MANAGING PERFORMANCE EVALUATION SYSTEMS: RELATIONAL INCENTIVES IN THE PRESENCE OF LEARNING-BY-SHIRKING*

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Abstract. An agent may privately learn which aspects of his responsibilities are more important by shirking on some of them and use that information in the future to shirk more effectively. In a model of long-term employment relationship, we characterize the optimal relational contract in the presence of such learning-by-shirking, and highlight how the performance measurement system can be managed to sharpen incentives. Two related policies are studied: intermittent replacement of existing measures, and adoption of new ones. In spite of the learning-by-shirking effect, the optimal contract is stationary and may involve stochastic replacement/adoption policies in order to dilute the agent's information rents from shirking.

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1. Introduction

A common problem in agency relationships is that the agent may attempt to cut corners at the principal's expense. While the literature on incentive theory typically assumes that the agent exactly knows the consequences of shirking, in many contexts, that may not be the case. The agent may lack information on the relative importance of his assigned tasks for his overall job performance, and, relatedly, he may not know which corners to cut so as to minimize the risk of getting caught.

There are numerous examples of agency relationships where the agent may have imperfect information about the consequences of shirking. A management consultant working for a client under a tight deadline may have to cut corners in some parts of his report but he may not know which parts of the report are more important to the client. Hospitals may require doctors to follow a checklist designed to reduce infection risks, but to save time, a doctor may skip some of the steps. He may not know, however, which items on the checklist are more crucial for preventing infections.[1] Upstream suppliers may not know whether their downstream buyers care more about product compatibility, durability, or reliability. And manufacturing plants facing government's workplace safety inspections may be required to follow a host of safety measures but may not know which measures are most critical.

In all these settings, if the agent cuts corners and such shirking goes unnoticed, the agent will privately learn which aspects of his job are relatively less important for ensuring a good performance. And he may then use this information to shirk more effectively in the future. This possibility of "learning by shirking" exacerbates the incentive problem, because shirking, when successful, provides the agent with valuable private information that he may use later on to cut corners that are harder to notice.

[1]Gawande (2010) provides several examples of complex production environments from a varied set of industries, e.g., healthcare, financial services, civil engineering, and aviation, where the workers are expected to follow a "check list" of tasks in order to ensure that the job is successfully completed.

The presence of learning-by-shirking also poses a challenge to the design and management of the performance measurement system. It is well documented in the literature that even though numerous performance metrics may be available to a firm, ideal performance measures are elusive, and most measurement systems tend to be more sensitive to some job aspects than others (Meyer, 2002). Thus, by "successfully" shirking on some aspects of the job, the agent may learn how to game the prevailing performance measurement system. As Meyer (2002) notes, perverse learning—i.e., learning how to game the system—is a major reason why performance measures run down and lose their ability to discriminate good from bad performances. And one way the organizations might respond to these issues is by replacing the old measures by new ones.[2] However, as such replacements may be costly (e.g., administrative costs associated with rolling out a new evaluation system), an organization must address when and how often it should replace the performance measures. We analyze this question and highlight how the performance measurement system may be optimally managed in order to mitigate the learning-by-shirking problem.

We model the long-term relationship between a firm and a worker as a relational contract where the firm offers incentives through a discretionary bonus that is tied to a set of non-verifiable performance measures and is sustained through a threat of future retaliation by the worker should the firm renege on its bonus payment (see Malcomson, 2013).[3] We consider an infinitely repeated employment relationship where the worker performs a job that consists of two tasks (or aspects). The first-best outcome requires the worker to exert effort in both tasks. However, the firm

[2]Meyer (2002; p. 78) notes that "...measures that have run down must be replaced by measures that have not. New measures with variability, in particular, must be different from run down measures having little variability. Thus, slugging percentage replaces batting average [in baseball], functional measures of hospital performance replaces gross mortality, new-car defects weighted by severity replace unweighted defects."

[3]Relational incentives are commonplace in many industries, particularly in complex jobs with multiple aspects, where verifiable performance measures well-aligned with the firm's goal could be difficult to obtain (see Baker, Gibbons, and Murphy, 1994; Levin, 2003).

cannot measure the worker's performance in each task but observes the overall job output. In addition, the firm can also rely on a performance measure that yields further information about the worker's effort.

Following Meyer (2002), we assume there is a host of such additional measures that the firm can choose from. And even though all tasks are equally important for ensuring a high output, any given performance measure is relatively more sensitive to the worker's effort in one of the two tasks. The identity of this "critical task" associated with a given performance measure is unknown to all parties. But if the worker shirks on a task and goes undetected, he privately learns the task identities and may use this information to shirk more effectively in future, as long as the same performance measure is still in place. The firm, however, may replace the existing performance measure by a new one at the end of any period (at a cost); and with such a replacement, the identity of the critical task also changes stochastically, as the task identities in any two measures are statistically independent.

We characterize the optimal relational contract and, relatedly, the firm's optimal policy for replacing the performance measure. A technical challenge that the analysis needs to address is that if shirking goes undetected, the beliefs about the identity of the critical task in the continuation game will cease to be common knowledge: the agent will know which task is critical without the principal knowing he has such information. Consequently, the worker enjoys an information rent off-equilibrium when he shirks but is not caught.[4] It is the presence of this information rent that aggravates moral hazard relative to standard incentive problems. And the replacement of the performance measure may help the firm precisely because it reduces such rent. When the performance measure is replaced by a new one, any information the agent might have about the old measure becomes obsolete and, therefore, worthless. But in spite of the aforementioned complexity, the optimal relational contract has a simple characterization and it is closely tied to the future surplus

[4]As a result, the standard recursive techniques à la Abreu et al. (1990) cannot be applied, and our analysis significantly differs from that of a canonical model of relational contract (see, e.g., Levin, 2003). We will elaborate on this later in our discussion on the related literature.

in the relationship—i.e., the firm's "reputational capital"—captured by the players' common discount rate $\delta \in (0, 1)$.

More specifically, for δ sufficiently large, the firm can credibly offer a large bonus that induces the worker to work on both tasks, even if the same performance evaluation system is used in all periods. Hence, the first-best surplus is attained, and the performance measure is never replaced since replacement is costly. In contrast, for sufficiently low δ, it is optimal to dissolve the relationship as no incentive could be sustained irrespective of how the performance measures are managed.

Our main result concerns with the intermediate value of δ. The optimal contract sharpens relational incentives through a stochastic replacement policy (provided the cost of replacement is not too large). At the end of every period, the firm replaces the existing performance measure by a new one with a constant probability, and the worker exerts effort in both tasks in every period. The possibility of replacement dissuades the worker from learning-by-shirking by diluting his information rents from privately learning how to cut corners under the current performance measurement system. As the worker's gains from superior information may only last for a short period of time, he becomes less inclined to shirk. The optimal replacement probability is driven by the trade-off between the cost of such replacement and the benefits of sharper incentives that it creates. Thus, the optimal way to respond to the learning-by-shirking problem is to be proactive: rather than waiting till a measure runs down, the firm stochastically replaces it to dissuade the workers from shirking at the first place.

Even though intermittent replacement of the existing performance measure can be an effective response to the leaning-by-shirking problem, finding alternative measures could be difficult. In some situations, output may be the only meaningful measure of overall performance obtainable at an acceptable cost. And while additional measures may exist, they may be highly correlated with output so that using them in addition to output is essentially worthless; also, replacing one measure by another in this case will be ineffective since by learning how to game one measure the agent learns to game all. To analyze such cases, we adapt the model by assuming that output is the only measure of overall performance available to the firm. So, learning about the performance measure effectively means learning about the production process. We therefore assume that some aspects of the worker's job are indeed more critical than the others for the overall job output. If shirking goes undetected, the worker privately learns which task is critical for production. As the underlying production technology does not change over time, the information on the critical task remains relevant in all future periods.

We argue that in such a setting, the firm can ameliorate the incentive problem by publicly revealing which tasks are more critical for production and asking the agent to focus on them. While doing so, the firm may adopt a new performance measure that reflects the worker's effort in the critical task only (if such a measure is available). As several authors have noted, by adopting performance measures firms can communicate specific goals and guide the workers accordingly (see, Gibbons and Kaplan, 2015, and references therein).[5] Information revelation can dissuade the worker from shirking as his information advantage from privately learning which task is critical disappears once the firm publicly reveals it (and asks the worker to perform that task only). The optimal revelation policy, therefore, trade offs the current gains from sharper incentives with the loss of future surplus due to reduced effort on the non-critical task.

When δ is too large or too low, the firm's disclosure policy is irrelevant: if δ is too large, effort in both tasks can be elicited even when the agent knows which task is critical; and if δ is too small, no effort can be induced. However, for moderate

[5]It is assumed the firm cannot monitor the agent's actions in all tasks simultaneously. That would be equivalent to having a perfect measure of performance. Since managerial attention is necessarily limited, monitoring is often imperfect and knowing which aspects the managers will focus can be valuable to the worker. Recent economic literature has emphasized the role of managers in affecting the productivity of the firm (Lazear, Shaw and Stanton, 2015; Hoffman and Tadelis, 2017), and a number of theoretical models have focused on managerial (in)attention (Dessein and Santos, 2016; Dessein, Galleotti, and Santos, 2016; Halac and Prat, 2016; also see Gibbons and Henderson, 2012, for a review).

values of δ, the optimal contract requires the firm to actively filter the information on the task identities. When δ is relatively large (but still within the intermediate range), opacity is essential for attaining efficiency, and hence, the firm never reveals information. In contrast, for δ relatively small (but still within the intermediate range), the optimal contract calls for full transparency. The firm reveals the identity of the critical task to the agent at the beginning of the game, and in all periods, the worker performs the critical task only. Finally, for an intermediate range of δ, the firm relies on a stochastic stationary policy that resembles the optimal replacement policy discussed earlier: at the end of every period, the firm reveals the identity of the critical task with a constant probability (if it has not yet been made public). The worker exerts effort on both tasks until the critical task is revealed, but once it is revealed, he works only on the critical task in all future periods.

One implication of the stochastic information revelation policy is that the performance of identical firms may differ over time. The information may be revealed (and performance decline) sooner in some firms than in others, even though they are identical to begin with. Another implication of that policy is that to an outside observer, the firm may appear to be failing in the long run as, almost surely, its performance declines over time. There is a vast literature on the causes of organizational failures (see Garicano and Rayo, 2016, for a review) that identifies the lack of proper incentives as a key factor. In contrast, our findings suggest that a gradual decline in organizational performance could be an unavoidable by-product of the incentive policy needed to sustain a higher surplus at the earlier stages of the relationship.

Related Literature: Following the seminal works by Eccles (1991) and Kalpan and Norton (1992), a vast literature on the design of performance evaluation systems has developed over the last few decades. This literature primarily explores how the managers may combine information on several financial and non-financial measures, as no single performance metric may adequately reflect the organization's performance (see Demski, 2008, for a review). Several authors have also studied how such collection of measures may be used in formulating incentive contracts (Ittner, Larcker and Rajan, 1997; Ittner, Larcker and Meyer, 2003), but there is little discussion on how the performance evaluation systems should be managed over time as the agents might eventually learn how to game the system (one exception is Meyer, 2002, as we have already mentioned in the introduction). The current article attempts to fill this gap.

Our paper is related to a few strands of literature in organization economics. Several authors have studied how different auxiliary instruments can be used in order to sharpen relational incentives. These studies have focused on formal contracts (Baker, Gibbons, and Murphy, 1994), integration decisions (Baker, Gibbons, and Murphy, 2002), ownership design (Rayo, 2007), job design (Schöttner, 2008; Mukherjee and Vasconcelos, 2011; Ishihara, 2016), design of peer evaluation (Deb, Li, and Mukherjee, 2016), and delegation decisions (Li, Matouscheck, and Powell, 2017). But, as mentioned before, the issue of design and management of performance evaluation systems has not received much attention.

There is a growing literature on strategic information disclosure in employment relationships, and it has primarily focused on two kinds of information: the employer's private information on the agents' performance (e.g., Fuchs, 2007; Aoyagi, 2010; Ederer, 2010; Mukherjee, 2010; Goltsman and Mukherjee, 2011; Zabojnik, 2014; Orlov, 2016; Fong and Li, 2017) and information on the compensation rule used by employers—i.e., what aspects of performance are measured, and how these measures affect the incentive pay (see Lazear, 2006, and Ederer, Holden and Meyer, 2014). In this literature, our paper is closest to Lazear (2006), who analyzes when it is optimal to reveal to the agent which aspects of his performance are being measured. While Lazear (2006) considers monitoring and information disclosure in a static setting, we explore the role of transparency in incentive provision in a dynamic context and focus on the optimal disclosure of information over time.

Our paper also relates to the literature on incentives for experimentation (see Bolton and Harris, 1999; Keller, Rady, and Cripps, 2005; Manso, 2011; Hörner and Samuelson, 2013; Bonatti and Hörner, 2015; Halac, Kartik, and Liu, 2016; Moroni, 2016; Guo, 2016). While most of these articles do not consider relational incentives, a recent exception is Chassang (2010). He shows that moral hazard in experimentation combined with the lack of commitment by the principal, can result in a range of different actions being adopted in the long run. In contrast to these settings, the incentive problem we focus on is how to design the relationship so as to dissuade the agent from experimentation (i.e., selectively perform only a subset of tasks to learn how to game the performance system). Indeed, experimentation does not occur along the equilibrium path in our model.

Finally, our paper contributes to the nascent but growing literature on long-term and relational contracts in which the posterior beliefs of the contracting parties diverge and cease to be common knowledge in the course of the play (Bergmann and Hege, 2005; Fuchs, 2007, Bonatti and Hörner, 2011; Bhaskar, 2014; Fong and Li, 2017; also see Malcomson, 2013, for a survey of the literature on relational contracts).[6] Our setup differs from the canonical model of relational contracting (Levin, 2003) due to the possibility of private learning by the agent, and the lack of common knowledge (following a deviation by the agent) implies that the standard recursive technique à la Abreu, Pearce, and Stacchetti (1990) cannot be readily applied. We recover the recursive structure of the problem by considering an augmented state variable that includes the agent's maximum off-equilibrium continuation payoff, and obtain a complete characterization of the optimal relational contract.

The rest of the paper is structured as follows. Section 2 describes our baseline model that focuses on strategic replacement of the performance measurement system. A benchmark case is analyzed in Section 3 which assumes away the possibility of such replacement. The optimal replacement policy is studied in Section 4. In Section 5 we adapt our baseline model to show how information revelation could be

[6]The lack of common knowledge is also a feature in models with persistent private information such as Battaglini (2005), Yang (2013), and Malcomson (2016).

used to address the moral hazard problem when the agent may learn about the production process. A final section concludes. All proofs are provided in the Appendix (and its online supplement).

2. Model

A principal (or "firm") P hires an agent (or "worker") A, where the two parties enter in an infinitely repeated employment relationship. Time is discrete and denoted as $t \in \{1, 2, ..., \infty\}$. In each period, the firm and the agent play the following stage game.

Stage game: We describe the stage game in terms of its four key components: *technology, performance measures, contracts,* and *payoffs.*

Technology: In any period t, the agent may be asked by the principal to perform a job that consists of two tasks: $\mathbb{A}$ and $\mathbb{B}$. The agent privately exerts an effort $e_t \in \{0, 1_{\mathbb{A}}, 1_{\mathbb{B}}, 2\}$ at a cost of $C(e_t)$ in order to complete the job. If the agent works on both tasks, $e_t = 2$, and his cost of effort is $C(2) = c_2$; but if he works on either one of the two tasks, $e_t = 1_{\mathbb{A}}$ or $1_{\mathbb{B}}$ (depending on whether he works on task $\mathbb{A}$ or $\mathbb{B}$), and his cost of effort is $C(1_{\mathbb{A}}) = C(1_{\mathbb{B}}) = c_1$ $(< c_2)$. Also, if he shirks on both tasks, $e_t = 0$, and his cost of effort is $C(0) = 0$.

The job output $Y_t \in \{-z, y\}$ is assumed to be observable but not verifiable. The job is successfully completed if the agent works on both tasks (i.e., $e_t = 2$), and yields an output $y > 0$. If the agent shirks on both tasks (i.e., $e_t = 0$) he fails at the job, leading to a negative output $-z$ (e.g., such a failure may lead to an erosion of the firm's market value). But if the agent performs exactly one of the two tasks (i.e., $e_t = 1_{\mathbb{A}}$ or $1_{\mathbb{B}}$), the output is y with probability μ (> 0) and $-z$ with probability $1 - \mu$.

Performance measures: In addition to the output, the principal also relies on a performance measure that yields further information on the agent's effort level. In the spirit of Meyer (2002), we assume that there are infinitely many performance measures $\{M^1, M^2, ...\}$ that the principal can choose from. But all measures are inherently noisy, and no measure is equally sensitive to the agent's effort in all aspects of his job.

