RULEBOOKS IN RELATIONAL CONTRACTS*

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Abstract. Firms can accrue large benefits by fostering worker initiative, but standardized work rules are still widely used. We present a model of relational incentives where the use of rules fluctuates over time as the firm faces shocks to its credibility. Worker initiative in adapting to local information can ensure production efficiency but also requires strong incentives. As shocks weaken relational incentives, the firm may adopt rules that yield satisfactory (though suboptimal) performance. Rules help the relationship survive the shocks, but the relationship becomes less efficient in the future. While the relationship may recover, its ability to weather future shocks deteriorates.

1. Introduction

Firms often refrain from setting detailed work rules and encourage worker initiative by imposing little or no requirements on how the workers should perform their jobs. Nordstrom, a leading departmental store chain based in the United States, offers a classic example of such lack of rules. For years Nordstrom's employee handbook simply stated that "Our number one goal is to provide outstanding customer service. Set both your personal and professional goals high [....] Nordstrom Rules: Rule #1: Use good judgment in all situations. There will be no additional rules" (Spector and McCarthy, 2012).

By leaving the rules vague, the firm effectively empowers the workers to exercise initiative, and such initiative is particularly beneficial when the workers need to acquire local information and cater to their clients' idiosyncratic needs that the top management may not be

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aware of (Rantakari, 2012). Arguably, many leading service-sector firms can attribute their success to employee empowerment and "empowerment in many ways is the reverse of doing things by the book" (Zemke and Schaaf, 1989, p. 68).

Though rules may stifle initiative, rules are still widely used. Even the organizations that emphasize worker discretion as a key ingredient for their overall success go through phases where the employees are urged to follow standardized work processes. In particular, during a time of crisis a common reaction of the top management is to take control by removing discretions given to the workers at the lower levels of the organizational hierarchy, asking them to instead follow new rules and standardized work processes (Slatter and Lovett, 1999).

However, organizations also go through phases where employee guidelines are relaxed and an enhanced emphasis is placed on worker initiative. In fact, the extent of worker initiative within an organization tends to fluctuate over time, and such oscillations are typically embedded in a broader swing between centralization and decentralization (Bartlett and Ghoshal, 1998). For example, through the 1970s, Hewlett-Packard (HP) was known for its highly decentralized structure ("The HP Way") where divisions were highly autonomous and employees were strongly encouraged to take initiative on actions that advanced the firm's goals. But in the early 1980s, the company found itself in turmoil and, in response, adopted a more centralized mode of operation. Nevertheless, within a decade of adopting this centralized structure, HP again moved back to a decentralized structure, and in subsequent years, the company has continued to go back and forth between the two organizational modes (Nickerson and Zenger, 2002).[1]

The goal of this article is to study the dynamics of work rules and worker initiative in long-term employment relationships. We explore when worker initiative should be encouraged, when it should give way to standardized work processes, and what the implications are of adopting standardized rules for the relationship's future. We show that rules can help the relationship survive shocks to its value (e.g., erosion of value due to financial crisis, shifts in demand or technological disruptions), but once such rules are put in place, they may

[1]Ford Motor Company also vacillated between these two modes in the 1990s, moving from a highly decentralized structure to a centralized one in order control runaway costs. However, it subsequently chose to decentralize globally so as to adapt more efficiently to local tastes and circumstances (Nickerson and Zenger, 2002). Yahoo! went through a similar process in the late 2000s. After a failed merger attempt with Microsoft, Yahoo!'s stock price plunged, and the company recentralized its structure in a attempt to reduce costs (Hill, Jones, and Schilling, 2014; p. 404). Some scholars describe these fluctuations between centralization and decentralization as "time-honored" cycles due to their prevalence across industries and over time (Eccles and Nohria, 1992).

critically change the nature of the on-going relationship by making it less efficient and more fragile to future shocks.

Our analysis is based on a key observation: Effective fostering of worker initiative requires trust. A firm that encourages its workers to have initiative must offer strong incentives to ensure that they exert consummate effort to advance the firm's objectives and do not misuse their autonomy (Milgrom and Roberts, 1988, 1992; Jensen and Meckling, 1992). And the workers must also trust the firm to acknowledge and reward their initiative, as court-enforceable contracts may not be feasible in such settings. Indeed, when worker initiative and local information is critical for production, verifiable measures of output are often elusive and incentives may be given through relational incentive contracts. The strength of the relational incentives depends on how much the employees can trust their firm.

But trust can also evolve over time. A firm's credibility in a relational contract depends on the future surplus generated in the employment relationship (Levin, 2003; also see Malcomson, 2012, for a survey). If economic shocks diminish the value of the relationship for the firm, trust erodes and incentive weakens, making it harder for the firm to foster initiative. The advantage of rules and standardized processes is that they help ensure adequate performance when strong incentives that could induce a superior performance (through worker initiative) are infeasible. Once the shocks pass, the firm may again encourage initiative by giving discretion back to its workers, but as we argue below, the adoption of work rules compromises the future of the employment relationship.

We formalize this argument by using a model of a long-term employment relationship between a firm and a liquidity-constrained worker. In every period, the worker privately takes an action in order to perform his job. Production efficiency requires the worker to adapt his action to the underlying circumstances so as to guarantee a high output. However, the firm can also put standardized work processes in place that can yield high output with some probability even if the worker ignores the underlying circumstances and simply takes an action per the standard rules. Such a "rigid" action (i.e., doing things by the rulebook) is less costly to the worker than the efficient "adaptive" one, and once the rigid action is made available, the worker can always use it in the future, even when the costly adaptive action might be called for.

Incentives are relational as the firm promises the worker a discretionary bonus tied to his performance. But, as in Li and Matouschek (2013), in every period, the firm may face a liquidity shock and fail to pay the worker due to a lack of funds. These shocks are assumed to be privately observed by the firm. They may stem from volatility in credit that the firm needs in order to pay its workers, or unexpected arrival of business opportunities that require moving funds intended for the payroll to new investments.

We show that the optimal relational contract gives rise to a dynamics that goes through three distinct phases. At the beginning of the relationship, the firm incentivizes the worker to take initiative and asks him to take the adaptive action, as the future surplus is large and trust is high. Moreover, the firm earns the maximum feasible payoff by appropriating all rents. However, if there is a liquidity shock, the firm cannot pay the worker and is forced to renege on its promise. Since the shock is privately observed by the firm, the worker must punish the firm when it fails to pay, and the firm is required to transfer a part of the future surplus to the worker. As a result, the firm's stake in the relationship is reduced and trust depletes. If the shocks are not too severe, the firm continues to encourage worker initiative. Once the shock passes and the firm makes good on its promise, the relationship recovers completely: the players' payoffs immediately revert back to what they were at the beginning of the relationship.

However, if there are too many consecutive shocks, trust erodes considerably and the relationship moves to its second phase. At the beginning of this phase, the optimal contract calls for standardization. The firms puts in place a standardized work process and the worker is asked to take the rigid action. Since the rigid action is less costly to the worker than the adaptive one, it can be induced with relatively weaker incentives. Once the shock passes and the firm pays its promised reward, trust is restored and discretion is given back to the worker. Hence, as trust evolves in response to shocks, the firm oscillates between allowing worker initiative and requiring standardized work rules.

But once standardization is put in place, the nature of the relationship changes: the relationship becomes less efficient and more vulnerable to future shocks. After the introduction of standardized processes, it becomes more difficult to encourage the worker to take initiative in the future. When discretion is given back to the worker, he can now deviate and take the cheaper rigid action (instead of the adaptive one, which is more costly), since he now knows that the rigid action might still yield a high output. Thus, the moral hazard problem aggravates, and in order to induce the adaptive action, the firm must offer rents to the worker. As the worker earns a rent, the firm's value of the relationship goes down. Consequently, the worker's trust in the firm deteriorates, and so does the relationship's ability to endure future shocks. It becomes more likely that an inefficient action (i.e., either the rigid action or termination) will be used if shocks arise in the future, and the joint surplus in the relationship shrinks even after the relationship recovers from the current shock.

Further shocks, if sufficiently severe, move the relationship to its final phase, where the firm's value of the relationship becomes so low that the firm cannot even credibly offer the incentives needed to elicit the rigid action. At this point, the relationship is terminated.

The dynamics of the optimal contract brings to the fore two novel aspects of standardized work rules. First, though rules help the relationship in times of stress, the resulting strain imposes a cost on the future relationship. Even after trust is regained and discretion is given back to the worker, the relationship continues to bear the scars of past shocks and never recovers completely.

Second, in response to the shocks, the firm may be compelled to use standardized work rules (i.e., the rigid action) when the strong incentives needed to encourage employee initiative are no longer feasible. However, standardization may also be called for as a precautionary measure, even when incentives for worker initiative (i.e., the adaptive action) are still feasible. This is due to the fact that shocks are more damaging to the relationship when the adaptive action is used. Since the adaptive action requires stronger incentives, the firm must promise a larger reward, and this promise is only credible if the firm is punished severely if it reneges. Thus, when shocks arise, the firm's stake in the relationship erodes faster, and so does the worker's trust in the firm. Standardization slows the relationship's decay caused by future shocks because the rigid action requires weaker incentives; for such incentives to be credible, punishments need not be too harsh. As a result, the relationship can survive a longer spell of consecutive shocks before it must face termination.

While we illustrate the long-term implications of standardization by using a model of employment relationship, one may consider several other contractual settings where similar dynamics can emerge. Indeed, a key aspect of our argument (i.e., the erosion of the relationship's value may necessitate the adoption of rules) is applicable in a wide range of environments. For example, in relationships between firms such as supply chains and joint ventures, production efficiency may require that the parties have flexibility to respond to local information. Consequently, it may not be optimal to stipulate rules if the parties can be incentivized to put in consummate effort and respond to underlying circumstances appropriately. But if the value of the relationship decreases, incentives for consummate effort may not be feasible, and the parties may optimally stipulate rules that can still elicit a moderate level of effort and also arrest the decay of the relationship. Similarly, a regulatory agency in charge of overseeing a firm's product safety may allow the firm considerable discretion on product design but impose large penalties if the product harms the consumers. But incentives through such large penalties may be feasible only if the regulator-firm relationship is sufficiently valuable as otherwise the firm may be liquidity constrained. Hence, if the value of the relationship depletes, the regulator may stipulate minimum safety standards that could be enforced with weaker incentives.

In all such settings, adoption of rules can be an effective way to ensure that all parties deliver a minimum level of performance as the relationship becomes less valuable. But, our findings highlight a long-term cost of introducing such standardized processes—once those involved learn that following these processes is an acceptable compromise, it becomes harder to incentivize them to go beyond the rules in the future.

Related literature: The extant literature on organizational design has largely focused on the trade-offs between adaptation and coordination. It has long been recognized that decentralization can enhance efficiency as economic agents adapt their actions to decision-relevant information that resides locally (Hayek, 1945). Centralization, on the other hand, facilitates coordination between different divisions of an organization and streamline production (Chandler, 1977). Several scholars have studied how decision rights should be allocated within an organization in order to balance this trade-off and, relatedly, how the junior managers may be incentivized to accurately report their local information to the top management (Aghion and Tirole, 1997; Alonso, Dessein, and Matouschek, 2008; also see Gibbons, Matouschek, and Roberts, 2012, for a survey and Bowen and Lawler, 1992, for a comparison of the rule-based and worker-empowerment approaches in the service-sector industries). Our article contributes to this literature, but we abstract away from the issues of coordination and communication. Instead, we focus on a different trade-off: we argue that the adoption of standardized work rules during a phase of centralization can shield a firm in times of crisis but compromises its future performance even after reverting to a decentralized mode.

Our findings relate to a large literature on organizational rigidities that both economists and management scholars have contributed to. It is argued that organizational rigidities may stem from several sources: the use of work routines that minimize misunderstanding and facilitate coordination (Nelson and Winter, 1982), adherence to norms meant to guide behavior and reaffirm reputation in unforeseen circumstances (Kreps, 1990), and agents' incentives to exploit existing business opportunities rather than to explore new ones so as to protect their current rents (Holmström, 1989; Henderson, 1993; Schaefer, 1998; also see Garicano and Rayo, 2016, for a survey). In contrast, our article offers a novel argument on how a firm's incentive structure may coevolve with its organizational design and make it harder for the firm to adapt its established routines and culture in the face of environmental changes. In fact, there is little work on the interplay between an organization's design and its culture, but one recent exception is Besley and Persson (2017). In their framework, the organizational culture is reflected by the relative sizes of different groups of managers where members in each group hold a set of common values. They show how the social identities of overlapping generations of managers can evolve with the organization's decentralization decision, leading to a co-movement of organizational design and its culture.

Our paper also contributes to a growing literature on the dynamics of employment relationships. Models of relational contracts have been used to show how relationships may improve over time as parties learn to coordinate more effectively (Watson, 1999, 2002; Chassang, 2010; Halac, 2014). Cooperation may also deteriorate due to a worsening production environment (Garrett and Pavan, 2012; Halac and Prat, 2016) or inefficient allocation of authority that emerges as a compromise for past events (Li, Matouschek, and Powell, 2017). Finally, relationships can cycle between phases of reward and punishment when parties may have private information (Li and Matouschek, 2013; Zhu, 2013; and Fong and Li, 2017). In our model, the relationship also oscillates between using an adaptive action and a rigid one, but the rigid action ushers in a structural change when it is first introduced; once this action is made available, the future surplus in the relationship is irrecoverably compromised.

