}{2} \gamma \phi (k_1 + k_2) / (k_1 + y k_1), \phi \gamma \right) \), say \( r^* \), such that \( \pi_t^r (r^*) = \pi_t^r (r^*) \). Moreover, as \( |\partial \pi_t^r / \partial r| < |\partial \pi_t^r / \partial r| \) for \( r \in \left( \frac{1}{2} \gamma \phi (k_1 + k_2) / (k_1 + y k_1), \phi \gamma \right) \), \( r^* \) must be unique.

**References**


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