In particular, for any $i \in \{1, 2, ...\}$, $M^i \in \{0, 1\}$ where $M^i = 1$ if the agent works on both tasks, and $M^i = 0$ if he shirks on both. But if he works on exactly one of the two tasks, the realization of M^i depends on which task M^i is more sensitive to—the "critical" task associated with the measure. If the agent only performs the critical task associated with M^i, $M^i = 1$ with probability θ (> 0) and 0 otherwise. But if the agent only performs the non-critical task, then $M^i = 0$ with certainty.

The identity of the critical task is an idiosyncractic feature of a measure, and remains unchanged over time. When a measure is first put in place, neither player knows which task is critical for that measure, and both players correctly believe that any of the two tasks could be critical with equal likelihood. In other words, the identity of the critical task is i.i.d. across different performance measures. Similar to the output, we assume that all performance measures are observable but not verifiable.

Let M_t be the measure that is used in period t, chosen from the set $\{M^1, M^2, ...\}$. At the end of any period, the principal can publicly replace the current performance measure by a new one at cost ψ (and, consequently, it randomly changes the identity of the critical task in the subsequent periods). We denote the principal's replacement decision as $\gamma_t \in \{0, 1\}$, where $\gamma_t = 1$ if the principal replaces the current measure at the end of period t, and $\gamma_t = 0$ otherwise.

Note that if the agent shirks by exerting effort in only one task, he may privately learn the identity of the critical task associated with the current performance measure. If he picks the critical task by chance, both the output and the measure turn out to be good (i.e., $Y_t = y$ and $M_t = 1$) with probability $p := \mu\theta$, and the principal would fail to detect the agent's shirking. As we will see later, this possibility of private learning-by-shirking has significant implications for the optimal relational contract. Also note that in our setting, the replacement of performance measures does not affect the agent's productivity. Hence, such a replacement is completely wasteful but for its incentive implications, on which we will elaborate below. Finally, we assume that at the beginning of the game, the principal already has a performance measure, M_1, in place.[7]

Contract: In each period t, the principal decides whether to offer a contract to the agent. We denote the principal's offer decision as $d_t^P \in \{0,1\}$, where $d_t^P = 0$ if no offer is made, and $d_t^P = 1$ otherwise. If the principal decides to make an offer, she offers a contract that specifies a commitment of wage payment w_t and a discretionary bonus $b_t = b_t(Y_t, M_t)$. The incentives are relational as the output and the performance measures are assumed to be non-verifiable.

The agent either accepts or rejects the contract. We denote the agent's decision as $d_t^A \in \{0,1\}$, where $d_t^A = 0$ if the offer is rejected and $d_t^A = 1$ if it is accepted. Upon accepting the offer, the agent decides on his effort level—whether to work on both tasks, shirk on both tasks, or choose one of the two tasks and work only on that.

Finally, as is typical in the repeated game literature, we assume the existence of a public randomization device to convexify the equilibrium payoff set. In particular, we assume that at the end of each period t, the principal and the agent publicly observe the realization x_t of a randomization device. This realization allows the players to publicly randomize their actions in period $t+1$. In addition, a realization x_0 is also assumed to be publicly observed at the beginning of period 1, allowing the players to randomize in period 1 as well.

The time line of the stage game is illustrated in Figure 1 below.

[7]This assumption streamlines the analysis and allows us to abstract away from the question of whether to adopt a performance measure at the first place, as we focus on the question of how to manage the existing performance measurement system in the face of the learning-by-shirking problem. Nevertheless, we explore the former issue in a related environment in Section 5.

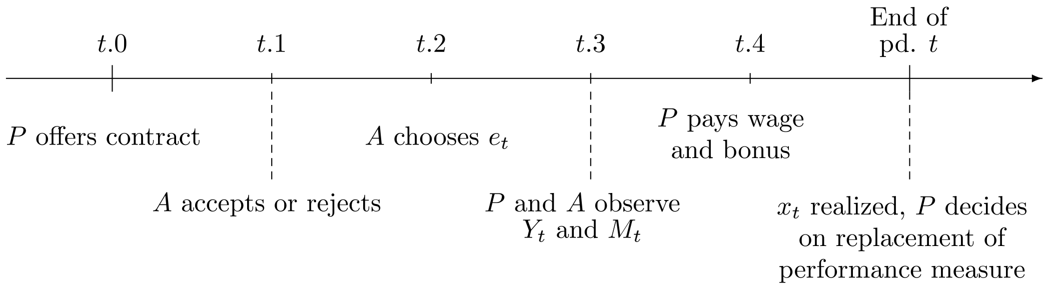

Figure 1. Timeline of the stage game.

Payoffs: Both the principal and the agent are risk neutral. If either d_t^A or d_t^P is 0, both players take their outside options in that period and the game moves on to period $t+1$. Without loss of generality, we assume that both players' outside options are 0. If $d_t^A = d_t^P = 1$, the expected payoffs for the agent and the principal are given as

$$\hat{u}_t = w_t + \mathbb{E}\left[b_t\left(Y_t, M_t\right) \mid e_t\right] - C\left(e_t\right) \text{ and } \hat{\pi}_t = \mathbb{E}\left[Y_t - w_t - b_t\left(Y_t, M_t\right) \mid e_t\right] - \psi\gamma_t,$$

respectively.

Repeated game: The stage game described above is repeated every period and players are assumed to have a common discount factor $\delta \in (0,1)$. At the beginning of any period t, the average payoffs of the agent and the principal in the continuation game are given by

$$u^t = (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left[d_\tau \hat{u}_\tau\right] \quad \text{and} \quad \pi^t = (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left[d_\tau \hat{\pi}_\tau\right],$$

respectively, where $d_\tau := d_\tau^A d_\tau^P$.

Strategies and equilibrium: The extant literature defines a relational contract as a pure strategy public Perfect Equilibrium (PPE) where the players only use public strategies and the equilibrium strategies induce a Nash Equilibrium in the continuation game starting from each public history (Levin, 2003). It is important to note that in our setting, we must account for the fact that the agent may privately learn about the identity of the critical task associated with a measure from his past deviation, and may find it profitable to use this information in future to game the performance metric (as long as the same measure is still in use). Thus, the restriction to pure strategy PPE may lead to some loss of generality, and hence, we focus on the perfect Bayesian Equilibrium (PBE) of the game defined as follows:

Let $h_t = \left\{d_\tau^A, d_\tau^P, Y_\tau, M_\tau, w_\tau, b_\tau, x_\tau, \gamma_\tau\right\}_{\tau=1}^{t-1}$ denote the public history of the game at the beginning of period t and H_t be the set of all such histories (note that, $H_1 = \{x_0\}$). The strategy of the principal consists of a sequence of functions $\sigma_P = \left\{D_t^P, W_t, B_t, \Gamma_t\right\}_{t=1}^{\infty}$, where her participation decision is given by $D_t^P : H_t \to \{0,1\}$, the contract offer is given as $W_t : H_t \to \mathbb{R}$ and $B_t : H_t \cup \{Y_t, M_t\} \to \mathbb{R}$, and finally, the replacement decision for the performance measurement system is given as $\Gamma_t : H_t \cup \left\{d_t^A, d_t^P, Y_t, M_t, w_t, b_t, x_t\right\} \to \{0,1\}$. The agent's strategy, however, may depend on his private history $\tilde{h}_t = \left\{d_\tau^A, d_\tau^P, e_\tau, Y_\tau, M_\tau, w_\tau, b_\tau, x_\tau, \gamma_\tau\right\}_{\tau=1}^{t-1}$, which not only records the public history but also includes information on the agent's past effort provisions. Let $\tilde{H}_t$ be the set of all such private histories. The agent's strategy is a sequence of functions $\sigma_A = \left\{D_t^A, E_t\right\}_{t=1}^{\infty}$, where his participation decision is given as $D_t^A : \tilde{H}_t \cup \left\{d_t^P, w_t, b_t\right\} \to \{0,1\}$, and his effort decision is given as $E_t : \tilde{H}_t \cup \left\{d_t^A, d_t^P, w_t, b_t\right\} \to \{0, 1_{\mathbb{A}}, 1_{\mathbb{B}}, 2\}$. Finally, denote $\mu_t = \Pr\left(\textit{task } \mathbb{A} \textit{ is crucial for } M_t\right)$ as the belief of the agent in period t about the identity of the critical task associated with the current performance measure M_t. Note that $\mu_t = \frac{1}{2}$ if the agent does not have any information on which task is critical, and it is either 0 or 1 if the agent has privately learned the identity of the critical task by shirking at some period in the past and the principal has not replaced the performance measure since then.

A profile of strategies $\sigma^* = \langle \sigma_P^*, \sigma_A^* \rangle$ along with a belief $\mu^* = \{\mu_t^*\}_{t=1}^{\infty}$ constitute a PBE of this game if σ^* is sequentially rational and μ^* is consistent with σ^* and derived using Bayes rule whenever possible. We define an "optimal" or "efficient" relational contract as a PBE of this game where the payoffs are not Pareto-dominated by any other PBE.

In what follows, we maintain a few restrictions on the parameters to focus on a more interesting modeling environment and to streamline the analysis.

Assumption 1. *(i)* $y - c_2 > 0$, *(ii)* $\frac{1}{2}pc_2 > c_1$, *and (iii)* $(1-\delta) \times ((\mu y - (1-\mu)z) - c_1) + \delta(y - c_2) < 0$.

Under Assumption 1 (i), efficiency requires the agent to exert effort on both tasks. Assumptions 1 (ii) and (iii) simplify the analytical tractability of the optimal contracting problem. Assumption 1 (ii) stipulates that the cost of exerting effort on both tasks is sufficiently large relative to the cost of working on only one of them. It ensures that the incentives needed to deter the agent from shirking on exactly one of the two tasks (i.e., choosing $e_t = 1_{\mathbb{A}}$ or $1_{\mathbb{B}}$ instead of $e_t = 2$) are also sufficient to deter him from shirking on both (i.e., choosing $e_t = 0$). Finally, Assumption 1 (iii) ensures that it is never optimal to ask the agent to cut corners—it is always better to dissolve the relationship than to have the agent perform only one of the two tasks in any given period (even if the efficient outcome is played in all future periods). This condition is trivially satisfied when z is sufficiently large.

3. A benchmark case and the feasibility of first-best

We begin our analysis by considering a benchmark scenario where the principal cannot replace the performance measure in the course of the play. That is, the measure available at the beginning of the game, $M_1 = M \in \{M^1, M^2, ...\}$, say, is the only performance measure (along with the output Y) that can be used to incentivize the agent in any future period.

This benchmark case is useful for our subsequent analysis for at least three reasons: First, it characterizes the optimal relational contract when incentives can only be offered through discretionary bonuses, and helps to illustrate how the replacement of an existing performance measure could sharpen incentives. Second, our analysis yields a necessary and sufficient condition for the feasibility of the "first-best" surplus—the surplus in the relationship when the agent exerts effort on both tasks in the job in every period and the principal never replaces the performance metric (i.e., $e_t = 2$ and $\gamma_t = 0$ for all t. Recall that in our setting replacement of an existing performance measure is completely wasteful but for its impact on the agent's effort incentives). Third, it also highlights an important technical aspect of our analysis. In our model, the optimal contract need not be stationary, and the game does not have a tractable recursive structure. Consequently, the standard method to characterize the equilibrium payoff set (à la Abreu, Pearce, and Stacchetti, 1990) no longer applies. Moreover, we cannot limit attention to the class of stationary contracts (as in Levin, 2003) without any loss of generality.

We present our analysis in three steps. First, we state the set of constraints that must hold in a given period under an efficient contract. Next, given the constraints, we derive a necessary and sufficient condition for an efficient contract to be feasible. And finally, using this condition, we present a complete characterization of the optimal contract.

Let the set of PBE payoffs for a given δ be $\mathcal{E}$. Take a $(u, \pi) \in \mathcal{E}$ that is efficient, i.e., the payoffs associated in an equilibrium where the agent exerts effort on both tasks in all periods and $u + \pi = y - c_2$. As the output and the performance measure are independently informative about the agent's effort, in any optimal contract the bonus is paid only if both indicate high effort, i.e., when $Y = y$ and $M = 1$.[8] Let w and b be the wage and bonus payment and $\left(u^N, \pi^N\right)$ be the continuation payoffs that sustain (u, π).[9] We assume that following any publicly observed deviation from the equilibrium actions, the relationship terminates.[10]

[8]The formal proof, however, is slightly elaborate (as it accounts for the learning-by-shirking effect that arises with the use of the performance metric) and is given in the online appendix.

[9]We use superscript N in the continuation payoffs to stress the fact that we are considering the case when there is *no* replacement of the performance metric. Also, note that w, b, u^N, and π^N are all functions of (u, π). We do not explicitly state these as functions in order to streamline the notation.

[10]The agent has a detectable deviation if the output or the performance metric do not conform to his equilibrium effort level. Similarly, the principal's detectable deviation consists of reneging on the bonus promise or failing to conform to the equilibrium play in the continuation game.

By virtue of being an equilibrium payoff with efficient actions, (u, π) must satisfy a set constraints. First, the agent and the principal's participation constraints must hold:

$$(IR) \qquad u \geq 0, \text{ and } \pi \geq 0.$$

Also, the consistency requirement of payoff decomposition implies that a player's payoff must be equal to the weighted sum of his current and continuation payoffs. Hence, we must have the following "promise-keeping" constraints:

$$(PK_A) \qquad u = (1-\delta)(w - c_2 + b) + \delta u^N,$$

$$(PK_P) \qquad \pi = (1-\delta)(y - w - b) + \delta \pi^N.$$

Next, the contract must satisfy the "incentive compatibility" constraints as the agent should not gain by deviating and shirking altogether or by performing exactly one of the two tasks. Recall that if the agent shirks on both tasks, $Y = -z$ and the relationship necessarily terminates. Hence, we must have:

$$(IC_0) \qquad u \geq (1-\delta) w.$$

But if the agent shirks by exerting effort on exactly one of the two tasks, the derivation of the incentive compatibility constraint is somewhat more involved. It must allow for the fact that upon deviating, the agent may privately learn the identity of the critical task associated with the performance measure M, and he may use this information to shirk again in the future. As a result, the principal and the agent (following a deviation) would have different beliefs on the task identities, and this lack of common knowledge renders the game devoid of a tractable recursive structure.

To address this issue, we proceed as follows. For any equilibrium payoff pair (u', π'), let $U(u', \pi')$ be the maximal continuation payoff of the agent where he privately knows which one is the critical task for the performance measure in use. That is, suppose (σ'_P, σ'_A) is the strategy profile of the players that gives rise to the payoff (u', π'). Now, $U(u', \pi')$ is the agent's payoff when he deviates from

σ'_A and plays his best-response to σ'_P using his knowledge on the identity of the critical task. If the payoff pair (u', π') could be supported with different equilibrium strategy profiles that are associated with different maximal deviation payoffs for the agent, without loss of generality, we choose the equilibrium strategy profile where the agent's maximal deviation payoff is the lowest (when the agent knows which task is critical for the current performance measure). This allows (u', π') to be a sufficient statistic for the agent's maximal deviation payoff. We can now state the agent's incentive compatibility constraints using the deviation payoff U:

$$(IC_1) \qquad u \geq (1-\delta)\left(w - c_1 + \frac{1}{2}pb\right) + \frac{1}{2}p\delta U\left(u^N, \pi^N\right).$$

Two remarks are in order: First, note that if the agent shirks by working on only one of the two tasks, he would pick the critical task associated with the performance measure and produce the on-equilibrium path outcome of $Y = y$ and $M = 1$ with probability $\frac{1}{2}p$ (and the principal would fail to detect such a deviation). Second, (IC_1) highlights the information value of shirking. The key difference between this constraint and its counterpart in the standard moral hazard is that the continuation payoff following shirking is $U\left(u^N, \pi^N\right)$ instead of u^N, and their difference, $U\left(u^N, \pi^N\right) - u^N$, reflects the agent's information rents from privately learning which task is critical.

Notice that for any $(u, \pi) \in \mathcal{E}$, $U(u, \pi) - u \geq 0$, since the agent can always disregard his superior information, and such rents from learning-by-shirking aggravate the moral hazard problem. Also, if the current performance metric could be replaced by a new one, then $U(u, \pi) - u = 0$ as the agent loses his information advantage. When this difference is strictly positive, it implies that there are further gains from shirking in the future. In other words, by acquiring knowledge about the performance metric via deviation, the agent may increase his gains from future deviation.[11]

[11]Note that when the agent privately learns which task is critical, it may not be the case that he always shirks by just performing the critical task whenever he is asked to put in effort on both tasks. The agent may want to wait for the right time to shirk. In particular, in a period when the

Next, the contract must satisfy the "dynamic enforceability" constraint to ensure that neither the principal nor the agent has incentives to renege on the bonus:

$$(DE_P) \qquad -(1-\delta)\,b + \delta\pi^N \geq 0,$$

$$(DE_A) \qquad (1-\delta)\,b + \delta u^N \geq 0.$$

Finally, we also have the "self-enforcing contract" constraints requiring the continuation payoffs themselves to be equilibrium payoffs in the continuation game:

$$(SE_N) \qquad \left(u^N, \pi^N\right) \in \mathcal{E}.$$

In light of the above constraints, we can now characterize the optimal contract in this benchmark case. While an analytical expression for the deviation payoff $U(u,\pi)$ is elusive, the deviation gains $U(u,\pi) - u$ can be bounded below by using the above set of constraints. Using this bound, we obtain a necessary and sufficient condition for the existence of such an efficient contract (i.e., the agent never shirks while the performance metric remains unchanged from period to period).