It is worth noting that the tradeoff that we explore here between rules and worker initiative presents an interesting contrast to Bernheim and Whinston's (1998) analysis of "strategic ambiguity" in contracts. Bernheim and Whinston argue that when only some aspects of an agent's job are verifiable and some are not, the formal contract may leave even the verifiable aspects of the job unspecified. Such contractual incompleteness allows the employer to adjust the contract terms ex post in response to the agent's performance on the non-verifiable aspects of the job, and it generates stronger incentives for all aspects of the job ex ante. In contrast, we consider a setting where the employer may opt to leave the work rules vague so as to encourage worker initiative and ensure production efficiency. In our model, job performance is unidimensional and non-verifiable, so incentives are always relational, and we study the dynamics of worker initiative in the face of shocks to the employer's credibility.

In fact, our tradeoff between rules and worker initiative—like the interaction between the verifiable and non-verifiable job aspects in Bernheim and Whinston—is reminiscent of the literature on the interaction between formal and informal incentives (see, e.g., Baker, Gibbons, and Murphy, 1994; Schmidt and Schnitzer, 1995; Che and Yoo, 2001). This literature assumes that an agent's private action is reflected in multiple performance signals, some of which are verifiable and some of which are not (a recent exception is Kvaløy and Olsen, 2009, where the extent of verifiability may be endogenously determined). The optimal incentive scheme, therefore, combines court-enforceable incentive contracts with implicit incentives that are often sustained through repeated interactions. Moreover, the two forms of incentives could be substitutes or complements depending on the underlying economic environment. However, in our setting, formal contracts are infeasible irrespective of whether rules are used or not, and the dynamics of worker initiative is driven by the dynamics of the optimal relational contract.

2. Model

A principal (or "firm") hires an agent (or "worker") where the two parties enter in an infinitely repeated employment relationship. Time is assumed to be discrete and denoted as $t \in \{1, 2, ..., \infty\}$. In each period, the principal and the agent play a stage game that is defined as follows.

Stage game: We elaborate on the stage game by describing its three key components: *Technology, contracts,* and *payoffs.*

Technology: The agent must take an (private) action a_t to perform a job with output $Y_t \in \{-z, 0, y\}$. There are two types of productive actions. First, the agent can always choose an "adaptive" action a_A, where the agent takes initiative to adapt his action to some underlying state of the world. When an adaptive action is chosen, the output is always high (i.e., $Y_t = y$).

Second, the agent may choose a "rigid" action a_R where he follows a set of standardized work rules and procedures irrespective of the underlying state of the world. The rigid action can be chosen only if these procedures are put in place by the principal. When the rigid action is chosen, the output is high ($Y_t = y$) with probability $p \in (0, 1)$ and low ($Y_t = 0$) with probability $1 - p$.

Both adaptive and rigid actions are costly to the agent. The agent incurs a cost of action $c(a_t)$ where $c(a_A) = C$ and $c(a_R) = c$. The agent may also shirk by choosing a costless action a_S, that leads to considerable damage to the firm, and $Y_t = -z$. We assume that $C > c > 0$; the adaptive action is costlier as it requires the agent to learn the underlying state in order to tailor his action accordingly.

The principal, in contrast, can establish the standard procedure at zero cost. Moreover, once the procedure has been established, it stays in place in all future periods. We assume that such a procedure is established by implementing a performance metric $M_t \in \{0, 1\}$ where the rigid action a_R can be conceived as the action that is targeted towards this measure. The metric $M_t = 1$ if the agent chooses either the adaptive or the rigid action (i.e., if $a_t \in \{a_A, a_R\}$) and $M_t = 0$ if he shirks (i.e., $a_t = a_S$). That is, M_t is a noisy signal of the agent's action—though it can detect if the agent is shirking, it cannot detect if the agent is exerting consummate effort by taking initiative or simply following the standard procedure.

While the agent's action is his private information, the output Y_t and the performance metric M_t (if the standard process has been established) are publicly observable, though non-verifiable. We denote the availability of the standard procedure as $\gamma_t \in \{0,1\}$, where $\gamma_t = 1$ if a procedure has been put in place and $\gamma_t = 0$ otherwise.

Contract: In each period t, the principal decides on whether to offer a contract to the agent. Let $d_t^P \in \{0,1\}$ denote the principal's offer decision where $d_t^P = 0$ if no offer is made and $d_t^P = 1$ otherwise. The principal also decides whether to put in place a standard work procedure (and measure M_t) at the beginning of the period, if it has not been done in the past.

As both Y_t and M_t are non-verifiable, explicit pay-per-performance contracts are not feasible. Instead, the principal offers a relational contract that specifies a discretionary bonus b_t that depends on Y_t and M_t (if measured). The agent is liquidity constrained, and b_t must be non-negative. We assume that the principal's ability to pay the agent is stochastic as she may be exposed to a liquidity shock. In absence of any shock the opportunity cost of a dollar is a dollar whereas if there is a shock, the opportunity cost is prohibitively high and the principal cannot make any payments to the agent. Let $\rho_t \in \{S, N\}$ be the realization of the liquidity shock in period t, where $\rho_t = S$ if there is a shock and $\rho_t = N$ if there is none. We assume that ρ_t is identically and independently distributed across periods and $\Pr(\rho_t = S) = \theta \in (0,1)$. The liquidity shock is privately observed by the principal after the realization of the output. As the principal cannot pay the agent anything if there is a liquidity shock, the agent's compensation does not include any contractual wage component.[2]

Upon receiving the contract offer, the agent decides whether to accept it or not. Let $d_t^A \in \{0,1\}$ denote the agent's decision where $d_t^A = 0$ if the offer is rejected and $d_t^A = 1$ if it is accepted. Upon accepting the offer, the agent decides on his action a_t.

Finally, we assume that there is a public randomization device, generating a realization $x_t \in [0,1]$ at the end of the period. We may assume that the public randomization device is also available at the beginning of the game.

The timing of the stage game is summarized below.

[2]Our assumptions on the liquidity shock are similar to those in Li and Matouschek (2013), and such shocks can emanate from the volatility in the credit market or unexpected arrival of new business opportunities that require a large investment. However, the specific source of the shock is not a critical aspect of the model. A similar trade-off with standardization and the associated contractual dynamics can potentially emerge due to any shock that erodes the value of the relationship (we revisit this issue later in Section 6).

- *Beginning of Period t.* The principal decides whether or not to offer a contract to the agent. If a contract is offered, the principal also decides on whether to establish a standardized procedure (if such procedure has not been set up yet) and the game moves to period $t.1$. If no contract is offered, the game moves to period $t+1$.
- *Period $t.1$.* The agent either accepts or rejects the contract offered by the principal. If he accepts, the game moves to period $t.2$. If he rejects, the games moves to period $t+1$.
- *Period $t.2$.* The agent chooses the action a_t. If a standardized procedure is in place $a_t \in \{a_A, a_R, a_S\}$ and if it is not $a_t \in \{a_A, a_S\}$.
- *Period $t.3$.* The output Y_t and the performance measure M_t, if it has been put in place, are observed.
- *Period $t.4$.* The principal privately observes the liquidity shock ρ_t.
- *Period $t.5$.* The principal decides on the bonus payment. A bonus may be paid if there is no shock.
- *End of Period t.* The outcome of the randomization device x_t is realized and the game moves to period $t+1$.

PAYOFFS: The principal and agent are risk neutral. If either d_t^P or d_t^A is 0, both receive their outside options in that period—which we assume to be 0 for both—and the game moves on to period $t+1$. If $d_t^A = d_t^P = 1$, for a given action action a_t of the agent, the agent and the principal earn:

$$\hat{u}_t = \mathbb{E}\left[1_{\{\rho_t=N\}} b_t\left(Y_t, M_t\right) \mid a_t\right] - c\left(a_t\right)$$

and

$$\hat{\pi}_t = \mathbb{E}\left[Y_t - 1_{\{\rho_t=N\}} b_t\left(Y_t, M_t\right) \mid a_t\right],$$

respectively, where the expectation is taken using the distribution of ρ_t, Y_t and M_t conditional on a_t.

Repeated game: The stage game described above is repeated every period and both the agent and the principal have a common discount factor $\delta \in (0,1)$. At the beginning of any period t, the normalized payoffs of the players in the continuation game are given as:

$$u^t = (1-\delta)\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_\tau \hat{u}_\tau \text{ and } \pi^t = (1-\delta)\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_\tau \hat{\pi}_\tau,$$

where $d_\tau := d_\tau^A d_\tau^P$.

Strategies and equilibrium: As standard in the literature (see, e.g., Levin, 2003) we define a relational contract as a pure strategy public Perfect Equilibrium (PPE) where the players only use public strategies, and the equilibrium strategies induce a Nash Equilibrium in the continuation game starting from each public history.

Let $h_t = \left\{d^A_\tau, d^P_\tau, \gamma_\tau, \mathbb{M}_\tau, b_\tau, x_\tau\right\}_{\tau=1}^{t-1}$ denote the public history of the game at the beginning of period t, where $\mathbb{M}_\tau = Y_\tau$ if $\gamma_\tau = 0$ and $\mathbb{M}_\tau = (Y_\tau, M_\tau)$ if $\gamma_\tau = 1$. Let H_t be the set of all such histories (we set $H_1 = \emptyset$; also note that if $\gamma_\tau = 1$, then $\gamma_t = 1$ for all $t > \tau$). A public strategy of the principal is a sequence of functions $\sigma_P = \left\{D^P_t, \Gamma_t, B_t\right\}_{t=1}^{\infty}$ where her participation decision is given by $D^P_t : H_t \to \{0,1\}$, the decision to put in place a standardized work procedure is given as $\Gamma_t : H_t \to \{0,1\}$, and the decision on the bonus payment (when there is no shock, i.e., when $\rho_t = N$) is given as $B_t : H_t \times \{-z, y\} \to \mathbb{R}_+$ if $\gamma_t = 0$ and $B_t : H_t \times \{-z, 0, y\} \times \{0,1\} \to \mathbb{R}_+$ if $\gamma_t = 1$.

Similarly, a public strategy for the agent is a sequence of functions $\sigma_A = \left\{D^A_t, A_t\right\}_{t=1}^{\infty}$. His participation decision is given as $D^A_t : H_t \times \{0,1\} \times \mathbb{R}_+ \to \{0,1\}$ (i.e., participation depends on the public history, whether or not the standardized work process has been established, and the bonus offer), and action decision is given as $A_t : H_t \times \mathbb{R}_+ \to \{a_A, a_O\}$ if $\gamma_t = 0$ and $A_t : H_t \times \mathbb{R}_+ \to \{a_A, a_R, a_O\}$ if $\gamma_t = 1$. We define an optimal relational contract as a PPE of this game where the payoffs are not Pareto-dominated by any other PPE.

In what follows, we maintain a few restrictions on the parameters in order to focus on a more interesting modeling environment.

Assumption 1. *(i)* $y - C > py - c > 0$; *(ii)* $pC > c$; *(iii)* $(1-\delta)(1-p) > \delta p \theta$ and $y > \max\left\{\frac{1}{\delta(1-\theta)}, \frac{1-\delta}{(1-\delta)(1-p)-\delta p\theta}\right\} K$, where $K := ((C-c) - \delta(1-\theta)(pC-c))/(1-p)$; *(iv)* $z > \frac{\delta}{1-\delta}(y - C)$.

Assumption 1 (i) ensures that the adaptive action (a_A) is more efficient than the rigid one (a_R), which is, in turn, more efficient than dissolving the employment relationship. Parts (ii) and (iii) ensure that both the adaptive and the rigid actions are used on the equilibrium path, and the optimal relational contract gives rise to a rich set of dynamics. Part (ii) requires the adaptive action to be sufficiently more costly than the rigid action, whereas part (iii) stipulates that neither δ nor p is too large, and when the job is successfully completed, the value of the output is sufficiently high (the term on the right gives a sufficient lower bound). Finally, part (iv) ensures that it is never optimal to ask the agent to shirk on the equilibrium path as the damage from shirking is sufficiently large.

Since the introduction of the standardized procedure is a part of the principal's strategy, the analysis of the optimal relational contract requires a complete characterization of the equilibrium payoff set both before and after the principal introduces this procedure (and establishes the performance measure M_t). The characterization results in these two scenarios are presented in the next two sections.

3. Equilibrium payoff set after establishing the standardized procedure

Suppose that the standardized work process has already been established by the principal and the rigid action (a_R) is available to the agent. Let $\mathcal{E}_r$ be the PPE payoff set (for a given δ). We characterize $\mathcal{E}_r$ using the recursive method à la Abreu et al. (1990). Any equilibrium payoff pair $(\pi, u) \in \mathcal{E}_r$ is supported either by a pure action profile in the stage game together with a set of continuation payoffs that the players expect to receive in the future, or by randomizing over a set of equilibrium payoff pairs that are themselves supported by some pure action profiles (in the stage game).

In a pure action profile in the stage game, the players can take the outside option, in which case both parties receive 0. The parties can also enter the relationship, in which case, the agent can take the adaptive action or the rigid action (by Assumption 1 (iv), it is never optimal for the principal to hire the agent and ask him to shirk). We denote these two action profiles, with a slight abuse of notation, as A and R, respectively. Also, we denote $a = O$ when the parties take the outside option. For any current period action $a \in \{A, R\}$, let b^a be the associated bonus to the agent when there is no liquidity shock. Also, let $(\pi_s^a, \pi_n^a, u_s^a, u_n^a)$ be the associated continuation payoffs where π_s^a and π_n^a are the principal's continuation payoff in the shock and no-shock states, respectively, and u_s^a and u_n^a are the same for the agent. Finally, let $\left(\pi^O, u^O\right)$ be the continuation payoffs of the two parties when $a = O$.