Lemma 1. *An efficient relational contract can be sustained if and only if*

$$(NSC^*) \qquad \frac{\delta}{1-\delta}\left(1 - \frac{p}{2-p\delta}\right)(y - c_2) \geq c_2 - c_1.$$

The characterization of the optimal contract follows directly from the (NSC^*) condition.

Proposition 1. ***(Optimal contract in benchmark case)*** *There exists a δ^* (δ at which (NSC^*) is binding) such that if $\delta \geq \delta^*$, the agent exerts effort in both tasks in all periods. Otherwise, no effort is induced and the players take their outside options.*

Proposition 1 suggests that when δ is sufficiently large the principal can rely on the discretionary bonus alone to generate adequate effort incentives and induce the agent to exert effort on all tasks. This is intuitive, as with a high δ, there is

agent's equilibrium payoff is high, he may not want to shirk because there will be too much to lose. But the agent may be more inclined to shirk when his equilibrium payoff is low.

enough surplus in the relationship such that the principal can credibly promise a large enough bonus to dissuade the agent from shirking even when the agent is sure to continue to work under the same performance measurement system. In spite of the fact that the information value of learning-by-shirking may be substantial, a sizable bonus payment mitigates the agent's temptation to shirk, as he would not risk terminating the relationship by choosing to work on only one task.

An immediate implication of this finding is that even when it is feasible to replace the existing performance metric—as in our main model—the first-best surplus is attained if and only if $\delta \geq \delta^*$: in the optimal relational contract, the principal would never replace the metric, but the agent would continue to exert effort on both tasks in all periods. But what is the optimal contract if $\delta < \delta^*$? In our benchmark setup, termination of the relationship is optimal. However, we next show that the principal can sharpen incentives and attain a higher surplus by replacing the performance metric with some frequency. We also characterize the optimal way to do so, i.e. the optimal way to manage the performance measurement system.

4. Optimal contract

Replacement of the existing performance metric may strengthen incentives as it depletes the information value of shirking. Any information on the performance metric that the agent may learn through shirking is rendered useless if his performance is assessed under a different metric in the future. Thus, if $\delta < \delta^*$ (and hence, effort in both tasks cannot be induced if the agent continues to work under the same performance measurement system), one may expect that rather than terminating the relationship, a larger surplus may be attained through strategic replacement of the existing performance measures. If so, what is the optimal replacement policy? Clearly, replacing the metric in every period would generate strong incentives, as the knowledge of the critical task that the agent may acquire by shirking in the current period would become irrelevant in the very next one. But such frequent replacements would lead to a large loss of surplus relative to the first-best as replacement is costly.

Thus, the optimal contract when the replacement of the performance measure is possible must minimize such cost while preserving the agent's effort incentives in every period. In the analysis of the optimal contract that follows, we limit our attention to the case where $\delta < \delta^*$, and we begin with a discussion on the constraints that a contract needs to satisfy if it is to induce effort on both tasks in a given period (i.e., $e_t = 2$).

Consider first a period t such that performance measure is the same as in the previous period. That is, the performance measure has not been changed at the end of period $t-1$. Suppose the critical task associated with the current measure M_t is not known to the agent, i.e., either $t = 1$ or the agent has not shirked successfully since the current measure has been put in use. Let the set of all PBE payoffs of the repeated game starting from period t be $\mathcal{E}$. Note that while $\mathcal{E}$ depends on δ, it is independent of t—as no information on the critical task is available to the agent, his belief on the tasks is the same as his prior. Consider a payoff pair $(u, \pi) \in \mathcal{E}$ that is sustained by eliciting effort on both tasks in the current period (i.e., $e_t = 2$).

At the end of period t (in fact, at the end of any period), the equilibrium strategies may call for one of the following three action profiles for the next period: (i) the agent exerts effort on both tasks while his performance is evaluated using the same measure that has been used in the previous period; (ii) the agent exerts effort on both tasks but faces a new performance metric (i.e., the principal has replaced the metric at the end of the previous period); and (iii) both players take their outside options in that period. For expositional clarity, we denote these three actions as $a = N$ ("no replacement"), R ("replacement"), and O ("outside option"), respectively. Recall that by Assumption 1 (iii), it is never optimal for the relationship to have the agent randomly select (and perform) exactly one of the two tasks. Also, using the public randomization device, the players could randomize over these three action profiles. Suppose that under the equilibrium strategy profile (that supports (u, π)), the action $a \in \{N, R, O\}$ is taken in the following period with probability α^a, and the corresponding continuation payoffs for the players are given as (u^a, π^a). If any player is caught deviating, without loss of generality, we may assume that the players take their outside options forever.

If a PBE induces effort on both tasks in period t, the associated equilibrium payoffs, contracts, and the continuation payoffs must satisfy a similar set of constraints to those we presented earlier in our benchmark analysis. However, the constraints need to account for the fact that the continuation game may call for any of the three possible action profiles in the following period: effort under the same performance metric ($a = N$), effort following replacement of the metric ($a = R$), or taking the outside option ($a = O$). The new set of constraints is given as follows:

The participation constraint remains the same as before:

$$(IR^*) \qquad u \geq 0, \text{ and } \pi \geq 0.$$

The "promise-keeping" constraints are modified as follows (notice that should the principal replaces the performance metric at the end of the period, the associated cost ψ is realized in the current period):

$$(PK_A^*) \qquad u = (1-\delta)(w - c_2 + b) + \delta\left(\alpha^N u^N + \alpha^R u^R + \alpha^O u^O\right),$$

$$(PK_P^*) \qquad \pi = (1-\delta)(y - w - b) + \delta\left(\alpha^N \pi^N + \alpha^R\left(\pi^R - \frac{1-\delta}{\delta}\psi\right) + \alpha^O \pi^O\right).$$

The "incentive compatibility" requires that:

$$(IC_0^*) \qquad u \geq (1-\delta) w,$$

$$(IC_1^*) \qquad \begin{aligned} u \geq \quad & (1-\delta)\left(w - c_1 + \tfrac{1}{2}pb\right) + \\ & \tfrac{1}{2}p\delta\left(\alpha^N U\left(u^N, \pi^N\right) + \alpha^R u^R + \alpha^O U\left(u^O, \pi^O\right)\right). \end{aligned}$$

Notice that when the performance metric is replaced, the agent's information on the critical task associated with the previous metric becomes irrelevant. As he loses the informative value from shirking, we have $U\left(u^R, \pi^R\right) = u^R$.

The "dynamic enforceability" ensures neither party reneges on the bonus and the principal would not renege on his promise to replace the metric (notice that a relational contract not only specifies the bonus payment but also the action profiles in each period):

$$(DE_P^*) \qquad -(1-\delta)\,b + \delta\left(\alpha^N\pi^N + \alpha^R\left(\pi^R - \frac{1-\delta}{\delta}\psi\right) + \alpha^O\pi^O\right) \geq 0,$$

$$(DE_P^*\text{-}R) \qquad -(1-\delta)\,\psi + \delta\pi^R \geq 0,$$

and

$$(DE_A^*) \qquad (1-\delta)\,b + \delta\left(\alpha^N u^N + \alpha^R u^R + \alpha^O u^O\right) \geq 0.$$

And finally, we need to impose a “self-enforcing contract” constraint for each of the three action profiles (i.e., under strategies that specify $a = N,\ R$, or O to be played in the next period):

$$(SE_N^*) \qquad \left(u^N, \pi^N\right) \in \mathcal{E},$$

$$(SE_R^*) \qquad \left(u^R, \pi^R\right) \in \mathcal{E},$$

and

$$(SE_O^*) \qquad \left(u^O, \pi^O\right) \in \mathcal{E}.$$

Consider now a period t where a new performance measure has just been introduced. Since the performance measure was replaced at end of period $t-1$, the critical task associated with the current measure M_t is again not known to the agent. If effort is elicited on both tasks in period t, the equilibrium payoffs, contracts and continuation payoffs must satisfy all the constraints above. However, in addition to those constraints, the principal's payoff must also satisfy $\pi \geq \frac{1-\delta}{\delta}\psi$ to ensure that she has an incentive to replace the performance measure at the end of period $t-1$. Thus, the set of PBE payoffs of the repeated game starting from any period t immediatley after a replacemeent of the performance measure is the set of payoff pairs $(u, \pi) \in \mathcal{E}$ such that $\pi \geq \frac{1-\delta}{\delta}\psi$.

4.1. **Preliminary analysis.** In order to characterize the optimal contract, we begin by presenting a set of lemmas that simplify our subsequent analysis. These lemmas state several observations about any PBE payoff that is sustained by $e_t = 2$ in a *given* period when the agent does not have any information on the critical task (for the current performance measure that is in use), and they allow us to restrict attention to a specific class of contracts without any loss of generality. The proofs are given in the online appendix. (The discussion below focuses on technical details; readers primarily interested in our key findings and economic intuition can omit this section and directly go to section 4.2.)

First, we show that the optimal contract need not use any bonus.

Lemma 2. *Consider a relational contract and take any period t and any history h_t. Suppose that the critical task for M_t is not known to the agent, and in the game starting from period t, the payoff profile (u, π) is sustained by $e_t = 2$ and $b_t \neq 0$. Then there exists another relational contract where (u, π) can be sustained by $e_t = 2$ and $b_t = 0$.*

The intuition behind this observation is as follows: First, suppose that (u, π) is supported by a contract that, in a given period t, specifies effort on both tasks and a negative bonus b. Such a contract is payoff equivalent to one where $e_t = 2$, but the wage (w_t) in that period is reduced by b and the bonus is set to 0. (It is routine to check that this new contract is also feasible). Next, suppose that the contract specifies effort in both tasks and a positive bonus b. Now, one may set $b = 0$ in period t and distribute the bonus amount among the continuation payoffs u^as (by raising the wages in period $t + 1$ that support each of the u^a payoffs) such that, in expectation, the agent continues to earn b.

Next, we present three lemmas that characterize the continuation payoffs in an optimal relational contract. Lemma 3 given below claims that without loss of generality, we can restrict attention to contracts that give zero continuation value to the principal (net of the cost of replacing the performance metric, if incurred) in each period.

Lemma 3. *Consider a relational contract and take any period t and any history h_t. Suppose that the critical task for M_t is not known to the agent, and in the game starting from period t, the payoff profile (u, π) is sustained by $e_t = 2$ and $\pi^a > 0$ for some $a \in \{N, R, O\}$. Then there exists another relational contract where (u, π) can be sustained by $e_t = 2$ and $\pi^N = \pi^R - \frac{1-\delta}{\delta}\psi = \pi^O = 0$.*

The intuition behind this observation is similar to that of Lemma 2 discussed earlier: any contract supporting (u, π) with a strictly positive continuation value (net of replacement cost) in some period t can be substituted by one that (i) sets $\pi^N = \pi^R - \frac{1-\delta}{\delta}\psi = \pi^O = 0$, (ii) increases the agent's continuation payoff u^a by raising the wage in the continuation game, w^a, that supports u^a (for all $a \in \{N, R, O\}$), and (iii) reduces the current period wage w by the (discounted) expected continuation payoff of the principal (π^a). It can be shown that for appropriate choices of w^as, such a contract is feasible and is payoff equivalent to the initial one. By virtue of Lemma 2 and 3, we can restrict attention to contracts where $b = 0$ and $\pi^N = \pi^R - \frac{1-\delta}{\delta}\psi = \pi^O = 0$, which automatically satisfy (DE_A^*), (DE_P^*), and (DE_P^*-R). In what follows, we therefore drop these constrains from set of constraints that the optimal contract must satisfy.

The next lemma suggests that without loss of generality, we can consider only those contracts that never specify $a = O$ on the equilibrium path.

Lemma 4. *If an optimal relational contract exists where the joint surplus is strictly positive, then there exists an optimal relational contract in which $\alpha^O = 0$ in all periods.*

The reason is that a strategy profile that calls for taking the outside option in period t is payoff-equivalent to an alternative strategy given as follows: in period t the strategy does not require the players to take their outside options but calls for termination of the relationship with probability $1 - \delta$. All other aspects of the new strategy are identical to the former one.

Finally, let v be the maximal joint payoff feasible in $\mathcal{E}$. Lemma 5 below suggests that following a replacement of the performance measure, we may set $u^R = v - \frac{1-\delta}{\delta}\psi$, i.e., the maximum surplus feasible in the continuation game net of the (discounted) cost of replacement.

Lemma 5. *In an optimal relational contract, in any period, if $\alpha^R > 0$, then $u^R = v - \frac{1-\delta}{\delta}\psi$.*

Lemmas 2–5 have the following important implication. In order to characterize the optimal contract, without loss of generality, we may restrict attention to contracts where, in any period, $b = 0$ and

$$w = \begin{cases} y & \text{if } a = N \text{ is played} \\ y - \frac{\psi}{\delta} & \text{if } a = R \text{ is played} \end{cases}.$$

That is, in the continuation game following every history, the principal's payoff (net of any costs of replacing the performance measure in the previous period) is zero. By constrast, the agent receives all of the surplus (net of the cost of replacing the performance measure in the previous period, if any). Note that under such a contract, (PK_P) is trivially satisfied. We focus on this class of contracts only for technical convenience, though other forms of implementation may be feasible.

4.2. **Characterization of the optimal relational contract.** Using the above lemmas, we can now simplify the optimal contracting problem. Notice that Lemma 4 implies that when deriving the optimal contract, we can restrict attention to contracts where in each period t the principal asks the agent to exert effort and either uses the same performance measure as before (i.e., set $a = N$) or adopts a new metric (i.e., set $a = R$). Let $1 - \alpha_t$ be the probability that the principal replaces the performance measure at the end of period t. Note that the optimal relational contract is completely determined by the sequence $\{\alpha_t\}_{t=1}^{\infty}$.

To streamline exposition, we present the optimal contracting problem in two steps: first, we derive the optimal contract for an arbitrary value of the agent's continuation payoff $u^R = s_1$, say, where s_1 is taken as a parameter that satisfies two conditions: (i) $s_1 < s_2 := y - c_2$, the surplus generated when there is no replacement of the performance measure and the agent exerts effort in both tasks;

and (ii) $\left(u^R, \pi^R\right) = \left(s_1, \frac{1-\delta}{\delta}\psi\right)$ can be sustained as an equilibrium payoff in the continuation game. Next, we characterize the optimal contract in our model by considering a specific value for s_1 as given in Lemma 5 (i.e., the value of s_1 when the performance metric may be replaced in order to sharpen incentives).

With a slight abuse of notation, let u^t be the agent's (average) payoff at the beginning of period t when (i) the critical task associated with the current performance measure is not known to the agent and (ii) the performance measure has not been replaced at the end of the previous period. Also, below we write $U(u)$ instead of $U(u, \pi)$ since the principal's continuation payoff (net of replcement costs) remains 0. Now, for a given s_1, we have the following recursive relationship for the agent's payoff:

$$u^t = (1-\delta)\, s_2 + \delta\left((1-\alpha_t)\, s_1 + \alpha_t u^{t+1}\right). \tag{1}$$

Therefore, if there exists a contract that implements effort in both tasks at least in the first period, solving for the optimal contract (in the class of such contracts) is tantamount to finding the optimal sequence $\{\alpha_t\}_{t=1}^{\infty}$ to maximize u^1. In other words, the optimal contract, for a given s_1, must solve the following program (denote $c := c_2 - c_1$):

$$\mathcal{P}: \begin{cases} \max_{\alpha_t \in [0,1]} \quad u^1 \quad s.t.\ \forall t, \\ \\ u^t = (1-\delta)\, s_2 + \delta\left((1-\alpha_t)\, s_1 + \alpha_t u^{t+1}\right) & (PK_A^*) \\ u^t \geq (1-\delta)\, y & (IC_0^*) \\ u^t \geq (1-\delta)(s_2 + c) + \frac{1}{2} p\delta\left((1-\alpha_t)\, s_1 + \alpha_t U\left(u^{t+1}\right)\right) & (IC_1^*) \\ \left(u^t, 0\right) \in \mathcal{E} \quad (SE_N^*) \text{ and } \left(s_1, \frac{1-\delta}{\delta}\psi\right) \in \mathcal{E} \quad (SE_R^*) \end{cases}.$$

Note that if $\alpha_t = 1$ for all t is feasible in $\mathcal{P}$, then the optimal relational contract is efficient. As we have argued in Lemma 1, this is the case if and only if (NSC^*) is satisfied. As we are interested in the case where (NSC^*) is violated, we consider below the case where $\alpha_t = 1$ for all t is not feasible.

The optimal contracting problem $\mathcal{P}$ presents an interesting technical challenge, as there is no standard method that could be used to directly compute $U(u)$, the agent's maximal deviation payoff in the continuation game (after he privately learns through shirking which task is critical for the current performance measure). The complexity stems from the fact that once the agent learns the task identities, the profitability of his future deviations would depend on the associated "replacement" policy, i.e., how the principal intends to replace the existing performance metrics over time.

For example, fix a production environment and suppose that the contract calls for replacing the existing performance measure after a certain number of periods, T (say), but the agent has shirked and learned the underlying critical task at an earlier period $t < T$. In such a scenario it may not be optimal for the agent to continue to shirk in all subsequent periods till T (i.e., as long as the same performance measure is in use). Notice that under the above replacement policy, the agent's continuation payoff decreases over time as we move closer to date T. Hence, it may be worthwhile for the agent to continue to exert effort on both tasks until some period $\bar{t}$, where $t < \bar{t} < T$ (as he stands to lose a larger continuation payoff if he shirks immediately after his first successful deviation) and then start to shirk again by working on the critical task only (when the continuation payoff is smaller).

We address this problem by considering a relaxed program that only allows for a specific form of deviation: if the agent deviates and (privately) learns the critical task, in all subsequent periods until the current performance measure is replaced, he always deviates by working on the critical task only (if he is not detected sooner). Notice that for a contract to be a part of an equilibrium, it must be robust to all forms of deviation, including the one specified above. Hence, the aforementioned deviation can be used to compute a lower bound on the agent's maximal deviation payoff $U(u)$ and we can characterize the optimal replacement policy that deters this type of deviation. We then show that this relaxed problem admits a stationary solution where at the end of each period, the principal replaces the performance measure at a fixed probability. We further argue that this policy, by virtue of being a stationary one, is robust to all deviations and, hence, a solution to the original problem $\mathcal{P}$. Lemma 6 reports this finding.

Lemma 6. *If there exists a solution to the optimal contracting problem $\mathcal{P}$, then there also exists a stationary solution to $\mathcal{P}$ where for all t, $\alpha_t = \alpha^*$ (which may vary with δ). That is, at the end of each period, the principal replaces the existing performance measure with a constant probability α^*.*

It is instructive to elaborate on the intuition behind the above lemma. Notice that even though the relaxed problem limits attention to a specific form of deviation, the exact time of deviation is still a choice variable for the agent. Hence, we have infinitely many incentive constraints: for every period t, we must have a constraint ensuring that no profitable deviation exists in that period.

It turns out that if the replacement policy were to deter deviation in period 1 only, it would take a form that features "early replacement": there is some T such that the principal would replace the current measure with a positive probability if $t < T$, but would never do so again afterwards. By replacing early, this policy backloads the rewards to the agent as much as possible. To see why it is useful to backload the rewards, note that compared to the agent who always works on both tasks, the agent who has shirked successfully and only works on the critical task is effectively less patient—the former discounts the future at rate δ, but the effective discount rate of the latter is $p\delta$, as the relationship is likely to terminate for him with probability $1 - p$. Since an agent who has successfully shirked in the past discounts the future more (relative to an agent who has not), an early replacement of the existing measure most effectively discourages the agent from shirking in period 1 by backloading the rewards as much as possible.

However, such an early replacement policy is necessarily time-inconsistent. While the agent is deterred from deviating in period 1, he may want to deviate in the later periods when the gains from shirking are larger. (As the principal is less likely to replace the measure in the later periods, the agent earns a larger information rent if he shirks and learns the critical task.) In other words, for every period t, the optimal policy would ideally induce an increasing sequence in the continuation payoff by increasing the current period's replacement probability and decreasing the probability of future replacements. But as this inducement is needed in every period, the resulting optimal policy becomes stationary and features a constant replacement probability in each period. Finally, by virtue of stationarity, this policy is necessarily robust to all possible deviations of the agent.

We are now ready to present a complete characterization of the optimal contract. From Lemma 6 we know that the optimal contract is stationary for any s_1, and Lemma 5 suggests that in the optimal contract where performance metrics may be replaced period to period, we must have $s_1 = v - \frac{1-\delta}{\delta}\psi$. By definition, v is the value of the optimal contracting problem under such a s_1 (notice that upon replacing the current measure, the principal would choose the optimal contract in the continuation game, and the continuation game is identical to the game at the beginning of period one). Using these two observations, along with Lemma 1, we obtain the following proposition.

Proposition 2. *(**Optimal contract with replacement of performance measures**) The optimal contract is characterized as follows. There exist two cutoffs, δ_R and δ^*, $\delta_R \leq \delta^*$(δ^* as defined in Lemma 1), such that the following holds:*

(i) *For all $\delta \geq \delta^*$, the principal never replaces the performance metric, and in every period the agent exerts effort on both tasks (i.e., he gets evaluated by the same performance measure that is in place since the beginning of the game). The optimal contract yields the first-best surplus.*

(ii) *For all $\delta \in [\delta_R, \delta^*)$, the optimal contract fails to attain the first-best surplus and is given as follows: At the end of every period, the principal replaces the existing performance measure with a constant probability α^* (which may vary with δ), and the agent exerts effort on both tasks in all periods. Moreover, $\delta_R < \delta^*$ if and only if the cost of replacement ψ is below a threshold.*

(iii) *Finally, for all $\delta < \delta_R$, no effort could be induced, and both parties take their outside options.*

The intuition for the above proposition is as follows: Recall that the optimal contract either induces effort on both tasks in all periods or no effort at all—by Assumption 1 (iii), dissolving the relationship is better than performing only one task chosen at random out of the two. Now, as noted in Lemma 1, for a large δ (i.e., if $\delta \geq \delta^*$), the first-best outcome is feasible: the principal never replaces the performance metric, and in every period the agent continues to exert effort on both tasks. In contrast, for δ sufficiently small (i.e., δ below δ_R), the optimal policy dissolves the relationship. The maximum bonus that the principal can credibly promise, no matter what replacement policy is used, fails to induce effort on both tasks.

But for a moderate δ—if $\delta \in [\delta_R, \delta^*)$—the principal may be able to induce effort on all tasks by adopting a stochastic replacement policy where at the end of each period, the principal may replace the existing performance metric with a fixed probability. As the agent knows that his information on the critical task for the current performance measure may become irrelevant in the near future, it dilutes the value of the information that he may obtain by shirking. Recall that a production environment with a new performance measure is assumed to be identical to its predecessors except for the identity of the critical task. Hence, if the threat of replacement provides an adequate incentive under the current performance measure, it must continue to provide similar incentives if the existing performance metric is replaced by a new one. Also notice that such a stochastic replacement policy would entail a loss of surplus relative to the first-best, as it is costly to replace an existing performance measure. Hence, such a policy could be optimal if and only if the cost of replacement is not too large. Otherwise, it is always better to dissolve the relationship than to attempt to induce effort with the threat of replacing the existing performance measure.

5. Learning about production and information revelation

In settings where finding new measures of overall performance of the agent is difficult, a policy of intermittent replacement of the performance measures is infeasible. Yet, in such cases, the principal may consider an alternative policy to mitigate the problem of learning-by-shirking, particularly when the information that the agent obtains by shirking is about the underlying production technology. Suppose some job aspects are more critical for the overall output than others and, by cutting corners, the agent may privately learn which tasks are more crucial for production. Then, the principal may at some point wish to reveal to the agent which jobs aspects are more critical and ask him to focus on them.

The incentive problem that emerges in this environment closely parallels our analysis in the previous section. But while our earlier analysis highlights how the firm may sharpen incentives through replacing existing performance measures, the analysis below informs us on whether and when the firm should reveal information about the critical tasks for production. The analysis also highlights how the performance evaluation system may be managed through adoption on new measure rather than by replacing the old one—in reality, firms often set up a new performance metrics that track the critical tasks so as to reveal which tasks are more important and guide the worker towards them (Gibbons and Kaplan; 2015).

The revelation of information involves a basic trade-off. It discourages the agent from shirking, as it dissipates the gains from privately learning the information on the critical tasks through shirking. But once the information is revealed, it becomes more difficult to incentivize the agent to execute all the tasks associated with his job. The optimal revelation policy must balance this trade-off.

Consider the following adjustments to our initial model. As before, let the agent's job output be $Y_t \in \{-z, y\}$ where $Y_t = y$ if $e_t = 2$ and $Y_t = -z$ if $e_t = 0$. But if the agent performs only one task, the output depends on which task he has shirked on as one of the two tasks is more critical for production. If the agent works on the "critical" task only, then, as in our initial model, $Y_t = y$ with probability μ (> 0)

and $-z$ with the remaining probability. But if he only performs the "non-critical" task, then $Y_t = -z$ for sure. At the beginning of the game, neither the principal nor the agent knows which task is critical, and both players correctly believe that either task could be critical with equal probability. The identity of the critical task is assumed to be the same over time.

At the end of each period, the principal can publicly access this information and disclose it to the agent at zero cost by putting in place a performance metric $M_t \in \{0,1\}$ where $M_t = 1$ if the critical task is completed and 0 otherwise.[12] As before, both Y_t and M_t are assumed to be observable but non-verifiable. And there are no other performance measures available to the firm. Let $\gamma_t = 1$ if the information on critical task is publicly available at the end of period t, and $\gamma_t = 0$ otherwise. Notice that $\gamma_t = 1$ if the performance measure M_t has been put in place in some period $\tau \leq t$ and (in contrast to our earlier model) the information on the critical task remains public in all periods $\tau > t$. The discretionary bonus $b_t = b_t(Y_t)$ if $\gamma_t = 0$ and $b_t = b_t(Y_t, M_t)$ otherwise.

We keep all other aspects of our initial model unchanged, and similar to Assumption 1, we impose the following restrictions on the parameters. (Note that the parameter μ plays the same role here as that of p in our main model: both parameters, in their respective settings, reflect the probability that if the agent only performs the critical task, his shirking would go undetected.)

Assumption 1A: (i) $y - c_2 > \mu y + (1-\mu)(-z) - c_1 > 0$, (ii) $\frac{1}{2}\mu c_2 > c_1$, and (iii) $(1-\delta)\left(\frac{1}{2}(-z + \mu y + (1-\mu)(-z)) - c_1\right) + \delta(y - c_2) < 0$.

The above restrictions have the exact same interpretation as their counterparts in Assumption 1, except in the case of part (i): here, we assume that while efficiency

[12]In our model, while the fact that the firm can adopt the measure M_t (that tracks the agent's effort on the critical task) helps the exposition, it is not essential for the results. All the results continue to hold (qualitatively) if such measure is not available and the firm simply reveals to the worker which of the two tasks is critical.

requires the agent to work on both tasks, working on the critical task only is better than dissolving the relationship and taking the outside options.[13]

Notice that if the task information is accessed and disclosed at the very beginning of the game, the game boils down to the canonical relational contracting model. The optimal contract in this setting has a simple characterization. (The proof is given in the online appendix.)

Lemma 7. *If the information on the critical task is made public at the beginning of the game, the optimal relational contract is characterized as follows: There exist two cutoffs, $\underline{\delta}$ and $\overline{\delta}$, where $\underline{\delta} \leq \delta^* < \overline{\delta}$ (δ^* as defined in Lemma 1 when $p = \mu$), such that (i) if $\delta \geq \overline{\delta}$, the agent exerts effort on both tasks in every period, (ii) if $\underline{\delta} \geq \delta \geq \overline{\delta}$, the agent exerts effort only on the critical task in every period, and (iii) if $\delta < \underline{\delta}$, no effort can be induced, and the parties take their outside options in every period.*

The intuition behind this result is straightforward: the bigger the δ, the larger is the principal's reputational capital; and hence, he can credibly offer stronger bonus incentives and elicit more effort. But Lemma 7 has an important implication: it is easier to induce effort on both tasks when the critical task is unknown to all than when it is public information (i.e., $\delta^* < \overline{\delta}$). When the task information is public, shirking yields a higher payoff to the agent, as he knows on which task to shirk (when the agent does not know the critical task, he expects to choose it only half of the time). Hence, the principal must offer a stronger incentive to elicit effort on both tasks. If $\delta \in [\delta^*, \overline{\delta})$, the maximum bonus that the principal can credibly promise is large enough to dissuade the agent from shirking when he does not know the critical task, but the bonus is too small to elicit effort on both tasks when the critical task is known to the agent.

Notice that if $\delta \geq \delta^*$, the optimal contract attains first-best but the underlying use of the task-specific measure M_t varies with δ. When $\delta \geq \overline{\delta}$, whether or not the

[13]The conditions in Assumption 1A jointly holds if both y and μ are relatively large and z is moderate.

principal put in place the performance measure is inconsequential—the first-best is attained irrespective of the principal's decision. But if $\delta \in [\delta^*, \overline{\delta})$, the optimal contract attains first-best only when the task identities are not known to the agent. Hence, the principal must not implement a performance measure that may reveal which task is critical.

But if $\delta < \delta^*$, when and how to implement the performance metric so as to reveal the task identities? As Lemma 7 suggests, the principal can induce effort on the critical task by revealing the task identities at the beginning of the game. But the principal may be able to attain a larger surplus by postponing the adaptation of the performance measure, so that the disclosure of information gets delayed. The analysis of the optimal adaptation policy bears strong resemblance to the optimal replacement policy studied in Section 4. Hence, for the sake of brevity, we omit the technical details and present the characterization of the optimal contract in the proposition below (the relevant details of the analysis are given in the proof).

Proposition 3. *(**Optimal contract with information revelation**) The optimal contract is characterized as follows. There exist four cutoffs $\underline{\delta} < \tilde{\delta} \leq \delta^* < \overline{\delta}$ (δ^* as defined in Lemma 1 when $p = \mu$) such that the following holds:*

(i) *For all $\delta \geq \overline{\delta}$, the optimal contract attains the first-best surplus (i.e., the agent exerts effort on both tasks in all periods) irrespective of the principal's decision on whether to reveal the critical task.*

(ii) *For all $\delta \in [\delta^*, \overline{\delta})$, the optimal contract attains the first-best surplus and the principal never uses any performance measure so as to conceal the identity of the critical task.*

(iii) *For all $\delta \in [\tilde{\delta}, \delta^*)$, the first-best surplus cannot be attained. In the optimal contract, the principal implements the performance measure for the critical task at the end of each period with a constant probability α^* (which may vary with δ). The* *agent works on both tasks until the measure is put in place and works only on the critical task afterwards. Moreover,* $\tilde{\delta} < \delta^*$ *if and only if*

$$\left(1 - \frac{\mu}{2}\right)(\mu y + (1-\mu)(-z) - c_1) > \left(1 - \frac{\mu}{2 - \mu\delta^*}\right)(y - c_2).$$

(iv) *For all* $\delta \in [\underline{\delta}, \tilde{\delta})$*, the first-best surplus cannot be attained. In the optimal contract, the principal implements the performance measure for the critical task at the beginning of the game and the agent only works on that task.*

(v) *Finally, for all* $\delta < \underline{\delta}$*, no effort could be induced and both parties take their outside options.*

The proposition above closely parallels the characterization of the optimal contract with replaceable performance measures that we discussed earlier, and illustrates how the adoption of the performance measure (for the critical task) relates to the amount of surplus generated by the employment relationship. A moderately large surplus calls for opacity (the principal does not adopt any performance measure), a moderately small surplus calls for full transparency (performance measure for the critical task is put in place at the very outset), and if the available surplus is in an intermediate range, the optimal contract calls for filtering of information through a stochastic adoption policy.

All parts of this proposition directly follow from Lemma 7, except for part (iii): As discussed in the context of Lemma 7, for a sufficiently large or sufficiently small δ—i.e., if $\delta < \underline{\delta}$ or $\delta \geq \overline{\delta}$—the principal's adoption policy plays little role in the optimal contract. But for an intermediate range of δ, i.e., for $\delta \in [\underline{\delta}, \overline{\delta})$, active management of information is critical.