Below, we first present the set of constraints that the bonus and the continuation payoffs must satisfy if an action $a \in \{A, R, O\}$ is used to support an equilibrium payoff pair $(\pi, u) \in \mathcal{E}_r$. Next, using these constraints, we characterize the frontier of $\mathcal{E}_r$.

3.1. The constraints.

For any equilibrium payoff pair (π, u) that is supported by an action profile $a \in \{A, R, O\}$ in the current period, the associated stage game play and the continuation payoffs must be such that: (i) the proposed course of the play indeed offers the said payoff (π, u) to the players, and (ii) neither party has any incentive to deviate from the proposed play in the stage game. These requirements give rise to a set of constraints for each one of the three pure action profiles in the stage game, A, R, and O.

Adaptive action. A payoff pair (π, u) can be supported by playing the adaptive action in the current period ($a_t = a_A$) if the following constraints are satisfied.

Promise-keeping: The consistency of the PPE payoff decomposition requires that the players' payoffs must be a weighted average of their current period payoff and the continuation payoff. Without loss of generality, we assume that when the principal wants to implement the adaptive action, the bonus b^A is paid if and only if $Y = y$. Hence, we must have:

$$(PK_A^A) \qquad u = \theta \left[(1-\delta)(-C) + \delta u_s^A \right] + (1-\theta) \left[(1-\delta) \left(b^A - C \right) + \delta u_n^A \right]$$

for the agent, and

$$(PK_P^A) \qquad \pi = \theta \left[(1-\delta) y + \delta \pi_s^A \right] + (1-\theta) \left[(1-\delta) \left(y - b^A \right) + \delta \pi_n^A \right]$$

for the principal.

No deviation: In equilibrium, neither party should have incentives to deviate from the proposed play, irrespective of whether such a deviation is publicly observed ("off-schedule") or not ("on-schedule"). Following an off-schedule deviation, without loss of generality, we may assume that the players take their outside options as it constitutes the harshest punishment for both players. Notice that the principal may deviate off-schedule by reneging on its bonus promise even after he reports a "no-shock" state. Consequently, we have the following non-reneging constraint on the principal:

$$(NR^A) \qquad -(1-\delta) b^A + \delta \pi_n^A \geq 0.$$

The principal may also deviate off-schedule by not offering a contract to the agent. The agent, on the other hand, deviates off-schedule if he rejects the principal's offer. Hence, the individual rationality constraints:

$$(IR) \qquad \pi \geq 0, \ u \geq 0.$$

But both the principal and the agent may also deviate on-schedule. The principal may be tempted to report a liquidity shock when there is none so as to save on the bonus pay. As a result, we have the following "truth telling" constraint:

$$(TT^A) \qquad -(1-\delta) b^A + \delta \pi_n^A \geq \delta \pi_s^A.$$

The agent, on the other hand, may deviate and choose the rigid action $(a_t = a_R)$ instead of the more costly adaptive action $(a_t = a_A)$. Therefore, the following incentive compatibility constraint must hold:

$$u \geq p\left[\theta\left((1-\delta)(-c)+\delta u_s^A\right)+(1-\theta)\left((1-\delta)\left(b^A-c\right)+\delta u_n^A\right)\right]+(1-p)(1-\delta)(-c)\,.$$

Using $\left(PK_A^A\right)$, we can simplify this constraint as:

$$(IC^A) \qquad u \geq \frac{1-\delta}{1-p}\,(pC-c) =: u^*.$$

Notice that as $pC - c > 0$ (by Assumption 1 (ii)), $\left(IC^A\right)$ implies that the agent must be given rents if the principal were to induce him to take the adaptive action when the rigid action is available to him. As $u \geq 0$, a deviation to completely shirking on the job $(a_t = a_S)$ is never profitable for the agent.

Feasibility: For the equilibrium payoff to be feasible, the associated bonus payment must be non-negative:

$$(NN^A) \qquad b^A \geq 0,$$

and the continuation payoffs themselves must be feasible, i.e., the following self-enforcing constraint must hold:

$$(SE^A) \qquad \left(\pi_\rho^A, u_\rho^A\right) \in \mathcal{E}_r,\ \rho \in \{S, N\}\,.$$

RIGID ACTION. Now suppose that a payoff pair (π, u) is supported by playing the rigid action in the current period. As in the case of adaptive action, a similar set of constraints must hold.

Promise-keeping: Without loss of generality, we assume that the bonus b^R is paid if and only if $M_t = 1$.[3] Hence, the promise-keeping constraints take the following form:

$$(PK_A^R) \qquad u = \theta\left[(1-\delta)(-c) + \delta u_s^R\right] + (1-\theta)\left[(1-\delta)\left(b^R - c\right) + \delta u_n^R\right],$$

and

$$(PK_P^R) \qquad \pi = \theta\left[(1-\delta)\,py + \delta \pi_s^R\right] + (1-\theta)\left[(1-\delta)\left(py - b^R\right) + \delta \pi_n^R\right].$$

No deviation: As discussed earlier, the non-reneging constraint on the principal and the individual rationality constraint on both players must hold in order to deter off-schedule deviations. That is, we require:

$$(NR^R) \qquad -(1-\delta)\, b^R + \delta \pi_n^R \geq 0,$$

and

$$(IR) \qquad \pi \geq 0,\ u \geq 0.$$

Similarly, the truth-telling constraint on the principal ensures no on-schedule deviation by misreporting the state:

$$(TT^R) \qquad -(1-\delta)\, b^R + \delta \pi_n^R \geq \delta \pi_s^R.$$

Notice that the agent's incentive compatibility constraint is trivially satisfied as deviating and taking the adaptive action would yield the same expected benefit (as that of taking the rigid action) but at a higher cost. Again, as $u \geq 0$, deviating to a_S (i.e., shirking) is never profitable.

Feasibility: As before, we have the non-negativity constraint and the self-enforcing constraint:

[3] In principle, the bonus and continuation payoffs can also depend on the output Y_t. But since performance measure $M_t = 1$ with certainty when the agent chooses the rigid action, having the bonus and continuation payoffs depending on Y_t would be redundant. We suppress their dependence here in order to simplify the exposition.

$$(NN^R) \qquad b^R \geq 0,$$

and

$$(SE^R) \qquad \left(\pi_\rho^R, u_\rho^R\right) \in \mathcal{E}_r,\ \rho \in \{S, N\}.$$

Outside option. Finally, if a payoff pair (π, u) is supported by players taking their respective outside options in the current period, the following set of constraints must hold.

Promise-keeping: We have:

$$(PK^O) \qquad u = \delta u^O \text{ and } \pi = \delta \pi^O.$$

Feasibility: The following self-enforcing constraint hold:

$$(SE^O) \qquad (\pi^O, u^O) \in \mathcal{E}_r.$$

3.2. **Properties of the PPE payoff frontier.** Define the PPE payoff frontier $U_r(\pi)$ as follows:

$$U_r(\pi) := \sup\{u \mid (\pi, u) \in \mathcal{E}_r\}.$$

The following lemma presents a set of general characteristics of the PPE payoff set.

Lemma 1. *The PPE payoff set $\mathcal{E}_r$ has the following properties: (i) it is compact, (ii) $U_r(\pi)$ is concave, and (iii) for any payoff pair $(\pi, U_r(\pi))$, the associated continuation payoffs (along the equilibrium path) remain on the frontier; i.e., for $a \in \{A, R\}$, $u_s^a = U_r(\pi_s^a)$; $u_n^a = U_r(\pi_n^a)$; and $u^O = U_r\left(\pi^O\right)$.*

For Part (i), the compactness of $\mathcal{E}_r$ follows from the fact that there are only a finite number of actions that the agent may be asked to undertake in any equilibrium (i.e., $a \in \{A, R, O\}$), and the transfer from the principal to the agent is essentially bounded by the total future surplus of the relationship. For Part (ii), the presence of the public randomization device ensures concavity of $U_r(\pi)$. The final part of the above lemma shows that, under an optimal relational contract, the continuation payoffs never fall below the frontier. Since both the principal's actions and the agent's performance are publicly observed, there is no need for joint punishment along the equilibrium path.

For our analysis, it is also useful to define the agent's highest payoff for a given payoff of the principal in the set of all PPE that are supported by a specific action. For any $a \in \{A, R, O\}$, let

$$u_r^a(\pi) := \max\{u \mid (\pi, u) \in \mathcal{E}_r \text{ and is supported by } a\}.$$

In particular, for $a \in \{A, R\}$, $u_r^a(\pi)$ satisfies the following:

$$u_r^a(\pi) = \max\nolimits_{b^a, \pi_s^a, \pi_n^a} \ (1-\delta)\left[(1-\theta)\, b^a - c(a)\right] + \delta\left[(1-\theta)\, U_r(\pi_n^a) + \theta U_r(\pi_s^a)\right]$$

$$\text{subject to } \ (PK_P^a),\ (NR^a),\ (TT^a),\ (NN^a),\ \text{and } (SE^a).$$

(Clearly, $u_r^a(\pi)$ is defined only for the values of π such that the corresponding (IC^a) constraint is satisfied.) Also notice that:

$$u_r^O(\pi) = \delta U_r\left(\pi^O\right), \text{ where } \pi^O = \pi/\delta.$$

Furthermore, notice that the PPE payoff frontier U_r is the function that satisfies the following: Let $\bar{\pi}_r := \max\{\pi \mid (\pi, u) \in \mathcal{E}_r\}$, i.e., $\bar{\pi}_r$ is the highest PPE payoff to the principal when the standardized work process has already been established. Then, for all $\pi \in [0, \bar{\pi}_r]$,

$$U_r(\pi) = \max\nolimits_{\alpha_a \geq 0,\ \pi_a \in [0, \bar{\pi}_r]} \sum\nolimits_{a \in \{A,R,O\}} \alpha_a u_r^a(\pi_a)$$

$$s.t. \ \sum\nolimits_{a \in \{A,R,O\}} \alpha_a = 1, \ and \ \sum\nolimits_{a \in \{A,R,O\}} \alpha_a \pi_a = \pi.$$

In order to characterize the frontier U_r, we first describe, for each action taken, the associated continuation payoffs for the principal.

Lemma 2. *Consider an equilibrium payoff pair (π, u) that is on the payoff frontier $U_r(\pi)$. The following holds:*

(i) If (π, u) is supported by the adaptive action, then

$$\pi_s^A(\pi) = \frac{1}{\delta}(\pi - (1-\delta)y) < \pi \text{ and } \pi_n^A(\pi) = \bar{\pi}_r \geq \pi,$$

and if there is no shock, the principal pays a bonus $b^A(\pi) = y - (\pi - \delta\overline{\pi}_r)/(1-\delta)$.

(ii) If (π, u) is supported by the rigid action, then

$$\pi_s^R(\pi) = \frac{1}{\delta}(\pi - (1-\delta)py) < \pi \text{ and } \pi_n^R(\pi) = \bar{\pi}_r \geq \pi,$$

and if there is no shock, the principal pays a bonus $b^R(\pi) = py - (\pi - \delta\overline{\pi}_r)/(1-\delta) > 0$.

(iii) If (π, u) is supported by the outside option, then

$$\pi^O(\pi) = \pi/\delta.$$

Part (i) and (ii) of the above lemma state that the principal's continuation payoff decreases in a shock state and increases in a no-shock state. Such a spread between the continuation payoffs in the two states induces the principal to report the state truthfully. However, as we will argue below, when the principal's continuation payoff is sufficiently low, following a shock, an inefficient action—either through the rigid action or the outside option—must be taken. To minimize the likelihood of that such inefficiency would arise, the principal's continuation payoff in a no-shock state jumps to the maximal PPE payoff ($\bar{\pi}_r$) which gives her the most cushion for future shocks. This pattern of movement in the continuation payoff also occurs in Li and Matouschek (2013) and follows from the same reasoning. Finally, notice that part (iii) directly follows from the principal's promise-keeping constraint when the outside option is used.

Proposition 1 below characterizes the payoff frontier U_r.

Proposition 1. *The payoff frontier U_r can be divided into four regions. There exist cutoffs $0 < \pi_r^O \leq \pi_r^R \leq \pi_r^A \leq \overline{\pi}_r$, with $\pi_r^O < \overline{\pi}_r$, such that:*

(i) For $\pi \in [0, \pi_r^O)$, the payoff frontier is linear and supported by randomization between $(0,0)$ and $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$. We have $U_r(0) = 0$ and the payoff $(0,0)$ is supported by $a = O$ (i.e., players taking the outside option).

(ii) For $\pi \in [\pi_r^O, \pi_r^R]$, $U_r(\pi) = u_r^R(\pi)$ (i.e., the payoff frontier is supported by the rigid action).

(iii) For $\pi \in (\pi_r^R, \pi_r^A)$, the payoff frontier is linear and supported by randomization between $\left(\pi_r^R, U_r\left(\pi_r^R\right)\right)$ and $\left(\pi_r^A, U_r\left(\pi_r^A\right)\right)$.

(iv) For $\pi \in [\pi_r^A, \overline{\pi}_r]$, $U_r(\pi) = u_r^A(\pi)$ (i.e., the payoff frontier is supported by the adaptive action), and $U_r(\overline{\pi}_r) = u^*$.