Within this range, when δ is relatively large ($\delta \in [\delta^*, \overline{\delta})$), full opacity is optimal, whereas a relatively small δ ($\delta \in [\underline{\delta}, \tilde{\delta})$) calls for full transparency. But for moderate values of δ ($\delta \in [\tilde{\delta}, \delta^*)$), the principal may do better by not revealing the task information at the beginning of the game. A larger surplus could be attained under a stochastic adoption policy where at the end of each period, the principal reveals the critical task with a fixed probability (by adopting a performance metric for that task). As the agent knows that the critical task is likely to become public information in the near future, it dilutes the value of the private information that he hopes to obtain through learning-by-shirking. Such a contract elicits effort on both tasks until the critical task is revealed, and hence, is more efficient than the one that reveals this information at the beginning of the game. However, a stochastic adoption policy can be optimal if and only if the condition given in part (iii) is satisfied—i.e., the surplus with effort on the critical task only ($s_1 := \mu y + (1-\mu)(-z) - c_1$) is not too small compared to the first-best surplus ($s_2 := y - c_2$). Indeed, the loss of surplus due to information revelation, $s_2 - s_1$, plays the same role as the cost of replacement, ψ, in our initial model.[14]

An important implication of the optimality of stochastic adoption is that the performance of the organization decreases over time. The agent performs both tasks at the beginning of the relationship. And once the critical task is revealed, he works on the critical task only, causing the performance to fall almost surely in the long run. Our model therefore adds to the broad literature of why organizations fail (Garicano and Rayo, 2016) and, in particular, to the recent relational contracting literature in which the long-run performance of the firms may be lower than their earlier performance (Barron and Powell, 2017; Fong and Li, 2017; and Li and Matouschek, 2013). In these papers, organizational performance declines because privately observed negative shocks in the past constrain the organization's ability to

[14] To see the intuition, observe that the revelation of the critical task has two effects. On the one hand, the benefit of revelation is that it reduces the agent's gains from shirking and learning the identity of the critical task. On the other hand, the cost of revelation is that the total surplus in the relationship is reduced—once the critical task is revealed, the agent will perform that task only. The larger is s_1, the smaller is the cost of revelation, whereas the agent's benefit of shirking is primarily linked to the surplus under first-best effort—if the shirking goes undetected, the agent per-period payoff is equal to the first-best surplus (s_2) plus the cost of effort saved ($c_2 - c_1$). As a result, the larger is s_1, the more likely it is that a partial revelation (through delayed adoption of the performance metric) will emerge as the optimal relational contract.

make promises to its employees and therefore to motivate its workforce—the organization is burned by its past promises. In contrast, there are no privately observed shocks in our setting. The decline in the performance is a by-product of information revelation, which is used to discourage the agent from learning to shirk at the beginning of the relationship.

6. Conclusion

This article explores the optimal provision of relational incentives when the worker may learn by shirking. Workers often hold jobs that involve multiple aspects (or a set of tasks) and the performance measures in place may be more sensitive to some job aspects than others. An interesting moral hazard problem emerges when the worker lacks information about the relative importance of the various job aspects: he may shirk on some aspects of the job not only to save on the costly effort but also to learn more about how they affect the underlying performance measures. Using a model of relational contracting, we study how the firm can sharpen incentives in such a setting by managing its performance measurement systems. We highlight two policies—frequent replacement of existing performance measures and adoption of new measures that guides the workers towards the more critical tasks. We show that both policies could be used as a strategic tool to strengthen relational incentives, and illustrate how the frequency of replacement (and, in the same vein, adoption of new measures) is tied to the amount of surplus generated in the relationship.

It is worth noting that even though our model focuses on how the firm may manage the performance measures in response to the learning-by-shirking problem, the incentive effects it highlight relate to any policy that a firm may adopt to "shake up" the production environment in the future in order to dissuade the agent from learning-by-shirking at the present. Indeed, firms often adopt job rotation and/or reorganization policies where workers expect to be moved to different division or assignments within the firm after every so few months in a given job. For example, in the classic study of the leveraged buyout of RJR Nabisco by Burrough and Helyar (1990; p. 26), the authors note that "[The CEO, F. Ross Johnson] reorganized

Standard Brands twice a year, like clockwork, changing people's jobs, creating and dissolving divisions, reversing strategic fields. To outsiders it seemed like movement for movement's sake. Johnson framed it as a personal crusade against specialization. 'You don't have a job,' he told [...], 'you have an assignment."' Similar policies are also common in government organizations in many countries where the civil servants are rotated among multiple locations as an anti-corruption measure (Bardhan, 1997). Insofar as such reorganizations are costly to the firm, our model sheds light on how such a policy should be used in the optimal incentive contract.

Appendix

Proof of Lemma 1. The proof is given in three steps. In the first step we derive a lower bound for $U(u,\pi) - u$ in any efficient equilibrium. In the second step we use this lower bound to show that (NSC^*) is a necessary condition for the existence of an efficient equilibrium. In the third step we show that this condition is also sufficient for the existence of an efficient equilibrium.

Step 1. (*Lower bound for $U(u,\pi) - u$ in an efficient equilibrium*) Suppose that an efficient equilibrium (i.e., an equilibrium where $e_t = 2$ and $\gamma_t = 0$ in all periods) exists. Recall that in an efficient equilibrium (and only in an efficient equilibrium), $\pi + u = y - c_2$. Define:

$$D := \min_{(u,\pi)\in\mathcal{E}} U(u,\pi) - u, \ \ s.t.\ \pi + u = y - c_2.$$

That is, D is the minimum of the agent's superior information across all the efficient equilibria. We next derive a lower bound for D. Take an arbitrary $(u,\pi) \in \mathcal{E}$ such that $\pi + u = y - c_2$. Then, there exist w, b, u^N, and π^N such that (PK_A) and (DE_P) are satisfied, i.e.,

$$u = (1-\delta)(w - c_2 + b) + \delta u^N, \tag{2}$$

and

$$(1-\delta)\, b \leq \delta\pi^N = \delta(y - c_2 - u^N), \tag{3}$$

where the last equality follows from the fact that in an efficient equilibrium also $u^N + \pi^N = y - c_2$. Moreover, observe that

$$U(u,\pi) \geq (1-\delta)(w + pb - c_1) + p\delta U\left(u^N, \pi^N\right). \tag{4}$$

(An inequality—and not necessarily an equality—holds as the agent may choose to exert effort in both tasks even if he knows the identity of the critical task.) Using (2) and (4), it then follows that

$$\begin{aligned}
&U(u,\pi) - u \\
&\geq (1-\delta)(c_2 - c_1) + p\left((1-\delta)b + \delta U\left(u^N,\pi^N\right)\right) - (1-\delta)b - \delta u^N \\
&= (1-\delta)(c_2 - c_1) - (1-p)\left((1-\delta)b + \delta u^N\right) + p\delta\left(U\left(u^N,\pi^N\right) - u^N\right) \\
&\geq (1-\delta)(c_2 - c_1) - (1-p)\delta(y - c_2) + p\delta D,
\end{aligned}$$

where the last inequality follows from (3) and the definition of D. As the inequality above holds for all (u,π), we therefore have $D \geq (1-\delta)(c_2 - c_1) - (1-p)\delta(y - c_2) + p\delta D$, or,

$$D \geq \frac{1}{1-p\delta}\left((1-\delta)(c_2 - c_1) - (1-p)\delta(y - c_2)\right). \tag{5}$$

Step 2. (*Necessity of NSC^**) Let $(u,\pi) \in \mathcal{E}$ such that $\pi + u = y - c_2$. Combining (IC_1) and (PK_A) one obtains:

$$(1 - \frac{1}{2}p)\left((1-\delta)b + \delta u^N\right) \geq (1-\delta)(c_2 - c_1) + \frac{1}{2}p\delta\left(U\left(u^N,\pi^N\right) - u^N\right).$$

Using (DE_P) and the fact that $u^N + \pi^N = y - c_2$, we obtain:

$$(1 - \frac{1}{2}p)\left((1-\delta)b + \delta u^N\right) \leq (1 - \frac{1}{2}p)\left(\delta\pi^N + \delta u^N\right) = (1 - \frac{1}{2}p)\delta(y - c_2).$$

Hence, we must have

$$\begin{aligned}
&(1 - \tfrac{1}{2}p)\delta(y - c_2) \\
&\geq (1-\delta)(c_2 - c_1) + \tfrac{1}{2}p\delta\left(U\left(u^N,\pi^N\right) - u^N\right) \\
&\geq (1-\delta)(c_2 - c_1) + \tfrac{1}{2}p\delta D \\
&\geq (1-\delta)(c_2 - c_1) + \tfrac{p\delta}{2(1-p\delta)}\left((1-\delta)(c_2 - c_1) - (1-p)\delta(y - c_2)\right),
\end{aligned}$$

(where the last inequality follows from (5)), or, equivalently,

$$(NSC^*) \qquad \frac{\delta}{1-\delta}\left(1-\frac{p}{2-p\delta}\right)(y-c_2) \geq c_2 - c_1.$$

Step 3. (*Sufficiency of* NSC^*) Consider the following stationary contract: in each period $w = y$ and $b = 0$, the agent is asked to exert effort in both tasks, and the relationship terminates if the agent is caught shirking. Under this arrangement, the principal's payoff is $\pi = 0$, the agent's payoff is $u = y - c_2$, and the only constraints that need to be checked in order for it to be sustained as an equilibrium are (IC_0) and (IC_1).

To check that (IC_0) is satisfied, note that under the proposed contract, $u^N = y - c_2$ and $b = 0$. Plugging these values in (IC_0), we get

$$y - c_2 \geq (1-\delta)y \Leftrightarrow -(1-\delta)\,c_2 + \delta\,(y-c_2) \geq 0 \Leftrightarrow \frac{\delta}{1-\delta}\,(y-c_2) \geq c_2,$$

which is satisfied when (NSC^*) is satisfied. To see this, observe that

$$\frac{\delta}{1-\delta}\,(y-c_2) \geq \frac{c_2-c_1}{1-p/\left(2-p\delta\right)} \geq \frac{c_2-c_1}{1-p/2} \geq c_2,$$

where the first inequality corresponds precisely to the (NSC^*), the second follows from the fact that $p\delta \in (0,1)$, and the third from the fact that $c_1 \leq \frac{1}{2}pc_2$ (Assumption 1 (ii)).

To check that (IC_1) is satisfied we need to analyze the agent's value from private information under the arrangement. Suppose the agent privately learns which task is critical. Given that the principal will continue to play according to the contract, the agent's problem is stationary, which implies that either the agent never shirks (by doing the critical task only) or he always shirks. Suppose first that the agent never shirks. Then, $U\left(u^N, \pi^N\right) = u^N$ and, since $u^N = y - c_2$ and $b = 0$, constraint (IC_1) collapses to:

$$\frac{\delta}{1-\delta}\left(1-\frac{1}{2}p\right)(y-c_2) \geq c_2 - c_1,$$

which is satisfied whenever (NSC^{*}) is satisfied. Suppose now the agent always shirks. Then $U\left(u^{N},\pi^{N}\right)=(1-\delta)\left(y-c_{1}\right)+p\delta U\left(u^{N},\pi^{N}\right)$, or,

$$U\left(u^{N},\pi^{N}\right)=\frac{1}{1-p\delta}\left(1-\delta\right)\left(y-c_{1}\right).$$

Given this and the fact that $u^{N}=y-c_{2}$ and $b=0$ under the proposed arrangement, (IC_{1}) is given by:

$$\left(1-\frac{1}{2}p\right)\delta\left(y-c_{2}\right)\geq(1-\delta)\left(c_{2}-c_{1}\right)+\frac{1}{2}p\delta\left(\frac{(1-\delta)\left(y-c_{1}\right)}{1-p\delta}-\left(y-c_{2}\right)\right),$$

or,

$$\left(1-\frac{1}{2}p\left(1+\delta\right)\right)\delta\left(y-c_{2}\right)\geq\left(1-\frac{1}{2}p\delta\right)\left(1-\delta\right)\left(c_{2}-c_{1}\right),$$

which is the same as the (NSC^{*}) above. ■

Proof of Proposition 1. From Lemma 1 and the observation that the term

$$\frac{\delta}{1-\delta}\left(1-\frac{p}{2-p\delta}\right)$$

is increasing in δ for $\delta\in(0,1)$ and $p\in(0,1)$, it follows that there exists a unique δ^{*} (at which NSC^{*} is satisfied with equality) such that for all $\delta\geq\delta^{*}$, in the optimal contract, the agent exerts effort in both tasks in all periods.

The fact that for $\delta<\delta^{*}$ no effort can be induced and it is optimal for the principal and the agent to take their outside options in every period follows directly from forthcoming results in this paper. Indeed, forthcoming Lemmas 2-4 in Section 4 of this paper are also valid when $\alpha^{R}=0$ in every period (i.e., when it is not possible to replace the existing performance measure). Thus, when looking for the optimal contract, it is without loss of generality to restrict attention to contracts where, in each period, no bonuses are used, the principal's continuation payoff is zero and the principal and agent permanently terminate the relationship with some probability. Given this, the optimal contract when replacement of performance measure is infeasible, must solve problem $\mathcal{P}$ that appears in Section 4.2 with $s_{1}=0$ (the continuation value of termination). However, as shown in Step 1 of the proof of Proposition 2, problem $\mathcal{P}$ has no solution when $s_{1}=0$, implying that effort cannot be sustained and it is optimal for the principal and agent to take their outside options in all periods. ■

Proof of Lemma 6. The proof is given by the following steps.

Step 1. (*Forming a relaxed problem by considering a specific deviation*) Let u_s^t be the agent's payoff when he privately knows which task is critical and always shirks by doing the critical task only (given that the principal continues to offer $w = y$ and $b = 0$) in all periods until the agent's deviation is detected or the performance measure is replaced. Note that $u_s^t \leq U(u_t)$ and satisfies the following recursive relation:

$$u_s^t = (1-\delta)(s_2 + c) + \delta p \left(\alpha_t u_s^{t+1} + (1-\alpha_t) s_1\right). \tag{6}$$

So, if one restricts attention to only this type of deviation, (IC_1^*) could be simplified as:

$$u^t \geq (1-\delta)(s_2 + c) + \frac{1}{2} p\delta \left(\alpha_t u_s^{t+1} + (1-\alpha_t) s_1\right), \tag{7}$$

or, equivalently,

$$2u^t \geq (1-\delta)(s_2 + c) + u_s^t. \tag{IC_1'}$$

Now, consider the following "relaxed" version of $\mathcal{P}$ where we replace (IC_1^*) with its weaker version (IC_1') and ignore the (IC_0^*) and (SE_N) constraints:

$$\mathcal{P}_R : \max_{\alpha_t \in [0,1]} \quad u^1 \quad s.t. \;\; (1),\; (6),\; \text{and} \;\; \left(IC_1'\right) \;\; \text{hold for all } t.$$

Step 2. (*Rewriting $\mathcal{P}_R$ in terms of α_t*) By using (1) and (6), one can eliminate u^t and u_t^s in $\mathcal{P}_R$ and consider an equivalent problem in terms of α_ts. Note that (1) can be rearranged as $u^t - s_1 = (1-\delta)(s_2 - s_1) + \delta\alpha_t \left(u^{t+1} - s_1\right)$. So, one obtains:

$$u^t - s_1 = (1-\delta)(s_2 - s_1)(1 + \delta S_t),$$

where $S_t = \alpha_t + \delta\alpha_t\alpha_{t+1} + \delta^2\alpha_t\alpha_{t+1}\alpha_{t+2} + ...$ Hence,

$$u^1 = s_1 + (1-\delta)(s_2 - s_1)(1 + \delta S_1). \tag{8}$$

Next, note that, $u_s^t - ps_1 = (1-\delta)(s_2 + c - ps_1) + \delta p \alpha_t \left(u_s^{t+1} - s_1\right)$, and hence,

$$\begin{aligned} u_s^t - s_1 &= u_s^t - ps_1 - (1-p)s_1 \\ &= (1-\delta)(s_2 + c - ps_1) + \delta p \alpha_t \left(u_s^{t+1} - s_1\right) - (1-p)s_1 \\ &= (1-p)\left((1-\delta)y - \delta s_1\right) + \delta p \alpha_t \left(u_{t+1}^s - s_1\right). \end{aligned}$$

So,

$$u_s^t - s_1 = (1-p)\left((1-\delta)y - \delta s_1\right)(1 + \delta p D_t), \tag{9}$$

where $D_t = \alpha_t + (\delta p)\alpha_t \alpha_{t+1} + (\delta p)^2 \alpha_t \alpha_{t+1} \alpha_{t+2} + ...$ Note that (IC_1') is equivalent to:

$$\begin{aligned} 2u^t - 2s_1 &\geq (1-\delta)(s_2 + c) - s_1 + u_s^t - s_1 \\ &= (1-\delta)(s_2 + c - s_1) - \delta s_1 + u_s^t - s_1, \quad \forall t, \end{aligned}$$

or,

$$k_0 (1 + \delta S_t) \geq k_1 + k_2 (1 + \delta p D_t) \quad \forall t.$$

where $k_0 = 2(1-\delta)(s_2 - s_1)$, $k_1 = (1-\delta)(s_2 + c - s_1) - \delta s_1$, and $k_2 = (1-\delta)(s_2 + c - s_1) - \delta(1-p)s_1$. Since we consider the case where $\delta \leq \delta^*$, and hence, (NSC^*) is violated, it routinely follows that $k_2 > 0$. Now, the above constraint can be rewritten in the following form:

$$D_t \leq A + BS_t \ \ \forall t, \tag{10}$$

where $A = (k_0 - k_1 - k_2)/k_2 \delta p$ and $B = k_0 / pk_2$. So, from (8) and (10), it follows that $\mathcal{P}_R$ is equivalent to the following program:

$$\mathcal{P}_R' : \max_{\alpha_t \in [0,1]} \ S_1 \ \ s.t. \ (10).$$

Step 3. (*Rewriting* $\mathcal{P}_R'$ *in terms of* α, S and D) Note the following: (i) Any sequence of $\{\alpha_t\}_{t=1}^\infty$ pins down a unique sequence $\{(S_t, D_t)\}_{t=1}^\infty$. (ii) S_t and D_t are non-negative and $S_t \geq D_t$ with equality holding if and only if $\alpha_t \alpha_{t+1} = 0$. (iii) S_t and D_t follow the recursive relations:

$$S_t = \alpha_t (1 + \delta S_{t+1}), \text{ and } D_t = \alpha_t (1 + \delta p D_{t+1}).$$

(iv) The set of $\{\alpha\}$ that satisfy (10) gives rise to a set of (S, D) that are feasible. Call this set $\mathcal{F}$. It is not necessary for the proof to characterize $\mathcal{F}$ but by standard argument we know that it must be compact. Now, we can rewrite $\mathcal{P}'_R$ as follows:

$$\mathcal{P}''_R : \begin{cases} \max_{\alpha\in[0,1],\ S,\ D,\ S',\ D'} \quad S \\ \\ s.t. \quad S = \alpha\left(1+\delta S'\right);\ D = \alpha\left(1+\delta p D'\right) \quad (PK_R) \\ D \le A + BS \quad (IC_R) \\ (S', D') \in \mathcal{F} \quad (SE_R) \end{cases}$$

(Note that the constraint (SE_R) implies (S', D') satisfies (IC_R), $D' \le S'$, and $D \le S$.) We will consider the case where $A > 0$. For $A \le 0$, we will later argue that the firm's program does not have a solution (and hence, the interval (δ_R, δ^*) does not exist).