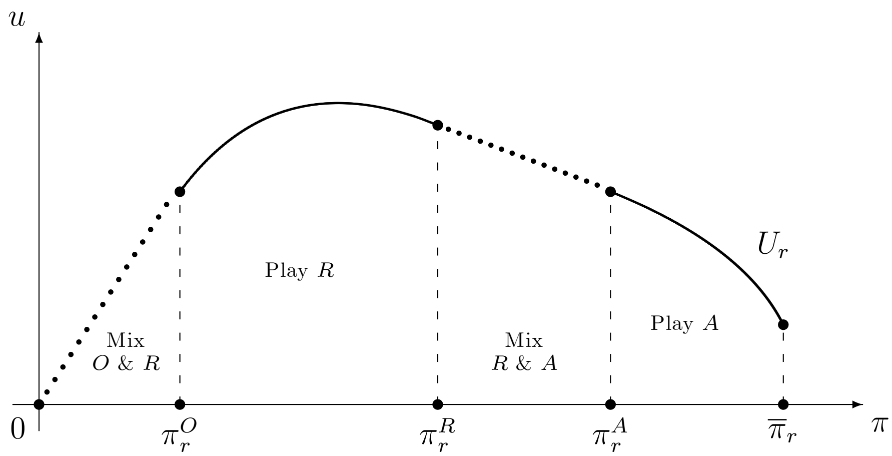

Figure 1. PPE payoff set and frontier when the standardized procedure has already been established.

Figure 1 illustrates the four regions described in Proposition 1. In two regions, one in the middle and one at the right-most end, the payoffs at the frontier are supported by pure actions, by playing $a = R$ and $a = A$, respectively. Also, the $(0, 0)$ payoff pair is the only point on the frontier that is supported by playing $a = O$. In the other two regions, the payoffs are sustained through randomization. Without loss of generality, we assume that in the regions where randomization is used, the players randomize only between the endpoints of the two adjacent regions that are sustained by pure actions.

One feature of the payoff frontier is that the more efficient action gets taken as the principal's payoff increases. When the principal's payoff (π) is sufficiently low, i.e., to the left of π_r^O, she does not have enough credibility to promise a bonus large enough to induce the adaptive or the rigid action. When π is above π_r^O, both the rigid and the adaptive action may be feasible. Note that while the adaptive action is more efficient, it gives the principal a lower continuation payoff in shock states (by Lemma 2, $\pi_s^A(\pi) < \pi_s^R(\pi)$), increasing the chance of termination of the relationship. When the principal's payoff is close to π_r^O, the threat of termination is more imminent, causing the parties to choose the rigid action. In contrast, for a large enough payoff for the principal, the termination is less of a concern, and the adaptive action is chosen.

A notable feature of the PPE frontier U_r is that, at the maximal payoff for the principal ($\bar{\pi}_r$), the agent's payoff is strictly positive. The reason is that when a standardized work process is established, the moral hazard problem becomes more severe as the agent may deviate and take the rigid action when asked to undertake the more costly adaptive action. To prevent the agent from doing so, the principal must offer him rents. In other words, rules stymie initiative—it gets harder to induce worker initiative once the work rules are standardized.

4. Equilibrium payoff set before establishing the standardized procedure

We now proceed to characterize the PPE payoff set $\mathcal{E}$, i.e., the payoff set when the principal is yet to put in place a standard work procedure but may implement it subsequently by establishing the performance measure M_t.

A key decision that the principal needs to make at the beginning of the period is whether or not to establish a standard work procedure (and implement the performance measure M_t). Notice that $\mathcal{E}_r \subseteq \mathcal{E}$ as for any payoff $(\pi, u) \in \mathcal{E}_r$ there always exists a PPE where the principal establishes the standard procedure at the beginning of the game and in the continuation game the parties play the same strategies that give rise to the payoff (π, u). Furthermore, once the principal decides to establishes the standard procedure, the analysis becomes identical to that discussed in the previous section.

But when the principal is yet to implement the procedure, there are only two actions available to the agent, the adaptive action and the outside option. In this case, any payoff pair $(\pi, u) \in \mathcal{E}$ is supported either by one of these two a pure action profiles or by a randomization over the two. We denote these two pure action profiles as $a = \mathcal{A}$, and $\mathcal{O}$, respectively. (We use different notations than before—$a = \mathcal{A}$ and $\mathcal{O}$, instead of A and O—in order to distinguish between the use of an action profile when the rigid action is available and when it is not.) In what follows, we only focus on this novel part of the analysis.

4.1. **The constraints.**

As in the previous section, we begin our analysis by presenting the set of constraints that the bonus and the continuation payoffs must satisfy if an action profile $a \in \{\mathcal{A}, \mathcal{O}\}$ is used to support an equilibrium payoff pair $(\pi, u) \in \mathcal{E}$.

Adaptive Action: Suppose $(\pi, u) \in \mathcal{E}$ is supported by the adaptive action ($a = \mathcal{A}$). As discussed in Section 3.1, the following promise-keeping constraints must hold:

$$(PK_A^{A*}) \qquad u = \theta\left[(1-\delta)(-C) + \delta u_s^{\mathcal{A}}\right] + (1-\theta)\left[(1-\delta)\left(b^{\mathcal{A}} - C\right) + \delta u_n^{\mathcal{A}}\right],$$

and

$$(PK_P^{A*}) \qquad \pi = \theta\left[(1-\delta)\, y + \delta \pi_s^{\mathcal{A}}\right] + (1-\theta)\left[(1-\delta)\left(y - b^{\mathcal{A}}\right) + \delta \pi_n^{\mathcal{A}}\right].$$

The associated no-deviation constraints include the non-reneging and individual rationality constraints for the off-schedule deviations:

$$(NR^{A*}) \qquad -(1-\delta)\, b^{\mathcal{A}} + \delta \pi_n^{\mathcal{A}} \geq 0,$$

and

$$(IR) \qquad u \geq 0;$$

as well as the truth-telling constraint for the on-schedule deviation:

$$(TT^{A*}) \qquad -(1-\delta)\, b^{\mathcal{A}} + \delta \pi_n^{\mathcal{A}} \geq \delta \pi_s^{\mathcal{A}}.$$

Finally, we have the two feasibility constraints (non-negativity and self-enforcing):

$$(NN^{A*}) \qquad b^{\mathcal{A}} \geq 0,$$

and

$$(SE^{A*}) \qquad \left(\pi_\rho^{\mathcal{A}}, u_\rho^{\mathcal{A}}\right) \in \mathcal{E},\ \rho \in \{S, N\}.$$

Note that, unlike the analysis in the previous section, the rigid action is not available. As a result, if the agent shirks he must choose $a_t = a_S$ and his shirking would be detected for sure. Therefore, the incentive compatibility constraint for the agent's action choice is always satisfied, and hence, we omit it here. Also, the $\left(SE^{\mathcal{A}}\right)$ differs from its counterpart in Section 3.1 by requiring the continuation payoffs to be in the payoff set $\mathcal{E}$ instead of $\mathcal{E}_r$, i.e., the PPE payoff set when the rigid action is available.

Outside option: If $(\pi, u) \in \mathcal{E}$ is supported by the parties taking the outside option in the current period ($a = \mathcal{O}$), the associated continuation payoffs $(\pi^{\mathcal{O}}, u^{\mathcal{O}})$ satisfy the following promise-keeping and self-enforcing constraints:

$$(PK^{O*}) \qquad u = \delta u^{\mathcal{O}} \text{ and } \pi = \delta \pi^{\mathcal{O}},$$

and

$$(SE^{O*}) \qquad (\pi^{\mathcal{O}}, u^{\mathcal{O}}) \in \mathcal{E}.$$

4.2. **Properties of the PPE payoff frontier.** As in the previous section, let $U(\pi)$ be the PPE payoff frontier at the beginning of the game (i.e., before the rigid action is made available), i.e.,

$$U(\pi) := \sup \{u \mid (\pi, u) \in \mathcal{E}\}.$$

and let $\bar{\pi} := \max \{\pi \mid (\pi, u) \in \mathcal{E}\}$. Also, for any $a \in \{\mathcal{A}, \mathcal{O}\}$, let

$$u^a(\pi) := \sup \{u \mid (\pi, u) \in \mathcal{E} \text{ and is supported by } a\}.$$

The following two lemmas present a set of general properties of the payoff frontier U that mirror those of U_r discussed above.

Lemma 3. *The PPE payoff set $\mathcal{E}$ has the following properties: (i) it is compact, (ii) $U(\pi)$ is concave, and (iii) for any payoff pair $(\pi, U(\pi))$ sustained by pure action $a \in \{\mathcal{A}, \mathcal{O}\}$, the associated continuation payoffs (along the equilibrium path) remain on the frontier; i.e., $u_s^{\mathcal{A}} = U(\pi_s^{\mathcal{A}})$, $u_n^{\mathcal{A}} = U(\pi_n^{\mathcal{A}})$ and $u^{\mathcal{O}} = U(\pi^{\mathcal{O}})$.*

Lemma 4. *Consider an equilibrium payoff pair (π, u) that is on the payoff frontier $U(\pi)$. The following holds:*

(i) If (π, u) is supported by the adaptive action, then

$$\pi_s^{\mathcal{A}}(\pi) = \frac{1}{\delta}(\pi - (1-\delta)y) < \pi \text{ and } \pi_n^{\mathcal{A}}(\pi) = \bar{\pi} \geq \pi.$$

If there is no shock, the principal pays a bonus $b^{\mathcal{A}}(\pi) = y - (\pi - \delta\overline{\pi}) / (1-\delta) > 0$.

(ii) If (π, u) is supported by the outside option, then

$$\pi^{\mathcal{O}}(\pi) = \pi/\delta.$$

The arguments behind the above lemmas closely parallel their counterpart in Lemma 1 and 2 (hence, we omit the formal proofs). Using these lemmas, we derive the following proposition that characterizes the PPE payoff frontier.

Proposition 2. *The payoff frontier U can be divided into four regions. There exist cutoffs $0 < \pi^O \leq \pi^R \leq \pi^A \leq \bar{\pi}$, with $\pi^O < \overline{\pi}$, such that (the notations $u_r^R(\pi)$ and $U_r(\pi)$ are as defined in Proposition 1):*

(i) For $\pi \in [0, \pi^O)$, the payoff frontier is linear and supported by randomization between $(0,0)$ and $\left(\pi^O, U\left(\pi^O\right)\right)$. We have $U(0) = 0$ and the payoff pair $(0,0)$ is supported by $a = \mathcal{O}$ (i.e., with players taking the outside option).

(ii) For $\pi \in [\pi^O, \pi^R]$, $U(\pi) = U_r(\pi) = u_r^R(\pi)$ (i.e., the payoff frontier is supported by the rigid action).

(iii) For $\pi \in (\pi^R, \pi^A)$, the payoff frontier is linear and supported by randomization between $\left(\pi^R, U\left(\pi^R\right)\right)$ and $\left(\pi^A, U\left(\pi^A\right)\right)$.

(iv) For $\pi \in [\pi^A, \overline{\pi}]$, $U(\pi) = u^{\mathcal{A}}(\pi)$ (i.e., the payoff frontier is supported by the adaptive action $\mathcal{A}$), and $U(\bar{\pi}) = 0$.

This proposition indicates that similar to U_r (the frontier when the rigid action is available), the frontier U can also be divided into four regions. A pure action is used to sustain payoffs in two of the four regions: The rigid action is made available and used to support a part in the middle, and the very right-most part of frontier is sustained through the adaptive action where the principal refrains from introducing rigid action. In addition, the outside option is used only to support the payoff $(0,0)$. The remaining two regions are supported through randomization. As before, we assume that in such regions the players randomize only between the endpoints of the two adjacent regions that are sustained by pure actions.

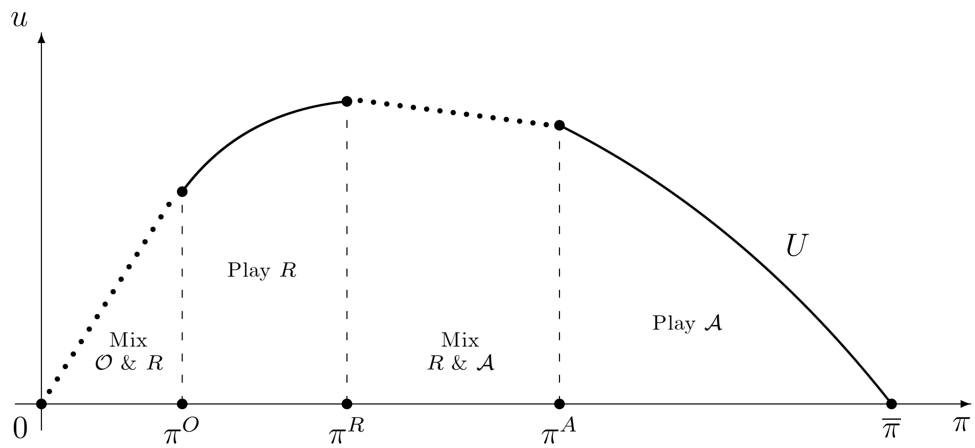

Figure 2. PPE payoff set and frontier before the standardized procedure is established.

The shape of the frontier U is also similar to its counterpart U_r where a more efficient action gets taken as the principal's payoff increases. However, there are some important differences. First, the frontier U indicates not only the agent's action but also the principal's decision on the implementation of the standardized procedure. In particular, whenever the principal's payoff falls below the cutoff π^R, the procedure is put in place with certainty. Second, at the principal's maximal payoff, $\bar{\pi}$, the agent does not earn any rents. This difference arises because at $\bar{\pi}$ the principal has not put in place the standardized procedure, and hence, the agent cannot take a rigid action. Consequently, the underlying moral hazard problem is less severe and the principal need not offer him rents in order to induce him to take the adaptive action. Finally, the cutoffs that define the four regions are, in general, different. This difference in the cutoffs is important as it affects the dynamics of the relationship, which we discuss in the next section.

5. The dynamics of the relationship

Using the results obtained in the previous two sections, we can now discuss the dynamics of the employment relationship as it may evolve over time in response to the liquidity shocks faced by the principal. We begin by presenting a lemma that helps us contrast the PPE payoff frontiers U_r and U—the ones when the principal has established the standardized procedure and when she has not.

Lemma 5. *We have the following:* $\pi^O = \pi_r^O$, $\pi^R \leq \pi_r^R$, *and* $\bar{\pi} > \bar{\pi}_r$. *In addition,* $U(\pi) = U_r(\pi)$ *for all* $\pi \leq \pi^R$ *and* $U(\pi) > U_r(\pi)$ *otherwise.*

This lemma highlights the efficiency loss that results from the establishment of the standardized procedure. The maximal joint surplus that could be obtained in any PPE where $\pi > \pi^R$ is strictly smaller if the standardized procedure is established from the beginning of the game. The loss of surplus stems from the fact that having the standardized procedure in place increases the likelihood that an inefficient action would be taken in the future (in response to shocks) even if the efficient adaptive action is chosen at present.

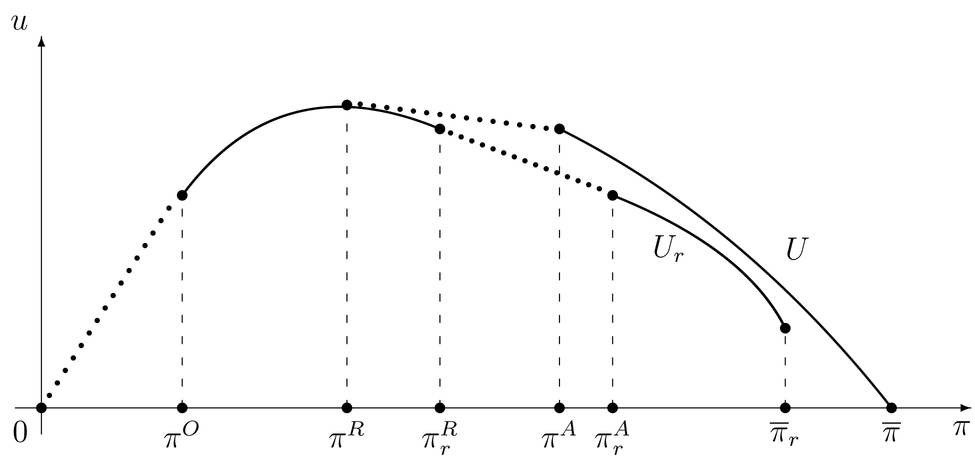

Figure 3. The PPE frontiers U_r and U.

Notice that the thresholds π^O and π^R indicate how far the principal's continuation payoff needs to fall before each of the two inefficient actions—the outside option and the rigid action, respectively—gets taken if the standardized procedure has not been established already (also, π_r^O and π_r^R represent the corresponding cutoffs when the procedure has been established). Though the threshold for taking the outside option is the same under the two cases, the threshold for taking the rigid action is (weakly) lower.

More importantly, the principal's maximal PPE payoff is strictly larger if the procedure is not in place than if it is (i.e., $\bar{\pi} > \bar{\pi}_r$). That is, the principal not only can extract more rents from the agent but also the joint surplus is larger if the standardized procedure is not in place to begin with. The argument for these two observations are closely interlinked. Recall that having the standardized procedure available makes the moral hazard problem more severe. In order to induce the agent to take the adaptive action, the principal must offer him rents (by Proposition 1). As the principal has a smaller continuation value to begin with, it lowers her credibility in promising a large bonus payment that is needed to induce the adaptive action. Consequently, the relationship becomes more vulnerable to shocks and the use of inefficient action becomes more likely in the future, lowering the total surplus in the relationship.

In contrast, if the procedure has not been established yet, the agent does not get any rents when the principal's payoff is $\bar{\pi}$. The principal is able to better extract rents, giving her more credibility in promising a bonus. Consequently, the relationship is more resilient to shocks and yields a strictly higher surplus.

Proposition 3. *(**The structure and the dynamics of the relationship**) The optimal relational contract contains the following three phases.*

(i) The relationship starts in Phase 1 where: (a) The standardized procedure is never established and the agent always chooses the adaptive action. (b) The principal's payoff π starts at $\bar{\pi}$. (c) For any π, when there is a shock, the principal's payoff decreases to $\pi_s^{\mathcal{A}}(\pi)$. But if there is no shock, the principal pays a bonus $b^{\mathcal{A}}(\pi)$ and his payoff moves to $\bar{\pi}$ (or remains at $\bar{\pi}$ if $\pi = \bar{\pi}$). The relationship stays in this phase as long as $\pi \geq \pi^A$. If $\pi < \pi^A$, the relationship transitions to Phase 2 with positive probability.

(ii) In Phase 2: (a) The standardized procedure is established and the agent starts the phase by choosing the rigid action. (b) Whenever the agent chooses the rigid action, if there is a shock, the principal's payoff decreases to $\pi_s^R(\pi)$. But if there is no shock, the principal pays a bonus $b^R(\pi)$ and his continuation payoff moves to $\overline{\pi}_r$ (where the agent chooses the adaptive action). (c) Whenever the agent chooses the adaptive action, if there is a shock the principal's payoff decreases to $\pi_s^A(\pi)$. And if there is no shock, the principal pays a bonus $b^A(\pi)$ and his payoff moves to $\overline{\pi}_r$ (or remains at $\overline{\pi}_r$ if $\pi = \overline{\pi}_r$). (d) The agent chooses the rigid action if $\pi \leq \pi_r^R$, and chooses the adaptive action with a positive probability if and only if $\pi > \pi_r^R$. The relationship stays in this phase as long as $\pi \geq \pi^O$. If $\pi < \pi^O$, the relationship transitions to Phase 3 with positive probability.

(iii) In Phase 3, the relationship is terminated.

The above result follows directly from the characterization of the PPE frontiers discussed in Propositions 1 and 2 (hence, we omit the formal proof). The relationship starts at the right-most point of U: the principal does not establish the standardized procedure, induces the agent to choose the adaptive action, and extracts all surplus (i.e., $\pi = \bar{\pi}$). If a shock occurs, the continuation payoffs move to the left along the PPE payoff frontier—in order to ensure truthful reporting, the principal must transfer rents to the agent following the announcement of a shock state. Though a no-shock state instantaneously moves the relationship to its initial starting point, as an arbitrarily long stretch of consecutive shocks occurs almost surely, the parties are eventually forced to take an inefficient action.

Once the principal's payoff falls below π^A, there is a positive probability that she would establish the standardized procedure, and the rigid action would be taken. In particular, for lower values of π, the principal may not have sufficient credibility to promise the bonus needed to induce the costly adaptive action, and the rigid action is used with certainty. But once the rigid action is made available, the relationship never fully recovers. Even after the shock has passed, the relationship moves to the right-most point of U_r instead of U, where the joint surplus is smaller than what it was at the beginning of the relationship. As before, following more shocks, the continuation payoffs move further to the left. Eventually, the principal loses so much credibility that she cannot even promise the bonus needed to induce the agent to take the rigid action, and the parties may terminate the relationship.

It is also worth noting that the size of the bonus goes down when a less efficient (i.e., less costly) action is taken. Moreover, for a given payoff of the principal, the bonus offered to inducing the adaptive action is larger when the principal is yet to establish the standardized procedure (i.e., $b^{\mathcal{A}}(\pi) > b^A(\pi) > b^R(\pi)$, by Lemma 4 and 2). Because the principal's maximal PPE payoff is larger when the standardized procedure is yet to be set (i.e., $\bar{\pi} > \bar{\pi}_r$), she can credibly transfer more rents to the agent in a no-shock state.

Two important implications of the above findings—as given in the proposition below—further illustrate the trade-offs with establishing the standardized procedure.

Proposition 4. *The optimal relational contract has the following features:*

(i) *The rigid action may be used when the adaptive action is still feasible. Moreover, the set of parameters for which this is the case is larger when the standardized procedure has already been established in the past than when it has not.*

(ii) *The number of consecutive shocks that guarantees that the rigid action is used when the relationship starts in Phase 1 (with $\pi = \bar{\pi}$ as the standard process is yet to be established)* *is at least as large as its counterpart when the relationship restarts after reaching Phase 2 (with $\pi = \bar{\pi}_r$ where the standard process has already been established).*

We have argued above that in response to a current shock the rigid action may be used as the principal may not have enough reputational capital to incentivize the agent to undertake the more costly adaptive action. But the first part of Proposition 4 states that in response to shocks the relationship may switch to rigid action as a "precautionary measure"—even if the principal could still induce the agent to take the adaptive action, the rigid action is called for. The intuition can be traced from the continuation payoffs in a shock state as given in Lemma 4 and 2. For a given payoff of the principal the associated continuation payoff in a shock state is smaller when the adaptive action is being used compared to the case when a rigid action is being used (i.e., $\pi_s^A(\pi) < \pi_s^R(\pi)$). So, by using the rigid action instead of the adaptive one the contracting parties can arrest the erosion of surplus in the relationship as shocks occur. Consequently, the likelihood of the relationship's survival increases and, under certain parameters, the resulting gains in the surplus outweighs the loss due to the use of the inefficient rigid action. Also, recall that if the rigid action has not yet been used in the past, there is an additional cost of using it as it reduces the surplus in the relationship even after the shock passes. Hence, the optimal contract is more likely to call for a "precautionary" use of the rigid action when it has already been in place.

The second part of Proposition 4 states that the relationship becomes more fragile following the introduction of the standardized procedure. A relationship that starts with the agent taking the adaptive action may move more quickly towards the phase where the rigid action is used when the rigid action is available from the very beginning than when it is not. The argument, again, relies on the fact that even if the relationship may recover after reaching Phase 2 (i.e., the agent can be induced to take the adaptive action), it becomes less valuable to the principal (i.e., $\bar{\pi}_r < \bar{\pi}$) as the rigid action remains available to the agent. Consequently, it is less resilient to shocks, and more likely to rely on the rigid action when the shocks arise in the future. In other words, by establishing the standardized procedure in the face of liquidity shocks, the principal can better weather the shock at present (and may save the relationship from termination), but it fundamentally changes the nature of the relationship in the future by compromising its resilience to future shocks.

6. Discussion and conclusion

It is well documented by economists and management scholars alike that the events of the past play a significant role in shaping an organization's future. In his seminal treatise on the limits of organization, Arrow (1974; p. 49) observes that "...the combination of uncertainty, indivisibility, and capital intensity associated with information channels and their use imply (a) that the actual structure and behavior of an organization may depend heavily upon random events, in other words on history, and (b) the very pursuit of efficiency may lead to rigidity and unresponsiveness to further change." Our analysis highlights a novel mechanism that speaks to this observation.

We show how the extent of worker initiative may evolve over time in response to private shocks to a firm's credibility. In a time of a crisis, a firm may avoid collapse by reducing employee discretion and asking the workers to follow a set of standardized processes. The implementation of such processes creates rigidities and weakens the firm's performance in the short run. However, it is easier to incentivize the workers to adhere to a set of rules than to take initiative and adapt their actions to the underlying circumstances.

But standardization is a double-edged sword. While it can help a firm weather shocks in the short term, it may change the nature of the employment relationship in the long term. Once standardization is put in place, it becomes more difficult to motivate the workers to exert consummate effort when discretion is given back to them. As the provision of incentives becomes more difficult, the value of the relationship decreases, making the firm more vulnerable to future shocks.

We conclude with the following remarks. First, in our analysis, the firm alternates between periods of work-process standardization and encouragement of worker initiative. The standardized processes are used in time of crisis, but after surviving the crisis the worker is again urged to take initiative. However, one can consider parameters in our model (by relaxing Assumption 1 (iii)) where the relationship is stuck with standardized processes once they are made available. In particular, when the value of the output is too low or the agent's rents under the adaptive action are too high (e.g., y is small and p is large), the maximum bonus the principal can credibly promise would no longer be enough to induce the agent to take the adaptive action when the rigid action is already available. This scenario reflects the so-called "structural inertia" in firms where they appear incapable of making significant changes to their organizational strategies in the face of changing business environments (Hannan and Freeman, 1989).

Second, in our model, the firm's credibility evolves as it faces liquidity shocks that might force it to renege on its promises to the workers. However, a similar contractual dynamics can be obtained even if the employment relationship experiences shocks of a different nature. For example, the productivity of the worker may be subject to shocks that are not observed by the firm, and the output may be low even if the worker puts in consummate effort. To provide the worker with the right incentives, the firm may have to punish the worker by reducing his continuation payoff whenever the output is low. However, if there are several consecutive shocks to the worker's productivity his continuation payoff may become too low, and it may not be feasible for the firm to reduce the worker's continuation payoff any further. And since the shocks also deplete the value of the relationship, promising large bonuses may become infeasible as well. Hence, it becomes more difficult for the firm to provide strong incentives to encourage initiative, and the firm may ask the worker to follow standardized rules instead.

Third, our model does not consider the issue of heterogeneity in workers' productivity. In reality, workers may vary in their innate abilities, and the low-ability workers may need more guidance on how to perform their jobs. If a standardized work process is conceived as a set of guidelines that minimize the risk of failure in a given job, then standardization may increase the productivity of the low-ability workers. As a result, a firm with a set of heterogeneous workers might be more prone to rely on standardization. However, as in our model, the incentives of the high-ability workers may still be negatively affected, and the same trade-off with standardization that we highlight in the paper is likely to emerge.