Step 4. (*Introducing $f(S)$ function and defining S^**) Note the following about $\mathcal{P}''_R$. (i) The recursive relations suggest:

$$\frac{D}{S} = \frac{1+\delta p D'}{1+\delta S'}.$$

(ii) For any (S, D), we have

$$\frac{D}{S} \le \frac{1+\delta p D}{1+\delta S} \text{ iff } D \le \frac{S}{1+\delta\left(1-p\right)S} =: f\left(S\right).$$

Observe that $f(S)$ is increasing (and concave) and $f(S)/S$ is decreasing in S. Also, under the first-best solution where all $\alpha_t = 1$, $(S, D) = \left(S^{FB}, D^{FB}\right) = \left(\frac{1}{1-\delta}, \frac{1}{1-\delta p}\right)$ and it satisfies $D^{FB} = f\left(S^{FB}\right)$. (iii) Since the first best is not feasible by assumption, we must have $D^{FB} > A + BS^{FB}$. Hence, the $D = f(S)$ curve must intersect $D = A + BS$ at some point (S^*, D^*) where $S^* < S^{FB}$, and $D^* < D^{FB}$ (since we have $A > 0$).

Step 5. (*S^* is the value of the program $\mathcal{P}''_R$.*) We claim that S^* is the value of the program $\mathcal{P}''_R$. The proof is given by contradiction. Suppose that the value of $\mathcal{P}''_R$ is $\bar{S}_1 > S^*$. Let $\mathcal{D}(S)$ be the minimal D associated with all solutions that yield the value S. As $\mathcal{F}$ is compact, $\mathcal{D}$ is well-defined. Consider the tuple

$\left(\bar{S}_1, \mathcal{D}\left(\bar{S}_1\right)\right)$. By the recursive relations, $\left(\bar{S}_1, \bar{D}_1\right) := \left(\bar{S}, \mathcal{D}\left(\bar{S}\right)\right)$ generates a sequence $\{\left(\bar{S}_2, \bar{D}_2\right), \left(\bar{S}_3, \bar{D}_3\right), ...\}$ such that each element of the sequence satisfies (i) $\bar{D}_n \leq A + B\bar{S}_n$ (if not, then (10) would be violated in some period) and (ii) the recursion relations (PK_R) for some associated sequence of $\alpha_t, \{\bar{\alpha}_t\}$ (say). We will argue in the next four sub-steps (steps $5a$ to $5d$) that such a sequence cannot exist.

Step 5a. We argue that $\bar{S}_1 > \bar{S}_2$ *and* $\bar{D}_1 > \bar{D}_2$. First, observe that for all $S \in (S^*, S^{FB})$, $f(S) > A + BS$. As $\bar{S}_1 > S^*$, $f\left(\bar{S}_1\right) > A + B\bar{S}_1 \geq \bar{D}_1 = \mathcal{D}\left(\bar{S}_1\right)$. Next, we claim that $f\left(\bar{S}_2\right) \geq \bar{D}_2$.

The proof is given by contradiction: suppose $f\left(\bar{S}_2\right) < \bar{D}_2$. But then we have $\bar{S}_2 < S^*$. The argument is as follows: Clearly, if $\bar{S}_2 = S^*$, the highest feasible $\bar{D}_2$ that could support $\bar{S}_2$ is $f(S^*)$ and hence there is no feasible $\bar{D}_2$ such that $f\left(\bar{S}_2\right) < \bar{D}_2$. Now suppose $\bar{S}_2 > S^*$. Since $f(S) > A + BS$ for all $S > S^*$ and $A + B\bar{S}_2 \geq \bar{D}_2$, it must be that $f\left(\bar{S}_2\right) > \bar{D}_2$. Hence, $f\left(\bar{S}_2\right) < \bar{D}_2 \Rightarrow \bar{S}_2 < S^*$.

Therefore, if $f\left(\bar{S}_2\right) < \bar{D}_2$, we obtain that:

$$\frac{\bar{D}_1}{\bar{S}_1} = \frac{1 + \delta p\bar{D}_2}{1 + \delta\bar{S}_2} > \frac{1 + \delta p f\left(\bar{S}_2\right)}{1 + \delta\bar{S}_2} = \frac{f\left(\bar{S}_2\right)}{\bar{S}_2} > \frac{f(S^*)}{S^*}, \tag{11}$$

where both equalities follow from (PK_R), the first inequality holds as $f\left(\bar{S}_2\right) < \bar{D}_2$ and the second inequality holds as $\bar{S}_2 < S^*$ (argued above) and $f(S)/S$ is decreasing in S. But as $\bar{S}_1 > S^*$ and $f\left(\bar{S}_1\right) > \bar{D}_1$ we must also have,

$$\frac{f(S^*)}{S^*} > \frac{f\left(\bar{S}_1\right)}{\bar{S}_1} > \frac{\bar{D}_1}{\bar{S}_1},$$

which contradicts (11). Hence, we must have $f\left(\bar{S}_2\right) \geq \bar{D}_2$.

As $f\left(\bar{S}_2\right) \geq \bar{D}_2$, we obtain:

$$\frac{\bar{D}_1}{\bar{S}_1} = \frac{1 + \delta p\bar{D}_2}{1 + \delta\bar{S}_2} \geq \frac{\bar{D}_2}{\bar{S}_2}.$$

As $\bar{S}_2 \leq \bar{S}_1$ (since $\bar{S}_1$ is assumed to be the highest S_1 feasible), the above inequality implies that we must have $\bar{D}_2 \leq \bar{D}_1$.

Step 5b. We must have $\bar{\alpha}_2 = 1$. We show this by contradiction. From (PK_R) we know that $\left(\bar{S}_2, \bar{D}_2\right) = \left(\bar{\alpha}_2\left(1 + \delta\bar{S}_3\right), \bar{\alpha}_2\left(1 + \delta p\bar{D}_3\right)\right)$. If $\bar{\alpha}_2 < 1$, increase $\bar{\alpha}_2$ to $\alpha'_2 := (1 + \varepsilon)\bar{\alpha}_2$ for some $\varepsilon > 0$. Let $(S'_2, D'_2) := (1 + \varepsilon)\left(\bar{S}_2, \bar{D}_2\right)$.

We argue that for sufficiently small ε, (S_2', D_2') is feasible. Since $(\bar{S}_3, \bar{D}_3) \in \mathcal{F}$ and (PK_R) is trivially satisfied by definition of (S_2', D_2'), it is enough to show that (S_2', D_2') satisfies (IC_R). To see this, recall that $\bar{D}_1/\bar{S}_1 \geq \bar{D}_2/\bar{S}_2$ (from step 5a) and $\bar{S}_2 \leq \bar{S}_1$. So, $(\bar{S}_2, \bar{D}_2)$ must lie on or below the line joining the origin to $(\bar{S}_1, \bar{D}_1)$. Now, there are two cases: (i) If (IC_R) is slack at $(\bar{S}_1, \bar{D}_1)$, all points on this line always lie strictly below the line $D = A + BS$. So, (IC_R) is also slack at $(\bar{S}_2, \bar{D}_2)$. (ii) If (IC_R) binds at $(\bar{S}_1, \bar{D}_1)$, this is the only point on the line at which (IC_R) binds, and it is slack at all other points. But, as $f(\bar{S}_1) > \bar{D}_1$, we have:

$$\frac{1 + \delta p \bar{D}_1}{1 + \delta \bar{S}_1} > \frac{\bar{D}_1}{\bar{S}_1} = \frac{1 + \delta p \bar{D}_2}{1 + \delta \bar{S}_2}.$$

So, $(\bar{S}_2, \bar{D}_2) \neq (\bar{S}_1, \bar{D}_1)$. Therefore, (IC_R) must be slack at $(\bar{S}_2, \bar{D}_2)$. Thus, for small enough ε, $(S_2', D_2') = (1 + \varepsilon)(\bar{S}_2, \bar{D}_2)$ always satisfies (IC_R).

Next, observe that,

$$\frac{\bar{D}_1}{\bar{S}_1} = \frac{1 + \delta p \bar{D}_2}{1 + \delta \bar{S}_2} > \frac{1 + \delta p (1 + \varepsilon) \bar{D}_2}{1 + \delta (1 + \varepsilon) \bar{S}_2} = \frac{1 + \delta p D_2'}{1 + \delta S_2'}.$$

Now, we reduce $\bar{\alpha}_1$ to some α_1' where $\alpha_1' (1 + \delta S_2') = \bar{S}_1$. Let $D_1' = \alpha_1' (1 + \delta p D_2')$. So, by the above inequality, we find that:

$$\frac{D_1'}{\bar{S}_1} = \frac{1 + \delta p D_2'}{1 + \delta S_2'} < \frac{\bar{D}_1}{\bar{S}_1}.$$

Hence, $D_1' < \bar{D}_1$. But this observation contradicts the fact that $\bar{D}_1$ is the lowest feasible D_1 that supports S_1 (as we have shown that the sequence $\{\alpha_1', \alpha_2', \bar{\alpha}_3, ...\}$ is feasible, and it yields $S_1 = \bar{S}_1$ and $D_1 = D_1' < \bar{D}_1$). Therefore, we must have $\bar{\alpha}_2 = 1$.

Step 5c. We must have $\bar{S}_3 < \bar{S}_2$ and $\bar{D}_3 < \bar{D}_2$. As $\bar{\alpha}_2 = 1$, (PK_R) implies $\bar{S}_2 = 1 + \delta \bar{S}_3$ and $\bar{D}_2 = 1 + \delta p \bar{D}_3$. As $\bar{S}_t < S^{FB} = 1/(1 - \delta)$ and $\bar{D}_t < D^{FB} = 1/(1 - \delta p)$ for any t, it is routine to check that $\bar{S}_3 < \bar{S}_2$ and $\bar{D}_3 < \bar{D}_2$.

Step 5d. Repeating steps 5b and 5c we can argue that $\bar{\alpha}_t = 1$ for all $t \geq 2$ and the sequence $\{\bar{S}_2, \bar{S}_3, ...\}$ is monotonically decreasing. So, we must have $\bar{S}_t = 1 + \delta \bar{S}_{t+1}$, $t = 2, 3, ...$ But such a sequence cannot exist. First, note that this sequence cannot converge. If it converges at some $\hat{S}$, we must have $\hat{S} = 1 + \delta \hat{S}$, or $\hat{S} = S^{FB} =$

$1/(1-\delta)$, which is not a feasible as all terms of the sequence is bounded away from $\bar{S}_1 < S^{FB}$. Therefore, some term of this sequence will be either negative or zero. But we know that $\bar{S}_t$ is non-negative. Also, suppose $\bar{S}_k = 0$. So, we must have $\bar{S}_{k-1} = \bar{D}_{k-1} = \bar{\alpha}_{k-1}$. But this is a contradiction as we know that $\bar{S}_{k-1} = \bar{D}_{k-1}$ only if $\bar{\alpha}_{k-1}\bar{\alpha}_k = 0$ but we have $\bar{\alpha}_{k-1} = \bar{\alpha}_k = 1$.

Step 6. (P''_R *does not have any solution if* $A \leq 0$) Note that in this case any feasible (S, D) must be such that $D < f(S)$. But then, by argument identical to one presented in step 5a to 5d we can claim that there cannot exist a solution to $\mathcal{P}''_R$.

Step 7. (S^* *can be implemented by a stationary contract*) As $D^* = f(S^*)$,

$$\frac{D^*}{S^*} = \frac{1+\delta p D^*}{1+\delta S^*}.$$

Define

$$\alpha^* := \frac{S^*}{1+\delta S^*} = \frac{D^*}{1+\delta p D^*}.$$

Notice that the stationary sequence $\alpha_t = \alpha^*$ for all t is a solution to $\mathcal{P}'_R$ as it yields $S_1 = S^*$ and the resulting sequence $\{(S_t, D_t)\} = \{(S^*, D^*)\}$ satisfies (10).

Step 8. (*If the original problem* $\mathcal{P}$ *has a solution, then* α^* *is a solution to* $\mathcal{P}$) We now show that if $\mathcal{P}$ has a solution, the optimal contract $\{\alpha^*\}$ satisfies (IC_1^*), (IC_0^*) and all (SE^*)s, and hence it is also a solution to $\mathcal{P}$. We show this in the following three sub-steps:

Step 8a. As the contract is stationary, the agent who privately learns the critical task does not have any deviation that is more profitable than always shirking by doing the critical task only. That is, we must have $u_s^t = U(u_t)$. Hence, the optimal contract $\{\alpha^*\}$ satisfies (IC_1^*).

Step 8b. As $\mathcal{P}'_R$ is a "relaxed" version of $\mathcal{P}$ and $\{\alpha^*\}$ is solution to $\mathcal{P}'_R$, then, for all t, the payoff u^* under the contract $\{\alpha^*\}$ must be at least as large as the payoff u^t under a contract that solves $\mathcal{P}$. Now, as any solution to $\mathcal{P}$ must satisfy (IC_0^*), i.e., it must satisfy $u^t \geq (1-\delta)y$ for all t, we must have $u^* \geq (1-\delta)y$. Hence, $\{\alpha^*\}$ also satisfies (IC_0^*).

Step 8c. Finally, to check that (SE^*)s are satisfied, note that: (i) By definition $\left(s_1, \frac{1-\delta}{\delta}\psi\right) \in \mathcal{E}$. (ii) In the proposed contract, $u^t = u^*$ for all t and $(u^*, 0) \in \mathcal{E}$ by construction given in the proof above. Hence, $\{\alpha_t\} = \{\alpha^*\}$ is a solution to the original problem if it has a solution. ■

Proof of Proposition 2. The proof is given in two steps. In the first step, we take s_1 as given (with $s_1 < s_2$), and derive a necessary and sufficient condition for $\mathcal{P}$ to have a solution. In the second step, we use this condition to prove the proposition.

Step 1. (*A necessary and sufficient condition* on s_1 *for* $\mathcal{P}$ *to have a solution*) We show that for a given $\delta < \delta^*$ problem $\mathcal{P}$ has a solution if and only if

$$\frac{\delta}{1-\delta}\left(1 - \frac{1}{2}p\right) s_1 \geq c_2 - c_1. \tag{12}$$

Step 1a. We first show that (12) is necessary for $\mathcal{P}$ to have a solution. If $\mathcal{P}$ has a solution, by Lemma 6 we know it is stationary: $\alpha_t = \alpha^*$ for all t. Moreover, $\alpha^* < 1$ as we are considering the case where $\delta < \delta^*$. Now, at the solution, the following two conditions must hold: (i) (IC_1^*) in period one holds with equality; and (ii) the following inequality holds:

$$u^{t+1} - s_1 < \frac{1}{2}p\left(U\left(u^{t+1}\right) - s_1\right) \tag{13}$$

for $t = 1$. Otherwise, it would be possible to increase α_1 from α^* (keeping $\alpha_t = \alpha^*$ for $t > 1$) and increase u^1 while preserving (IC_1^*) and all other constraints in $\mathcal{P}$, contradicting the fact that α^* is solution. Now, observe that (13) implies that if under the optimal contract (IC_1^*) in period one is satisfied for $\alpha_1 = \alpha^*$ (which must be the case), then it is also satisfied for $\alpha_1 = 0$, i.e.,

$$(1-\delta)\, s_2 + \delta s_1 \geq (1-\delta)\,(s_2 + c) + \frac{1}{2}p\delta s_1,$$

which is equivalent to (12).