Finally, in our model, the firm and the worker always have a common understanding of what is expected out of the worker. This is a natural assumption when rules are in place. As mentioned above, rules can serve as guidelines to the worker on how to do his job. In absence of rules however, this assumption becomes more important. When the worker initiative is desired, it is necessary that the worker understand what the firm's objectives are and what the worker needs to do in order to attain those objectives. In a complex production environment, it is conceivable that this understanding is difficult to establish, leading to a "problem of clarity" (Gibbons and Henderson, 2012a, 2012b). Thus, the strength of relational incentives depends not only on the extent of trust between the contracting parties but also on their ability to communicate clear expectations about their respective roles in the relationship. The interplay between the problems of "credibility" and "clarity" can have important implications for the optimal use of rules in relational contracts. A formal treatment of this issue is beyond the scope of this paper, and we leave it for future research.

Appendix

This appendix contains the proofs omitted in the text.

Proof of Lemma 1. Part (i) follows from standard arguments (as in Abreu et al. 1990) since the action space for each player in the stage game is finite. Part (ii) immediately follows from the availability of the public randomization device. The argument for part (iii) is given as follows.

Notice that it is sufficient to show this property for a payoff supported by a pure action. Without loss of generality, assume that $(\pi, u) = (\pi, U_r(\pi))$ and it is supported by the adaptive action $(a = A)$, and the continuation payoffs (in the shock and no-shock states) are $\left(\pi_s^A, u_s^A\right)$ and $\left(\pi_n^A, u_n^A\right)$. Suppose that $u_n^A < U_r\left(\pi_n^A\right)$. Now consider an alternative strategy that also specifies $a = A$ and offers continuation payoffs $\left(\pi_s^A, u_s^A\right)$ and $\left(\pi_n^A, u_n^A + \varepsilon\right)$ where $\varepsilon > 0$ and $u_n^A + \varepsilon < U_r\left(\pi_n^A\right)$. Under this strategy, $\left(PK_A^A\right)$ and $\left(PK_P^A\right)$ imply that the principal's payoff remains at π whereas the agent's payoff is $u + (1-\theta)\,\delta\varepsilon > U_r(\pi)$. It is routine to check that this strategy profile also satisfies all other constraints, and hence, constitutes a PPE. But this observation contradicts the fact that u is the highest PPE payoff to the agent when the principal's payoff is π (as we have assumed that (π, u) is on the frontier U_r). Hence, we must have $u_n^A = U_r\left(\pi_n^A\right)$. An identical argument holds in the case of all other continuation payoffs. ■

Proof of Lemma 2. *Part* (i): **Step 1**. We claim that without loss of generality, we can assume that $\left(TT^A\right)$ binds. We prove this by contradiction. Given a strategy profile where $\left(TT^A\right)$ is slack, consider a new strategy where π_n^A is reduced by $\theta\varepsilon$ $(\varepsilon > 0)$ and π_s^A is increased by $(1-\theta)\,\varepsilon$, and all other aspects of the initial strategy profile are kept unchanged. Now, for ε sufficiently small, this new strategy satisfies all constraints that a PPE payoff must abide by when it is supported by the adaptive action, and yields a payoff $(\pi, \hat{u})$ where $\hat{u} \geq u$. To see this, note that as $b^A \geq 0$, we have $\pi_n^A > \pi_s^A$ (as $\left(TT^A\right)$ is slack). So, as U_r is concave, for ε sufficiently small we have

$$\theta U_r\left(\pi_s^A + (1-\theta)\,\varepsilon\right) + (1-\theta)\,U_r\left(\pi_n^A - \theta\varepsilon\right) \geq \theta U_r\left(\pi_s^A\right) + (1-\theta)\,U_r\left(\pi_n^A\right).$$

From Lemma 1 we know that the continuation payoffs are always on the frontier U_r. Hence, $\left(PK_A^A\right)$ implies that under the new strategy profile the agent's payoff $\hat{u} \geq u$. By construction, $\left(PK_P^A\right)$ remains unaltered; so $(\pi, \hat{u})$ satisfies (IR) and $\left(IC^A\right)$ is (weakly) relaxed. Also by construction, $\left(NN^A\right)$ is unaffected, and $\left(TT^A\right)$ continues to hold as it was slack to begin with. Finally, as $\pi_n^A > \pi_s^A$ and since the PPE payoff set $\mathcal{E}_r$ is convex, both $\left(\pi_s^A + (1-\theta)\,\varepsilon, U_r\left(\pi_s^A + (1-\theta)\,\varepsilon\right)\right)$ and $\left(\pi_n^A - \theta\varepsilon, U_r\left(\pi_n^A - \theta\varepsilon\right)\right)$ are in $\mathcal{E}_r$. So, $\left(SE^A\right)$ holds. Hence, if $\left(TT^A\right)$ is slack, either (π, u) is not on the frontier of $\mathcal{E}_r$, which is a contradiction, or one can construct a PPE where $\left(TT^A\right)$ binds and the players get the exact same payoff as before.

Step 2. Given that $\left(TT^A\right)$ binds, we have from $\left(PK_P^A\right)$ that

$$\begin{aligned} \pi &= \theta\left[(1-\delta)\,y + \delta\pi_s^A\right] + (1-\theta)\left[(1-\delta)\left(y - b^A\right) + \delta\pi_n^A\right] \\ &= (1-\delta)\,y + \delta\pi_s^A. \end{aligned}$$

This gives that

$$\pi_s^A = \frac{1}{\delta}\left(\pi - (1-\delta)\, y\right). \tag{A1}$$

As $\pi < y$ (note that the highest surplus that can be attained in a stage game is $y - C$), $\pi_s^A < \pi$.

Step 3. Next, we determine π_n^A. As (π, u) is on the frontier of $\mathcal{E}_r$ and is supported by $a = A$, we have $u = u_r^A(\pi)$. Moreover, $\pi + u_r^A(\pi)$ is the maximum joint payoff attainable in any PPE that uses the adaptive action in the current period and gives a payoff of π to the principal. From Lemma 1 we know that the continuation payoffs are on the frontier, and Step 1 and 2 of this proof show that in any such PPE, we can assume that $\left(TT^A\right)$ binds and π_s^A is constant (given π). Hence, we must have

$$\pi + u_r^A(\pi) = \max_{b^A;\ \tilde{\pi}_n^A} \quad (1-\delta)(y - C) + \delta\left[\theta\left(\pi_s^A + U_r\left(\pi_s^A\right)\right) + (1-\theta)\left(\tilde{\pi}_n^A + U_r\left(\tilde{\pi}_n^A\right)\right)\right]$$

$$\begin{aligned}
s.t.\ \ \theta\left[(1-\delta)(-c) + \delta U_r\left(\pi_s^A\right)\right] + (1-\theta)\left[(1-\delta)\left(b^A - c\right) + \delta U_r\left(\tilde{\pi}_n^A\right)\right] &\geq u^* && \left(IC^A\right)\\
-(1-\delta)\, b^A + \delta\tilde{\pi}_n^A &= \delta\pi_s^A && \left(TT^A\right)\\
b^A &\geq 0 && \left(NN^A\right)\\
0 \leq \tilde{\pi}_n^A &\leq \bar{\pi}_r, && \left(SE^A\right)
\end{aligned}$$

and the solution to the above program yields $\pi_n^A(\pi)$. Define

$$\bar{\pi}_r^* =: \sup\{\pi : U'_{r-}(\pi) \geq -1\}.$$

Below, we show that $\pi_n^A(\pi) = \bar{\pi}_r^* = \bar{\pi}_r$.

Step 4. First, we show that $\bar{\pi}_r^* = \bar{\pi}_r$. Suppose to the contrary that $\bar{\pi}_r^* < \bar{\pi}_r$. First note that $(\bar{\pi}_r^*, U_r(\bar{\pi}_r^*))$ is an extreme point and is therefore sustained by a pure action. Notice that it cannot be sustained by O. If so, then the associated continuation payoffs would be $(\bar{\pi}_r^*/\delta, U_r(\bar{\pi}_r^*)/\delta) >> (\bar{\pi}_r^*, U_r(\bar{\pi}_r^*))$, and it contradicts the definition of $\bar{\pi}_r^*$. Hence, $(\bar{\pi}_r^*, U_r(\bar{\pi}_r^*))$ is sustained either by A or R.

Step 5. Suppose that $(\bar{\pi}_r^*, U_r(\bar{\pi}_r^*))$ is sustained by A. Since $\left(TT^A\right)$ binds, we have

$$\pi_s^A(\bar{\pi}_r^*) = (\bar{\pi}_r^* - (1-\delta)\, y)/\delta < \bar{\pi}_r^*.$$

Step 5a: Now, if $b^A = 0$, from $\left(TT^A\right)$ we have $\pi_n^A(\bar{\pi}_r^*) = \pi_s^A(\bar{\pi}_r^*)$. Consider in this case a perturbation in which $\tilde{\pi}_s^A = \tilde{\pi}_n^A = \pi_s^A(\bar{\pi}_r^*) + \varepsilon$, and b^A unchanged. This perturbation satisfies all the constraints. It increases the payoff of the principal by $\delta\varepsilon$ and changes the agent's payoff by

$$\delta\left(U_r\left(\pi_s^A(\bar{\pi}_r^*) + \varepsilon\right) - U_r\left(\pi_s^A(\bar{\pi}_r^*)\right)\right) \geq -\delta\varepsilon,$$

where the inequality follows as U_r is concave and $\pi_s^A(\bar{\pi}_r^*) < \bar{\pi}_r^*$. But this implies (along with concavity of U_r) that $U_r(\bar{\pi}_r^* + \varepsilon\delta) \geq U_r(\bar{\pi}_r^*) - \varepsilon\delta$, contradicting the definition of $\bar{\pi}_r^*$.

Step 5b: Next, if $b^A > 0$, we then consider a perturbation in which $\tilde{\pi}_s^A = \pi_s^A(\bar{\pi}_r^*) + (1-\delta)\varepsilon$, $\tilde{b} = b^A(\bar{\pi}_r^*) - \delta\varepsilon$ and $\pi_n^A(\bar{\pi}_r^*)$ unchanged. This perturbation again satisfies all the constraints. It increases the payoff of the principal by $(1-\delta)\delta\varepsilon$ and changes the agent's payoff by

$$\theta\delta\left(U_r\left(\pi_s^A(\bar{\pi}_r^*) + (1-\delta)\varepsilon\right) - U_r\left(\pi_s^A(\bar{\pi}_r^*)\right)\right) - (1-\theta)\delta(1-\delta)\varepsilon \geq -\delta(1-\delta)\varepsilon.$$

The inequality again follows because U_r is concave and $\pi_s^A(\bar{\pi}_r^*) < \bar{\pi}_r^*$. And this again implies that $U_r(\bar{\pi}_r^* + \varepsilon\delta(1-\delta)) \geq U_r(\bar{\pi}_r^*) - \varepsilon\delta(1-\delta)$, contradicting the definition of $\bar{\pi}_r^*$.

Step 6. Next, suppose that $(\bar{\pi}_r^*, U_r(\bar{\pi}_r^*))$ is sustained by R. In this case, if $b^R > 0$ or if $b^R = 0$ and $\pi_n^A(\bar{\pi}_r^*) = \pi_s^A(\bar{\pi}_r^*) < \bar{\pi}_r^*$, the same perturbations as described above lead to contradictions. It remains to derive contradiction for $b^R = 0$ and $\pi_n^R(\bar{\pi}_r^*) = \pi_s^R(\bar{\pi}_r^*) \geq \bar{\pi}_r^*$. Notice that if $\pi_n^R(\bar{\pi}_r^*) = \pi_s^R(\bar{\pi}_r^*) = \bar{\pi}_r^*$, we then have $U_r(\bar{\pi}_r^*) = -c$, which is impossible. If $\pi_n^R(\bar{\pi}_r^*) = \pi_s^R(\bar{\pi}_r^*) > \bar{\pi}_r^*$, consider a deviation where $\tilde{\pi}_s^R = \tilde{\pi}_n^R = \pi_s^R(\bar{\pi}_r^*) - \varepsilon$, and b^R remain to be 0. This perturbation satisfies all the constraints. It decreases the payoff of the principal by $\delta\varepsilon$ and increases the agent's payoff by

$$\delta\left(U_r\left(\pi_s^A(\bar{\pi}_r^*) - \varepsilon\right) - U_r\left(\pi_s^A(\bar{\pi}_r^*)\right)\right) > \delta\varepsilon,$$

where the strict inequality follow from the definition of $\bar{\pi}_r^*$ and that both $\pi_s^A(\bar{\pi}_r^*) > \bar{\pi}_r^*$ and U_r is concave. But this implies $U'_{r-}(\bar{\pi}_r^*) < -1$, which is a contradiction. This finishes showing that $\bar{\pi}_r^* = \bar{\pi}_r$ so that $U'_{r-}(\pi) \geq -1$ for all π.

Step 7. Given that $U'_{r-}(\pi) \geq -1$ for all π, it is then without loss of generality to choose $\pi_n^A(\pi) = \bar{\pi}_r$. To see this, suppose to the contrary that $\pi_n^A(\pi) < \bar{\pi}_r$. Now consider an alternative profile where $\tilde{\pi}_n^A = \pi_n^A(\pi) + (1-\delta)\varepsilon$ and $\tilde{b}^A = b^A + \delta\varepsilon$. Under this perturbation, the principal's payoff is preserved and so are all the constraints. The agent's payoff changes by

$$\delta\left(U_r\left(\pi_n^A(\pi) + (1-\delta)\varepsilon\right) - U_r\left(\pi_n^A(\pi)\right)\right) + (1-\delta)\delta\varepsilon \geq 0,$$

where the inequality holds because $U'_{r-}(\pi) \geq -1$ for all π. This implies that this perturbation yields a payoff that is also on the payoff frontier (U_r), and therefore, we can keep increase $\pi_n^A(\pi)$ (adjusting b^A accordingly) until $\pi_n^A(\pi) = \bar{\pi}_r$.

Step 8. Finally, we show that $U'_{r-}(\pi) > -1$ for all π, so the choice of $\pi_n^A(\pi)$ is unique. To see this, suppose to the contrary that there exists a exists a $\pi^* < \bar{\pi}_r$ where $U'_{r-}(\pi) = -1$ for $\pi \in [\pi^*, \bar{\pi}_r]$, where π^* is the left end point of this line segment. Notice that $(\pi^*, U(\pi^*))$ is an extreme point, and Assumption 1 ensures that this point is sustained by A. Now $\left(TT^A\right)$ again implies that $\pi_s^A\left(\pi^*\right) < \pi^*$. Also note that

$$b^A\left(\pi^*\right) = y - \left(\pi^* - \delta\overline{\pi}_r\right)/\left(1-\delta\right) > y - \left(\overline{\pi}_r - \delta\overline{\pi}_r\right)/\left(1-\delta\right) > 0.$$

Now consider the following perturbation: decrease $b^A\left(\pi^*\right)$ by $\delta\varepsilon$, increase $\pi_s^A\left(\pi^*\right)$ by $\left(1-\delta\right)\varepsilon$ and keep the rest unchanged. Under this perturbation, all constraints are satisfied. The principal's payoff increases by $\delta\left(1-\delta\right)\varepsilon$. The agent's payoff changes by

$$\theta\delta\left(U_r\left(\pi_s^A\left(\pi^*\right) + \left(1-\delta\right)\varepsilon\right) - U_r\left(\pi_s^A\left(\pi^*\right)\right)\right) - \left(1-\theta\right)\left(1-\delta\right)\delta\varepsilon > -\delta\left(1-\delta\right)\varepsilon,$$

implying that this perturbation generates a payoff that exceeds $U_r\left(\pi^* + \delta\left(1-\delta\right)\varepsilon\right)$. This is a contradiction. This implies that we must have $\pi_n^A\left(\pi\right) = \bar{\pi}_r$.

Step 9. As $\left(TT^A\right)$ binds, $(A1)$ implies

$$b^A = y - \frac{\pi - \delta\bar{\pi}_r}{1-\delta}.$$

This observation completes the proof of Part (i).

Part (ii): The proof is identical to that of Part (i). As above, we may assume that $\left(TT^R\right)$ binds with equality; and thus, $\left(PK_P^R\right)$ implies

$$\pi = \left(1-\delta\right)py + \delta\pi_s^R.$$

This gives that

$$\pi_s^R = \frac{1}{\delta}\left(\pi - \left(1-\delta\right)py\right). \tag{A2}$$

Next, because $U'_{r-}\left(\pi\right) > -1$ for all π, the same argument as above gives that

$$\pi_n^R = \bar{\pi}_r.$$

Now, the formula for b^R follows from $\left(TT^A\right)$.

Part (iii): Immediate from $\left(PK^O\right)$. ■

In order to prove Proposition 1, we first prove the following lemma.

Lemma 6. *$U_r\left(0\right) = 0$ and $\left(0,0\right)$ is sustained by $a = O$. Furthermore, if for some $\tilde{\pi} > 0$, $\left(\tilde{\pi}, U_r\left(\tilde{\pi}\right)\right)$ is sustained by $a = O$, then for all $\pi \leq \tilde{\pi}$, $U_r\left(\pi\right) = u_r^O\left(\pi\right)$. Hence, there exists a cutoff π_r^O such that U_r is a straight line between $\left(0,0\right)$ and $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$, and $U_r\left(\pi\right) = u_r^O\left(\pi\right)$ if and only if $\pi \leq \delta\pi_r^O$.*

Proof. **Step 1**. As $(0, U_r(0))$ is an extreme point, it must be sustained by a pure action. But it is routine to check that $(0, U_r(0))$ cannot be sustained by $a = A$ or R, as the promise-keeping and truth-telling constraints cannot be satisfied simultaneously. As only $a = O$ is feasible, from $\left(PK^O\right)$ we have $\pi^O = 0$. So, the unique PPE that supports $(0, U_r(0))$ is one where both players take their outside options in all periods. Hence, $(0, U_r(0)) = (0, 0)$.

Step 2. From Lemma 1 and $\left(PK^O\right)$ we have

$$u_r^O(\pi) = \delta U_r\left(\pi^O(\pi)\right) = \delta U_r(\pi/\delta)$$

for all $\pi \in [0, \delta\bar{\pi}_r]$ (i.e., for all π where $u_r^O(\pi)$ is well-defined). Hence,

$$u_{r-}^{O\prime}(\pi) = U_{r-}^{\prime}(\pi/\delta) \leq U_{r-}^{\prime}(\pi)$$

for all $\pi \in (0, \delta\bar{\pi}_r)$, where the inequality follows from the concavity of U_r (notice that by virtue of being concave, the left- and right-derivative of U_r always exist in the interior of its domain). But as $u_r^O(\tilde{\pi}) = U_r(\tilde{\pi})$, this implies that $u_r^O(\pi) \geq U_r(\pi)$ for all $\pi \leq \tilde{\pi}$. But as $U_r(\pi) \geq u_r^O(\pi)$, we have $U_r(\pi) = u_r^O(\pi)$ for all $\pi \leq \tilde{\pi}$.

Step 3. As $U_r(\pi) = u_r^O(\pi)$ for all $\pi \leq \tilde{\pi}$, we have $U_{r-}^{\prime}(\pi) = u_{r-}^{O\prime}(\pi)$. So, from step 2 above, $u_{r-}^{O\prime}(\pi) = U_{r-}^{\prime}(\pi/\delta) = U_{r-}^{\prime}(\pi)$, and since U_r is concave, this implies that U_r is a straight line passing through $(0, 0)$ and extends at least up to the point $(\tilde{\pi}, U_r(\tilde{\pi}))$. Denote the right-most end-point of this line as $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$.

Step 4. Take any $(\pi, U_r(\pi))$ such that $\pi/\delta \leq \pi_r^O$. We claim that such a payoff is sustainable by $a = O$. Note that the associated continuation payoffs $\left(\pi^O, u^O\right) = (\pi/\delta, U_r(\pi/\delta))$ (using Lemma 1 and $\left(PK^O\right)$), and hence, $\left(SE^O\right)$ is satisfied. Finally, $\left(PK^O\right)$ for the agent holds as $U_r(\pi) = \delta U_r(\pi/\delta)$ since U_r is linear.

Step 5. But if $\pi/\delta > \pi_r^O$, then the payoff $(\pi, U_r(\pi))$ cannot be sustained by $a = O$. The argument is as follows. If $\pi_r^O < \bar{\pi}_r$, we have

$$U_r(\pi) > (1 - \delta) U_r(0) + \delta U_r(\pi/\delta) = \delta U_r(\pi/\delta).$$

The inequality follows from the fact that $U_r(\pi')$ is concave and the segment starting from $(0, 0)$ is linear if only if $\pi' < \pi_r^O$ whereas $\pi/\delta > \pi_r^O$. Also the equality follows from $U_r(0) = 0$. But this implies that $\left(PK^O\right)$ for the agent is violated, and hence, $(\pi, U_r(\pi))$ cannot be supported by $a = O$. And if $\pi_r^O = \bar{\pi}_r$, the proof is immediate as by $\left(PK^O\right)$ any point sustained by $a = O$ requires $\pi^O(\pi) = \pi/\delta \leq \bar{\pi}_r = \pi_r^O$. ∎

Lemma 7. *Both the rigid and the adaptive action are used on the payoff frontier. In particular,* $\pi_r^O < \bar{\pi}_r$*, and* $(\bar{\pi}_r, U_r(\bar{\pi}_r))$ *is sustained by the adaptive action whereas* $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$ *is sustained by the rigid action.*

Proof. First, consider the case of the *adaptive action.* We prove this by constructing a stationary PPE with associated payoffs (π^*, u^*) where $\pi^* > py$ (though the payoffs need not be on the frontier). As py is an upper bound on the principal's payoff in any PPE where the adaptive action is never used, the adaptive action must be used on the payoff frontier. The proof is given by the following steps.

Step A1. Consider the following stationary strategy profile where in each period, the agent chooses the adaptive action, receives a bonus of $b^* \geq 0$ in the no-shock state and gets a payoff of $u^* = \frac{1-\delta}{1-p}(pC - c)$. When the principal claims that it is a shock state, the relationship terminates. Denote the principal's associated payoff as π^*. For this strategy profile to be a PPE, (π^*, u^*) must satisfy all constraints given in Section 3.1 for the case of adaptive action. Note that $\left(IC^A\right)$ and (IR) for the agent are trivially satisfied when $u = u^*$. Also notice that the (NR) constraint in this case is the same as $\left(TT^A\right)$. Hence, it remains to check if the following constraints are satisfied:

$$\begin{aligned} u^* &= (1-\delta)(-C) + (1-\theta)((1-\delta)b^* + \delta u^*), && \left(PK_A^A\right) \\ \pi^* &= (1-\delta)y + (1-\theta)(-(1-\delta)b^* + \delta\pi^*), && \left(PK_P^A\right) \\ &-(1-\delta)b^* + \delta\pi^* \geq 0, && \left(TT^A\right) \\ \pi^* &\geq 0. && (IR) \end{aligned}$$

Since the proposed strategy profile is stationary, if (π^*, u^*) satisfies the above constraints, it also satisfies $\left(SE^A\right)$.

Step A2. From $\left(PK_A^A\right)$, we obtain

$$(1-\theta)b^* = C + \frac{1-(1-\theta)\delta}{1-\delta}u^* =: K. \tag{A3}$$

And using $\left(PK_A^A\right)$ and $\left(PK_P^A\right)$ we have

$$\pi^* + u^* = (1-\delta)(y - C) + (1-\theta)(\delta u^* + \delta\pi^*).$$

Hence,

$$\pi^* = \frac{(1-\delta)(y-C)}{1-(1-\theta)\delta} - u^* = \frac{(1-\delta)(y-K)}{1-(1-\theta)\delta}. \tag{A4}$$

Step A3. We claim that u^*, b^* as given in $(A3)$, and π^* as given in $(A4)$ satisfy all four constraints given above. Trivially $\left(PK_A^A\right)$ and $\left(PK_P^A\right)$ are satisfied by construction. To see that $\left(TT^A\right)$ holds (and hence (IR) holds as well), note that using $(A3)$ and $(A4)$, $\left(TT^A\right)$ can be written as

$$\frac{\delta\left(1-\delta\right)\left(y-K\right)}{1-\left(1-\theta\right)\delta} \geq \left(1-\delta\right)b^* \Leftrightarrow y \geq \frac{K}{\left(1-\theta\right)\delta}.$$

But this is true by Assumption 1 (iii). Hence, the above strategy profile constitutes a PPE.

Step A4. Finally, we have $\pi^* > py$ as using $(A4)$ it boils down to

$$\left(\left(1-\delta\right)\left(1-p\right)-p\theta\delta\right)y > \left(1-\delta\right)K,$$

which is the case by Assumption 1 (iii). But this implies that $\left(\bar{\pi}_r, U_r\left(\bar{\pi}_r\right)\right)$ must be supported by adaptive action. Notice that as $\left(\bar{\pi}_r, U_r\left(\bar{\pi}_r\right)\right)$ is an extreme point, it must be sustained by a pure action. But it cannot be sustained by $a = O$, as then by Lemma 2 we have

$$\pi^O\left(\bar{\pi}_r\right) = \frac{1}{\delta}\bar{\pi}_r > \bar{\pi}_r,$$

which is a contradiction. Also, if $\left(\bar{\pi}_r, U_r\left(\bar{\pi}_r\right)\right)$ is sustained by $a = R$ instead, we have

$$\pi_s^R\left(\bar{\pi}_r\right) = \frac{1}{\delta}\left(\bar{\pi}_r-\left(1-\delta\right)py\right) > \bar{\pi}_r,$$

(the last inequality follows because $\bar{\pi}_r \geq \pi^* > py$) and this is a contradiction as well.

Next, consider the case of the *rigid action*. The proof is given by the following steps (the reader may note that the steps R1 and R2 below are more elaborate than what is necessary for this proof, but we adopt this approach as it remains applicable for the proof of Lemma 9 below, and hence, it allows us to avoid repetition).