Step 1b. To see the sufficiency of (12), observe that if it is satisfied then clearly a contract in which $\alpha_t = 0$ for all t satisfies (IC_1^*). Moreover, such contract also satisfies (IC_0^*). To see this, observe that IC_0^* is given by

$$u \geq (1-\delta)y \Leftrightarrow (1-\delta)\, s_2 + \delta s_1 \geq (1-\delta)y \Leftrightarrow \frac{\delta}{1-\delta}s_1 \geq c_2,$$

which is implied by (12). Thus, at least the contract in which $\alpha_t = 0$ for all t is feasible, meaning that $\mathcal{P}$ has a solution.

Step 2. (*A necessary and a sufficient condition for $\delta_R < \delta^*$*) First observe that $v \in [s_2 - \psi, s_2)$ when $\delta < \delta^*$, since $v = s_2 - \psi$ when the performance measure must be replaced in every period and $v = s_2$ when the measure is never replaced.

Step 2.1. (Necessary condition) Since $s_1 = v - \frac{1-\delta}{\delta}\psi$, the highest possible value s_1 can take is $s_1 = s_2 - \frac{1-\delta}{\delta}\psi$. From this and Step 1 it follows that $\mathcal{P}$ has a solution (i.e., $\delta_R < \delta^*$) only if

$$\frac{\delta^*}{1-\delta^*}\left(s_2 - \frac{1-\delta^*}{\delta^*}\psi\right) \geq \frac{c_2 - c_1}{1 - p/2}$$

or, equivalently,

$$\frac{\delta^*}{1-\delta^*}(y - c_2) \geq \frac{c_2 - c_1}{1 - p/2} + \psi.$$

Step 2.2 (Sufficient condition) Problem $\mathcal{P}$ has a solution for a given $\delta < \delta^*$ if (12) is satisfied when s_1 is the lowest possible, i.e. when

$$\frac{\delta}{1-\delta}(y - c_2) \geq \frac{c_2 - c_1}{1 - p/2} + \frac{\psi}{1-\delta}. \tag{14}$$

Let $u^1(\widehat{v})$ be the value associated with that solution when $s_1 = \widehat{v} - \frac{1-\delta}{\delta}\psi$ for any $\widehat{v} \in [s_2 - \psi, s_2)$. Since (i) $u^1(\widehat{v})$ is continuous, (ii) $u^1(\widehat{v}) \geq \widehat{v}$ when $\widehat{v} = s_2 - \psi$ and (14) is satisfied, and (iii) $u^1(\widehat{v}) < \widehat{v}$ when $\widehat{v} = s_2$ (recall $\delta < \delta^*$), then there exists at least one $\widehat{v}$ such that $u^1(\widehat{v}) = \widehat{v}$. The value associated with the optimal contract $v = \max\{\widehat{v} \in [s_2 - \psi, s_2) : u(\widehat{v}) = \widehat{v}\}$.

The rest of the proof follows from the fact that when problem $\mathcal{P}$ has no solution then effort in both tasks cannot be elicited (even in period one) and therefore it is optimal for the principal and agent to take their outside options in every period and from Proposition 1. ■

Proof of Proposition 3. The first three steps of the proof argue that the optimal contracting problem is same as $\mathcal{P}$ studied in section 4.2, with suitable reinterpretation of the some notations. The final step completes the proof by using our findings on $\mathcal{P}$.

Step 1. Notice that if $\delta < \delta^*$, in any period, there are three possible action profiles on the equilibrium path: (i) agent exerts effort on both tasks while no measure is adopted, and hence, no information is revealed; (ii) the principal adopts a measure to reveal the critical task, and the agent exerts effort on that task only; and finally, (iii) both parties take their outside options. As before, with a slight abuse of notation, we continue to denote these three cases as $a = N$, R, and O, respectively. Let α^a be the probability of choosing action prfile a in the subsequent period and let (u^a, π^a) be the continuation payoffs where $a \in \{N, R, O\}$.

Step 2. Next, consider the constraints that a contract must satisfy in order to sustain effort on both tasks in a given period when the critical task remains unknown to all. These constraints are identical to their counterpart in section 4 except for the following two differences: (i) in the principal's promise-keeping constraint (PK_P^*) and dynamic enforceability constraint (DE_P^*), the term $\pi^R - \frac{1-\delta}{\delta}\psi$ is replaced by π^R (notice that the use of a performance measure is assumed to be costless); and (ii) the sequential enforceability constraint following replacement of existing performance measure (SE_R^*) is replaced by

$$(SE_C^*) \qquad \left(u^R, \pi^R\right) \in \mathcal{E}_K,$$

where $\mathcal{E}_K$ denotes the set of equilibrium payoffs (for a given δ) when the critical task is publicly known.

Step 3. Now, as discussed in Lemmas 2–5 in section 4, it is routine to check that the following conditions continue to hold even in the current setting: without loss of generality, we can restrict attention to a class of contracts where (i) no bonus is used (i.e., $b = 0$), (ii) the principal's continuation payoff is always 0 (i.e., $\pi^N = \pi^R = \pi^O = 0$), (iii) termination is never used (i.e., $\alpha^O = 0$), and finally, (iv) in the optimal contract, if $\alpha^R > 0$ in any period, then $u^R = \mu y + (1 - \mu)(-z) - c_1$ (i.e., the maximal surplus in the relationship when the critical task is revealed and the agent works on that task only). That is, we may restrict attention to contracts where, in any period, $b = 0$ and

$$w = \begin{cases} y & \textit{if } a = N \textit{ is played} \\ \mu y + (1-\mu)(-z) & \textit{if } a = R \textit{ is played} \end{cases}.$$

Hence, the optimal contracting problem is the same as the principal's program $\mathcal{P}$ studied in section 4.2, except for the following modifications: (i) the parameter p in $\mathcal{P}$ is replaced by μ, (ii) we now denote $1 - \alpha_t$ as the probability that the principal adopts a performance measure to reveal the critical task at the end of period t, given that it has not yet been adopted in the past; and (iii) the agent's continuation payoff $s_1 = \mu y + (1-\mu)(-z) - c_1$.

Step 4. Now consider the characterization of $\tilde{\delta}$. From Step 1 in the proof of Proposition 2, it follows that problem $\mathcal{P}$ has a solution if and only if:

$$\frac{\delta}{1-\delta}\left(1 - \frac{p}{2}\right) s_1 \geq c_2 - c_1. \tag{15}$$

Let $\tilde{\delta}$ be the value of δ for which (15) is binding. Substituting μ for p, we obtain that $\tilde{\delta} < \delta^*$ if and only if:

$$\frac{\delta^*}{1-\delta^*}\left(1 - \frac{\mu}{2}\right) s_1 > \frac{\delta^*}{1-\delta^*}\left(1 - \frac{\mu}{2-\mu\delta^*}\right) s_1 = c_2 - c_1,$$

that simplifies to the condition given in part (iii) (recall that (NSC^*) is binding at δ^*.) The rest of the proof immediately follows from Proposition 1 and Lemmas 6 and 7. ■

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Online Appendix

This appendix is organized as follows. We start by formally stating and proving the claim made in the article that when searching for the optimal contract we can restrict attention, without loss of generality, to contracts where the principal uses the performance measure M_t, along with the output Y_t. We next present the proofs of several lemmas that are omitted in the article as they are primarily technical in nature.

Lemma: *In the model given in Section* 2, *it is (weakly) optimal to tie the agent's bonus to the performance measure* M_t.

Proof. We present the proof for the benchmark case where the performance measure is invariant over time. (The proof for the case with replacement of measures is analogous.)

Step 1. Suppose the bonus pay in the optimal contract is independent of M_t. Fix an arbitrary period t with history h_t such that the optimal contract induces $e_t = 2$ and pays a bonus $b_t = b$ (> 0) if $Y_t = y$. By virtue of being an optimal contract, the associated payoffs satisfy all constraints given in Section 3. Without loss of generality, we assume that the employment relation terminates if the agent is caught shirking.

Step 2. Now consider a new contract that keeps all aspects of the initial contract unaltered, except the following: at every instance where a bonus pay requires $Y_t = y$ (in the original contract), the new contract requires both $Y_t = y$ and $M_t = 1$. As $e_t = 2$ ensures both $Y_t = y$ and $M_t = 1$, the new contract yields the same equilibrium payoff as the initial one. Hence, for any period t, all of the aforementioned constraints remain unaltered, except for the incentive compatibility constraint (IC_1).

Step 3. We claim that (IC_1) gets relaxed under the new contract. To see this, note that (IC_1) under the initial contract is given by

$$(IC_1\text{-}Y) \qquad u \geq (1-\delta)(w - c_1 + \mu b) + \mu\delta u^C,$$

where u^C is the agent's continuation payoff. In contrast, the same constraint under the modified contract is as stated in Section 2:

$$(IC_1) \qquad u \geq (1-\delta)\left(w - c_1 + \frac{1}{2}\mu\theta b\right) + \frac{1}{2}\mu\theta\delta U\left(u^N, \pi^N\right),$$

where u^N and π^N are the continuation payoffs of the agent and the principal, respectively. Since the two contracts are exactly identical except for the fact that in the latter case, the agent's deviation is more likely to get detected (and hence, the agent faces a higher likelihood of termination should he decide to shirk in the future with or without the information on the critical task), we have $u^C \geq U\left(u^N, \pi^N\right)$. As $\theta < 1$, therefore, (IC_1-Y) implies (IC_1). Hence, the new contract is feasible; moreover, it remains feasible under some parameters where the initial contract is not. ■

Proof of Lemma 2. Consider a relational contract where, for some period t and history h_t, the critical task for the period is not known to the agent and the payoff profile (u, π) is sustained by effort in both tasks ($e = 2$) and bonus $b \neq 0$ in period t. We construct another contract where, in the same period and for the same history, (u, π) is sustained by $e = 2$ and supported by $b = 0$.

Step 1. (*If* (u, π) *is supported by a contract with* $b < 0$, *then it is supported by a contract with* $b = 0$.) Suppose (u, π) is supported by a contract in which $w_t = w$ and $b_t < 0$. Consider now a new contract (strategy) with wage and bonus (w', b') in period t, where $w' = w + b$ and $b' = 0$. All other aspects of the contract remain the same, including past and future play. Observe that the new contract keeps (PK_P^*) and (PK_A^*) unaffected as $w' + b' = w + b$. Hence, the players' payoff remains (u, π). We claim that this contract satisfies all other constraints as well, and hence, gives a payoff (u, π) in the game starting from period t by inducing $e = 2$ in that period.

Step 1a. Notice the following about the constraints in period t: The new contract makes (IC_0^*), (IC_1^*) and (DE_A^*) slack and (DE_P^*) remains satisfied as $\pi^a \geq 0$ for all $a \in \{N, O\}$ and $\pi^R - (1-\delta)/\delta \geq 0$. Finally, this change also preserves the (IC_1^*) for all periods prior to t, ensuring that past play continues to be consistent with equilibrium (and hence the agent did not have any incentives to deviate in the past and learn the identity of the task). To see this, observe that since the PK_A^* is preserved, the (IC_1^*) of each one of the periods until the last replacement of the performance measure is automatically satisfied. Regarding the (IC_1^*) of the periods from the last replacement of the performance measure, observe that under the original contract:

(16)
$$U(u,\pi) = \max \left\{ \begin{array}{l} (1-\delta)(w+b-c_2) + \delta \left(\alpha^R u^R + \sum_{a \in \{N,O\}} \alpha^a U(u^a, \pi^a) \right), \\ (1-\delta)(w+pb-c_1) + p\delta \left(\alpha^R u^R + \sum_{a \in \{N,O\}} \alpha^a U(u^a, \pi^a) \right) \end{array} \right\},$$

and that the corresponding payoff under the new contract, denoted here by U', is obtained by substituting w and b in these expressions by b' and w', respectively. Clearly, with the proposed change in the contract, the first element (inside the curly brackets) remains the same and the second becomes smaller. This implies $U' \leq U(u,\pi)$. Moreover, since (16) holds for any period in which $a = N$, and

$$U(u,\pi) = \alpha^R u^R + \alpha^N U(u^N, \pi^N) + \alpha^O U(u^O, \pi^O)$$

in any period in which $a = O$, then for any τ and $a \in \{N, O\}$, $U(u_\tau, \pi_\tau)$ is non-decreasing in $U(u_\tau^a, \pi_\tau^a)$. Thus, $U_\tau' \leq U_\tau$ for all period $\tau \leq t$ since the last replacement of the performance measure. Thus, in any period prior to t, the agent's payoff on-the-equilibrium path remains the same and the payoff from deviating does not increase.

Step 2. (*If* (u,π) *is supported by a contract with* $b > 0$, *then it is supported by a contract with* $b = 0$.) Suppose now that (u,π) is supported by a contract in which $b > 0$. We show, again by construction, that it can also be supported by a contract in which $b = 0$.

Step 2a. Define

$$b^R = b \times \frac{\pi^R - \frac{1-\delta}{\delta}\psi}{\alpha^N \pi^N + \alpha^R \left(\pi^R - \frac{1-\delta}{\delta}\psi\right) + \alpha^O \pi^O},$$

and

$$b^a = b \times \frac{\pi^a}{\alpha^N \pi^N + \alpha^R \left(\pi^R - \frac{1-\delta}{\delta}\psi\right) + \alpha^O \pi^O},$$

for all $a \in \{N, O\}$. By construction, $\alpha^N b^N + \alpha^R b^R + \alpha^O b^O = b$. Furthermore,

$$0 \leq b^R \leq \frac{\delta}{1-\delta}(\pi^R - \frac{1-\delta}{\delta}\psi) \text{ and } 0 \leq b^a \leq \frac{\delta}{1-\delta}\pi^a \tag{17}$$

for all $a \in \{N, O\}$, where the second inequality in each of these two sets of inequalities follows from (DE_P^*).

Step 2b. Now, in the new contract, set the bonus equal to zero and adjust the continuation play as follows. First, suppose $\left(u^N, \pi^N\right)$ and $\left(u^R, \pi^R\right)$ are supported, respectively, by wages w^N and w^R. Now set the new wages

$$w^{a\prime} = w^a + \frac{b^a}{\delta}$$

for $a = N, R$. The principal's continuation payoffs become

$$\pi^{a\prime} = \pi^a - \frac{1-\delta}{\delta} b^a$$

for $a = N, R$. Observe that, by (17), $w^{a\prime} \geq w^a$, $\pi^{N\prime} \geq 0$ and $\pi^{R\prime} - \frac{1-\delta}{\delta}\psi \geq 0$, which ensures that when the continuation play calls for $a = N$ or $a = R$ both the agent and the principal will again accept the contract. Second, consider $\left(u^O, \pi^O\right)$. If $\pi^O = 0$, then nothing needs to be done in the new contract and we continue with the same continuation play dictated by $\left(u^O, \pi^O\right)$. If, otherwise, $\pi^O > 0$, then we know that players will engage in the relationship at some point. Let w^O be the wage the principal pays the agent the first time the relationship resumes, and assume that the parties have taken the outside option t periods before that. Note that when the relationship resumes, the principal's payoff is π^O/δ^t. Now let

$$w^{O\prime} = w^O + \frac{1}{\delta^{t+1}} b^O,$$

and this gives

$$\pi^{O\prime} = \delta^t \left[\frac{\pi^O}{\delta^t} - (1-\delta)\frac{1}{\delta^{t+1}} b^O\right] = \pi^O - \frac{1-\delta}{\delta} b^O.$$

Once again, by (17), $w^{O\prime} \geq w^{O}$ and $\pi^{O\prime} \geq 0$, which implies that both the principal and the agent accept the contract if continuation play calls for $\left(u^{O}, \pi^{O}\right)$. Hence, continuation play is again an equilibrium for $a \in \{N, R, O\}$.

Step 2c. Next, note that this change leaves (PK_P^*) and (PK_A^*) unchanged. Regarding (IC_1^*), under the new contract it is given by

$$(18) \quad u \geq (1-\delta)(w-c_1) + \frac{1}{2}p\delta\left(\alpha^N U\left(u^{N'}, \pi^{N'}\right) + \alpha^R u^{R'} + \alpha^O U\left(u^{O'}, \pi^{O'}\right)\right).$$

Since under the new contract, in any future periods, only the wage w^a is affected, we obtain that $U(u^{a\prime}, \pi^{a\prime}) = U(u^a, \pi^a) + (1-\delta)b^a/\delta$ for all $a \in \{N, O\}$ and $u^{R'} = u^R + (1-\delta)b^R/\delta$. Using this and the fact that $b = \sum \alpha^a b^a$, it is easy to see that (18) is equivalent to the (IC_1^*) in the original contract.

Step 2d. Finally, (IC_1^*) for all periods prior to t is also satisfied under the new contract. Under the original contract, $U(u, \pi)$ is again as stated in (16). The corresponding payoff under the new contract is obtained by substituting, in that expression, b with 0, u^R with $u^{R'}$, and $U(u^a, \pi^a)$ with $U\left(u^{a'}, \pi^{a'}\right)$ for all $a \in \{N, O\}$. It is easy to see that $U' = U(u, \pi)$. Since, as shown above, for any period τ, $U(u_\tau, \pi_\tau)$ is non-decreasing in $U(u_\tau^a, \pi_\tau^a)$ for all $a \in \{N, O\}$, it follows that for any period $\tau \leq t$, $U'_\tau \leq U_\tau$. Hence, in any period prior to t, the agent's payoff on-the-equilibrium path remains the same and the payoff from deviating does not increase. This observation completes the proof. ■

Proof of Lemma 3. Consider a relational contract where, for some period t and history h_t, the critical task for the period is not known to the agent and the payoff profile (u, π) is sustained by effort in both tasks ($e = 2$), wage w and bonus $b = 0$ in period t. There is no loss of generality by Lemma 2 in assuming that $b = 0$. Let w^a be the next period wage that supports the continuation payoffs (u^a, π^a) for all $a \in \{N, R\}$ in this equilibrium. Similarly, let w^O denote the wage paid the first time the relationship resumes (in case it resumes) that supports the continuation payoffs (u^O, π^O).

Next consider a strategy that is identical to the above equilibrium, except for the following changes in the current and next period wages. For all $a \in \{N, R\}$, let the new wage in the continuation game be

$$w^{N\prime} = w^N + \frac{\pi^N}{1-\delta} \text{ and } w^{R\prime} = w^R + \frac{1}{1-\delta}\left(\pi^R - \frac{1-\delta}{\delta}\psi\right).$$

If $\pi^O > 0$, then the players will engage in the relationship at some point in the future. Suppose that the parties take the outside option t periods before engaging again in the relationship. Note that when the relationship resumes, the principal's payoff is π^O/δ^t. In this case, let

$$w^{O\prime} = w^O + \frac{\pi^O}{\delta^t(1-\delta)}.$$

Finally, let the new current wage be

$$w' = w - \frac{\delta}{1-\delta}\left(\alpha^N\pi^N + \alpha^O\pi^O + \alpha^R\left(\pi^R - \frac{1-\delta}{\delta}\psi\right)\right).$$

Under these changes, $\pi^{a\prime} = 0$ for all $a \in \{N, O\}$, $\pi^{R'} - (1-\delta)\psi/\delta = 0$, and all the relevant constraints remain satisfied. It is easy to see that (PK_P^*) and (PK_A^*) are preserved. Constraints (DE_P^*) and (DE_A^*) are automatically satisfied since $b = 0$. Also, the proposed changes increase the agent's continuation payoff and relax (IC_1^*). More specifically, the (IC_1^*) under the original contract is given by

$$u \geq (1-\delta)(w - c_1) + \frac{1}{2}p\delta\left(\alpha^N U\left(u^N, \pi^N\right) + \alpha^R u^R + \alpha^O U\left(u^O, \pi^O\right)\right).$$

Under the new contract, the left-hand side of the constraint remains the same since PK_A^* is preserved. The right-hand side is obtained by replacing w with w', $U(u^a, \pi^a)$ with $U\left(u^{a'}, \pi^{a'}\right) = U(u^a, \pi^a) + \pi^a$ for all $a \in \{N, O\}$, and u^R with $u^{R'} = u^R + (\pi^R - \frac{1-\delta}{\delta}\psi)$. Hence, it is equal to that under the original contract minus

$$\delta(1 - \frac{1}{2}p)\left(\alpha^N\pi^N + \alpha^O\pi^O + \alpha^R\left(\pi^R - \frac{1-\delta}{\delta}\psi\right)\right).$$

Finally, under the proposed changes, the (IC_1^*) constraint for all periods prior to t remains satisfied, ensuring that past play continues to be consistent with equilibrium. To see this, observe that under the original contract

$$U(u,\pi) = \max \left\{ \begin{array}{l} (w-c_2)(1-\delta) + \delta \left(\alpha^R u^R + \sum_{a=N,O} \alpha^a U(u^a,\pi^a) \right), \\ (w-c_1)(1-\delta) + \delta p \left(\alpha^R u^R + \sum_{a=N,O} \alpha^a U(u^a,\pi^a) \right) \end{array} \right\}.$$

The corresponding payoff under the new contract, U', is obtained by replacing in this expression, w with w', $U(u^a,\pi^a)$ with $U\left(u^{a'},\pi^{a'}\right)$ for all $a \in \{N,O\}$, and u^R with $u^{R'}$. The first element inside the curly brackets remains the same under the new contract. The second element is the same minus

$$\delta(1-p)\left(\alpha^N \pi^N + \alpha^O \pi^O + \alpha^R(\pi^R - \frac{1-\delta}{\delta}\psi)\right),$$

which implies that $U' \leq U(u,\pi)$. Since, as shown in the proof of Lemma 2, for any period τ, $U(u_\tau,\pi_\tau)$ is non-decreasing in $U(u_\tau^a,\pi_\tau^a)$ for $a = N, O$, it follows that for any period $\tau \leq t$, $U'_\tau \leq U_\tau$. Hence, in any past period, the agent's payoff on-the-equilibrium path remains the same and the payoff from deviation does not increase.

■

Proof of Lemma 4. Suppose there is an optimal contract that generates positive joint surplus. Such contract cannot begin with $a = O$, since a contract beginning with period two of that contract would have a higher associated payoff. Let t be the first period in which $\alpha^O > 0$ and let u be the agent's payoff at the beginning of that period. By Lemmas 2 and 3, we can restrict attention without loss of generality to contracts where, in any period, $b = 0$ and the principal's continuation payoff (net of costs of replacing the performance measure) is zero. (Note that in such contracts, in any period, $w = y$ if $a = N$ is played and $w = y - \psi/\delta$ if $a = R$ is played.) Hence, if u is sustained by playing $a = N$ in period t, (PK_A^*) implies that

$$u = (1-\delta)(y-c_2) + \delta\left(\alpha^N u^N + \alpha^O u^O + \alpha^R u^R\right),$$

and if it is sustained by playing $a = R$, (PK_A^*) implies that

$$u = (1-\delta)(y - \psi/\delta - c_2) + \delta\left(\alpha^N u^N + \alpha^O u^O + \alpha^R u^R\right),$$

where u^a for $a \in \{N, R, O\}$ are the appropriate continuation payoffs. The analysis that follows is valid for either case.

When the continuation play calls for exit, note that

$$u^O = \delta u_c,$$

where u_c is the agent's expected continuation payoff. Now consider the following alternative strategy. The new strategy is the same as that in the optimal contract we are considering here, except that in period t, if continuation play calls for exit (which happens with probability α^O), then the game continues in the following way: with probability $1-\delta$, players terminate the relationship forever; and with probability δ, the game continues with u_c (which could be sustained by randomization).

Under this alternative strategy, the agent's payoff (following the contingency that exit is called for in the original equilibrium) is given by

$$u^{O\prime} = \delta u_c = u^O.$$

This implies that (PK_A^*) is preserved and the agent's continuation payoff at the beginning of the period under the alternative strategy, u', satisfies $u' = u$. In addition,

$$U\left(u^{O\prime}\right) = \delta U(u_c) = U\left(u^O\right).$$

(We omit the principal's continuation payoffs π^a for $a = N, O$ as argument of U since they are zero in the contracts considered in this proof.) Since $u' = u$ and $U\left(u^{O\prime}\right) = U\left(u^O\right)$, clearly (IC_1^*) is preserved under the alternative strategy.

Finally, if u is sustained by playing $a = R$ in period t, then for all periods prior to t, the (IC_1^*) constraint must be satisfied since $u' = u$. If instead u is sustained by playing $a = N$ in period t, then following an approach identical to that used in the proof of Lemmas 2 and 3, we obtain again that for all the periods prior to t the (IC_1^*) constraint is also satisfied. Therefore, the alternative strategy is also an equilibrium that gives the agent the same payoff as that originally considered. This implies that if the equilibrium asks players to take their outside options in the next period, we can replace this with a probability of permanent exit. Finally, in an optimal contract, permanent exit cannot be played with a positive probability since it is dominated by replacement of the performance measure. Thus, in an optimal contact, $\alpha^O = 0$ in all periods. ■

Proof of Lemma 5. Suppose there is an optimal contract that generates positive surplus. Such contract must begin with $a = N$. Let t be the first period in which $\alpha^R > 0$, and let the agent's continuation payoff at the beginning of that period be u. By Lemmas 2-4, we can restrict attention without loss of generality to contracts with no bonuses, in which the principal's continuation payoff (net of costs of replacing the performance measure) are zero, and where players do not take their outside option. Hence, since u is sustained by playing $a = N$ in period t, (PK_A^*) implies that

$$u = (1-\delta)(y - c_2) + \delta\left(\alpha^R u^R + \left(1-\alpha^R\right)u^N\right),$$

where u^R and u^N are the continuation payoffs.

Suppose $u^R < v - (1-\delta)\psi/\delta =: s_1$. Then, we can consider an alternative strategy profile in which u^R is replaced with

$$u^{R\prime} = s_1.$$

Under this new new strategy, the agent's continuation payoff at the beginning of period t is

$$u' = (1-\delta)(y-c_2) + \delta\left(\alpha^R s_1 + \left(1-\alpha^R\right)u^N\right) = u + \delta\alpha^R\left(s_1 - u^R\right) > u. \tag{19}$$

In addition, (IC_1^*) in period t is satisfied. To see this note that under the original contract (IC_1^*) in period t can be written as:

$$\left(\alpha^R u^R + \left(1-\alpha^R\right)u^N\right) + \frac{1}{2}p\left(\left(1-\alpha^R\right)\left(u^N - U\left(u^N\right)\right)\right) \geq \frac{1-\delta}{\left(1-\frac{1}{2}p\right)\delta}\left(c_2 - c_1\right). \tag{20}$$

Following the change, (IC_1^*) in period t can be written as:

(21)
$$\left(\alpha^R s_1 + \left(1-\alpha^R\right) u^N\right) + \frac{1}{2}p\left(\left(1-\alpha^R\right)\left(u^N - U\left(u^N\right)\right)\right) \geq \frac{1-\delta}{\left(1-\frac{1}{2}p\right)\delta}\left(c_2 - c_1\right).$$

Since (20) is satisfied and $s_1 > u^R$, then (21) must also be satisfied. We next show that the proposed change also relaxes (20) for all $\tau < t$, so that the agent does not deviate in any past period under the new strategy. In what follows, let u_τ denote the agent's payoff in period τ, u'_τ the same payoff under the new strategy, and $\Delta = \delta\alpha^R\left(s_1 - u^R\right)$, i.e. Δ is the change in the agent's payoff in period t (see 19). Thus, $u'_t = u_t + \Delta$. Moreover, since period t is the first in which $\alpha^R > 0$, we can write

$$u_{t-k} = (1-\delta)\left(y - c_2\right) + \delta u_{t-k+1}$$

and

$$u'_{t-k} = (1-\delta)\left(y - c_2\right) + \delta u'_{t-k+1},$$

for all $k = 1, .., t-1$. This means that $u'_{t-k} = u_{t-k} + \delta^k\Delta$, or, equivalently,

$$u'_{t-k} - u_{t-k} = \delta^k\Delta. \tag{22}$$

Next observe that

$$U(u_t) = \max\left\{\begin{array}{c} (1-\delta)(y-c_2) + \delta\left(\alpha^R u^R + \left(1-\alpha^R\right)U(u^N)\right), \\ (1-\delta)(y-c_1) + \delta p\left(\alpha^R u^R + \left(1-\alpha^R\right)U(u^N)\right) \end{array}\right\}$$

and that $U(u'_t)$ is the same except that u^R is replaced with s_1. It follows that $U(u'_t) - U(u_t) \leq \Delta$. Moreover,

$$U(u_{t-k}) = \max\left\{(1-\delta)(y-c_2) + \delta U(u_{t-k+1}),\, (1-\delta)(y-c_1) + \delta p U(u_{t-k+1})\right\}$$

and $U(u'_{t-k})$ can be obtained by replacing $U(u_{t-k+1})$ with $U(u'_{t-k+1})$ in this expression. Hence,

$$U(u'_{t-k}) - U(u_{t-k}) \leq \delta^k\Delta. \tag{23}$$

Next, observe that IC_1^* in any period $t-k-1$ under the original strategy can be written as

$$(1-\delta)(y-c_2)+\delta u_{t-k} \geq (1-\delta)(y-c_1)+\delta pU(u_{t-k+1}) \tag{24}$$

and under the new strategy it can be written as

$$(1-\delta)(y-c_2)+\delta u'_{t-k} \geq (1-\delta)(y-c_1)+\delta pU(u'_{t-k+1}). \tag{25}$$

Since the former is satisfied and by (22) and (23), $u'_{t-k}-u_{t-k} \geq U(u'_{t-k})-U(u_{t-k})$, the latter must also be satisfied. Finally, observe that the proposed change of strategy increases the agent's payoff at the beginning of the game. This shows that in any optimal contract $u^R = v-(1-\delta)\psi/\delta$ in the first period in which $\alpha^R > 0$. Applying a similar procedure recursively we obtain that $u^R = v-(1-\delta)\psi/\delta$ the second time $\alpha^R > 0$, and in any other period in which $\alpha^R > 0$. ■

Proof of Lemma 7. **Step 1.** When the critical task is publicly known, we can restrict attention to stationary contracts (Levin, 2003). That is, we can assume that the principal offers the same contract and the agent chooses the same effort level every period. There are three possible actions profiles that could be supported in an optimal stationary contract: (i) the agent exerts effort on both tasks; (ii) the agent exerts effort on the critical task only; and (iii) both players exit the relationship and take their outside option in each period. Recall that by Assumption 1A (iii), it is never optimal for the relationship to have the agent exert effort only on the non-critical task.

Step 2. We begin by deriving the conditions under which effort $e=2$ in every period can be sustained in (a stationary) equilibrium. Let (w,b) be the wage and bonus in a stationary contract. The bonus is paid whenever $Y_t = y$ and $M_t = 1$. As transfers between players are frictionless, without loss of generality, we assume that in the optimal contract, the principal extracts all surplus. Thus, the agent's individual rationality constraint binds and it is given as:

$$w+b-c_2 = 0. \tag{26}$$

The agent's incentive compatibility constraint is:

$$(1-\delta)(-c_2+b) \geq \max\{(1-\delta)(-c_1+\mu b),\ 0\},$$

or,

$$b \geq \max\left\{\frac{c_2-c_1}{1-\mu}, c_2\right\} = \frac{c_2-c_1}{1-\mu}, \tag{27}$$

as $(c_2-c_1)/(1-\mu) > c_2$ by Assumption 1A (ii). Now, given (26), on the equilibrium path, the principal earns the entire surplus. So, for the principal to not renege on the bonus, we must have the following dynamic enforceability constraint:

$$\delta(y-c_2) \geq (1-\delta)\, b. \tag{28}$$

Hence, the optimal contract sustaining $e=2$ must be a solution to the following program:

$$\mathcal{P}_E:\ \max_{w,b}\ \hat{\pi}_t = y - c_2\ \ s.t.\ (27),\ (28)\ \text{and}\ (26).$$

Note that by combining (27) and (28), we get that the necessary and sufficient condition to sustain $e=2$ is:

$$\frac{\delta}{1-\delta}(y-c_2) \geq \frac{c_2-c_1}{1-\mu}. \tag{29}$$

This condition is also sufficient because it allows the implementation of $e=2$ through the following feasible contract:

$$b = \frac{c_2-c_1}{1-\mu},\ \text{and}\ w = c_2 - b.$$

Thus, $\overline{\delta}$ is the value of δ for which (29) is satisfied with equality.

Step 3. Consider now equilibria in which the agent works on the critical task only. The analysis is identical to the analysis of the case of $e=2$, but with two exceptions. First, now the bonus is paid whenever $M_t = 1$. This is because M_t is a perfect measure of whether or not the agent works on the critical task. And since the only relevant deviation for the agent is to not work at all, the agent's incentive compatibility constraint boils down to $b \geq c_1$. Second, the per-period surplus is now $\mu y - c_1$ and hence, the principal's dynamic enforceability constraint becomes $\delta\left(\mu y-c_{1}\right) \geq(1-\delta) b$. Combining the two, we can derive the necessary and sufficient condition for sustaining effort in the critical task only:

$$\frac{\delta}{1-\delta}\left(\mu y-c_{1}\right) \geq c_{1}. \tag{30}$$

This condition is sufficient as it allows for the following feasible contract that implements effort in the critical task only on the equilibrium path: $b=c_{1}$ and $w=0$. Thus, $\underline{\delta}$ is the value of δ for which (30) is satisfied with equality. ■