Step R1. Suppose to the contrary that rigid action is not used. So by Lemma 6, it follows that $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$ must be sustained by the adaptive action, and hence, $u_r^A\left(\pi_r^O\right) = U_r\left(\pi_r^O\right)$. Let s be the slope between $(0,0)$ and $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$. As $\pi_s^A\left(\pi_r^O\right) < \pi_r^O$, by APS bang-bang result and Lemma 6 we have $\pi_s^A\left(\pi_r^O\right) = 0$ as $(0,0)$ is the only extreme point to the left of π_r^O. Hence, we have $\pi_r^O = \left(1-\delta\right)y$, and therefore,

$$s := \frac{U_r\left(\left(1-\delta\right)y\right)}{\left(1-\delta\right)y}.$$

Furthermore, from $\left(PK_A^A\right)$ and $\left(PK_P^A\right)$ (and using the fact that $\pi_s^A\left(\pi_r^O\right) = 0$ and $\pi_n^A\left(\pi\right) = \bar{\pi}_r$) we have

$$\begin{aligned} U_r\left(\left(1-\delta\right)y\right) &= -\left(1-\delta\right)C+\left(1-\theta\right)\delta\left(\bar{\pi}_r+U_r\left(\bar{\pi}_r\right)\right) \\ &\leq -\left(1-\delta\right)C+\left(1-\theta\right)\delta\left(y-C\right), \end{aligned}$$

where the inequality follows because $y - C$ is the aggregate surplus under efficiency. So,

$$s \leq \frac{-(1-\delta)C + (1-\theta)\delta(y-C)}{(1-\delta)y}.$$

Step R2. Next, consider a strategy profile where the agent chooses the rigid action, bonus payment is b^R as given in Lemma 2, and the continuation payoffs following shock and no-shock states are $(0,0)$ and $(\bar{\pi}_r, U_r(\bar{\pi}_r))$ respectively. Under this strategy profile, the principal's payoff is $(1-\delta)py$ (from $(A2)$), and the agent's payoff $u = u_r^R((1-\delta)py)$ satisfies

$$u_r^R((1-\delta)py) + (1-\delta)py = (1-\delta)(py-c) + (1-\theta)\delta(\bar{\pi}_r + U_r(\bar{\pi}_r)).$$

It follows that

$$S := \frac{u_r^R((1-\delta)py)}{(1-\delta)py} = \frac{-(1-\delta)c + (1-\theta)\delta(\bar{\pi}_r + U_r(\bar{\pi}_r))}{(1-\delta)py}.$$

Step R3. We claim that $S > s$. Since $(\bar{\pi}_r, U_r(\bar{\pi}_r))$ is sustained by the adaptive action, we have $U_r(\bar{\pi}_r) \geq u^*$ (by $\left(IC^A\right)$). Also, by definition $\bar{\pi}_r \geq \pi^*$. Hence, using $(A4)$ we obtain

$$\bar{\pi}_r + U_r(\bar{\pi}_r) \geq \pi^* + u^* = \frac{(1-\delta)(y-C)}{1-(1-\theta)\delta}.$$

So,

$$S \geq \frac{1}{(1-\delta)py}\left(-(1-\delta)c + \frac{(1-\theta)\delta(1-\delta)(y-C)}{1-(1-\theta)\delta}\right)$$

Now,

$$-(1-\delta)c + \frac{(1-\theta)\delta(1-\delta)(y-C)}{1-(1-\theta)\delta} > p(-(1-\delta)C + (1-\theta)\delta(y-C)),$$

as it can be rearranged as

$$(1-\delta)(pC-c) + \frac{(1-\theta)\delta(y-C)}{1-(1-\theta)\delta}((1-\delta)(1-p) - p\delta\theta) > 0,$$

which is the case by Assumption 1 (ii) and (iii).

Step R4. As $S > s$ we have

$$\frac{u_r^R((1-\delta)py)}{(1-\delta)py} > \frac{U_r((1-\delta)y)}{(1-\delta)y} = \frac{U_r((1-\delta)py)}{(1-\delta)py}$$

(recall that U_r is a straight line between $(0,0)$ and $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$, and $(1-\delta)py < (1-\delta)y \leq \pi_r^O$). But this implies

$$u_r^R((1-\delta)py) > U_r((1-\delta)py),$$

which is a contradiction. Therefore, $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$ must be sustained by the rigid action.

Notice that as $\pi_r^O \leq \bar{\pi}_r$ and $\bar{\pi}_r$ can only be sustained by the adaptive action whereas π_r^O can only be sustained by the rigid action, we have $\pi_r^O \neq \bar{\pi}_r$. Hence, $\pi_r^O < \bar{\pi}_r$. ■

Lemma 8. *If $u_r^A(\pi') \geq u_r^R(\pi')$ for some π', then $u_r^A(\pi) \geq u_r^R(\pi)$ for all $\pi \geq \pi'$.*

Proof. Adding (PK_P^A) and (PK_A^A) we obtain that

$$\pi + u_r^A(\pi) = (1-\delta)(y-C) + \theta\delta\left(\pi_s^A + U_r\left(\pi_s^A\right)\right) + (1-\theta)\delta\left(\bar{\pi}_r + U_r(\bar{\pi}_r)\right),$$

where

$$\pi_s^A = \frac{1}{\delta}\left(\pi - (1-\delta)y\right).$$

Similarly, adding (PK_P^R) and (PK_A^R) we obtain that

$$\pi + u_r^R(\pi) = (1-\delta)(py-c) + \theta\delta\left(\pi_s^R + U_r\left(\pi_s^R\right)\right) + (1-\theta)\delta\left(\bar{\pi}_r + U_r(\bar{\pi}_r)\right),$$

where

$$\pi_s^R = \frac{1}{\delta}\left(\pi - (1-\delta)py\right).$$

This implies that

$$\begin{aligned} u_r^A(\pi) - u_r^R(\pi) = & \quad (1-\delta)\left((1-p)y - C + c\right) + \theta\delta\left(\pi_s^A + U_r\left(\pi_s^A\right)\right) - \theta\delta\left(\pi_s^R + U_r\left(\pi_s^R\right)\right) \\ = & \quad (1-\delta)\left[(1-p)y(1-\theta) - C + c\right] + \theta\delta\left(U_r\left(\pi_s^A\right) - U_r\left(\pi_s^R\right)\right). \end{aligned}$$

As a result,

$$u_{r+}^{A\prime}(\pi) - u_{r+}^{R\prime}(\pi) = \theta\left(U_r'\left(\pi_s^A\right) - U_r'\left(\pi_s^R\right)\right) \geq 0$$

since $\pi_s^A = \frac{1}{\delta}\left(\pi - (1-\delta)y\right) < \frac{1}{\delta}\left(\pi - (1-\delta)py\right) = \pi_s^R$ and U_r is concave. In other words, if $u_r^A(\pi') \geq u_r^R(\pi')$ for some π', then $u_r^A(\pi) \geq u_r^R(\pi)$ for all $\pi \geq \pi'$. ■

Proof of Proposition 1. *Part* (i): That $U_r(0) = 0$, the existence π_r^O, and the linearity of U_r between $(0,0)$ and $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$ are proved in Lemma 6. By virtue of linearity, any payoff in this line segment can be supported by randomization between the two end points. Hence, without loss of generality, we can also assume that $a = O$ is played on the frontier only to support $(0,0)$ payoff.

For expositional clarity, below we prove parts (ii) and (iv) first, and then prove part (iii).

Part (ii) and Part (iv): Recall from Lemma 7 (step R3) that $\left(\pi_r^O, U_r\left(\pi_r^O\right)\right)$ is supported by $a = R$ and that $\pi < \pi_r^O$ is supported by randomization (by Part (i) above). Lemma 7 also shows that the adaptive action is used on the frontier, and Lemma 8 implies (along with the fact that $u_r^A(\pi)$ and $u_r^R(\pi)$ are concave functions) that the set of π values such that

$(\pi, U_r(\pi))$ is supported by each of these two actions ($a = A$ and R) are intervals (potentially containing a single point only) on $[0, \bar{\pi}_r]$. Moreover, if $(\pi, U_r(\pi))$ is supported by $a = R$ and $(\pi', U_r(\pi'))$ is supported by $a = A$, then it must be that $\pi' > \pi$. Hence, there exists two cutoffs π_r^R and π_r^A where $\pi_r^O \leq \pi_r^R \leq \pi_r^A \leq \bar{\pi}_r$ (but $\pi_r^O < \bar{\pi}_r$) such that $(\pi, U_r(\pi))$ is supported by $a = R$ for $\pi \in [\pi_r^O, \pi_r^R)$ and by $a = A$ for $\pi \in [\pi_r^A, \bar{\pi}_r]$.

It remains to show that $U_r(\bar{\pi}_r) = u^*$. Suppose on the contrary $U_r(\bar{\pi}_r) > u^*$ (from (IC^A) we must have $U_r(\bar{\pi}_r) \geq u^*$). Since $(\bar{\pi}_r, U_r(\bar{\pi}_r))$ is supported by $a = A$, by Lemma 7, we know that the associated continuation payoffs for the principal in the shock and no-shock states are $\pi_s^A < \bar{\pi}_r$ and $\bar{\pi}_r$, respectively. And the associated bonus payment $b^A = y - \bar{\pi}_r > 0$ as $y - C$ is an upper bound on $\bar{\pi}_r$. Now, consider an alternate strategy profile where the agent is asked choose A, the principal's continuation payoffs in the shock and no-shock states are $\hat{\pi}_s^A = \pi_s^A + \varepsilon$ and $\bar{\pi}_r$, respectively, and the bonus is $\hat{b}^A = b^A - \delta\varepsilon/(1-\delta)$ where $\varepsilon > 0$. From (PK_A^A) and (PK_P^A) we obtain the associated payoffs as

$$\hat{u} = \theta\left[(1-\delta)(-C) + \delta U_r\left(\pi_s^A + \varepsilon\right)\right] + (1-\theta)\left[(1-\delta)\left(b^A - \delta\varepsilon/(1-\delta) - C\right) + \delta U_r(\bar{\pi}_r)\right]$$

and

$$\hat{\pi} = \theta\left[(1-\delta)y + \delta\left(\pi_s^A + \varepsilon\right)\right] + (1-\theta)\left[(1-\delta)\left(y - b^A + \delta\varepsilon/(1-\delta)\right) + \delta\bar{\pi}_r\right]$$

Observe that under the new strategy profile (NR^A) is relaxed, (TT^A) remains unaltered by construction. Moreover, for ε sufficiently small, both (IC^A) and (NN^A) remain slack (i.e., $\hat{u} > u^*$ and $\hat{b}^A > 0$), and $\hat{\pi}_s^A < \bar{\pi}_r$ so that (SE^A) is satisfied as well. Hence, the proposed strategy profile constitutes a PPE where the principal's payoff is $\hat{\pi} = \bar{\pi}_r + \delta\varepsilon > \bar{\pi}_r$, which is a contradiction.

Part (iii): If $\pi_r^R < \pi_r^A$, it implies that any payoff $(\pi, U(\pi))$ where $\pi \in [\pi_r^R, \pi_r^A)$ cannot be supported by any of the three pure actions. But such $(\pi, U(\pi)) \in \mathcal{E}_r$ as $\mathcal{E}_r$ is convex. So, $(\pi, U(\pi))$ must be supported by randomization between a payoff that is supported by $a = R$ and one that is supported by $a = A$. Consequently, $U_r(\pi)$ is linear on this interval. Also, it is without loss of generality to assume that we randomize between the end points $\left(\pi_r^R, U\left(\pi_r^R\right)\right)$ and $\left(\pi_r^A, U\left(\pi_r^A\right)\right)$. ■

In order to prove Proposition 2, we first prove the following lemma.

Lemma 9. *The PPE frontier U satisfies the following properties:*

(i) $U(0) = 0$ *and if for some* $\tilde{\pi} > 0$, $(\tilde{\pi}, U(\tilde{\pi}))$ *is sustained by* $a = \mathcal{O}$, *then for all* $\pi \leq \tilde{\pi}$, $U(\pi) = u^{\mathcal{O}}(\pi)$. *Hence, there exists a cutoff* π^O *such that* U *is a straight line between* $(0,0)$ *and* $(\pi^O, U(\pi^O))$, *and* $U(\pi) = u^{\mathcal{O}}(\pi)$ *if and only if* $\pi \leq \delta\pi^O$. Moreover, $\pi^O = \pi_r^O = (1-\delta)py$.

(ii) $U(\bar{\pi}) = 0$, $\bar{\pi} > \bar{\pi}_r$, and $U'_-(\pi) > -1$.

(iii) The adaptive action ($a = \mathcal{A}$) *is used on* U. *Moreover, if* $U(\pi') = u^{\mathcal{A}}(\pi')$ *for some* π' *then* $U(\pi) = u^{\mathcal{A}}(\pi)$ *for all* $\pi \geq \pi'$.

Proof. Part (i): To see that $\pi^O = \pi_r^O$, note that by the same argument as given in Lemma 7, π^O must be supported by $a = R$ (as is the case for π_r^O). Also, note that both U and U_r are straight lines where the left-most point is $(0,0)$ in both cases, and the right-most points are $(\pi^O, U(\pi^O))$ and $(\pi_r^O, U(\pi_r^O))$, respectively. Hence, by APS bang-bang result, $\pi_s^R(\pi^O) = \pi_s^R(\pi_r^O) = 0$. But as $\pi_s^R(\pi) = \frac{1}{\delta}(\pi - (1-\delta)py)$, we have $\pi^O = \pi_r^O = (1-\delta)py$. The proof of all other statements of this part are identical to the proof of part (i) of Lemma 6.

Part (ii): The proof of $U(\bar{\pi}) = 0$ is similar to that of $U_r(\bar{\pi}_r) = u^*$ given above. To see that $\bar{\pi} > \bar{\pi}_r$, notice that $U(\pi) \geq U_r(\pi)$ and, hence, $\bar{\pi} \geq \bar{\pi}_r$ as the principal always has the option of revealing the rigid action at the beginning of the game. In addition, $U(\bar{\pi}_r) \geq U_r(\bar{\pi}_r) > U(\bar{\pi}) = 0$. So, it must be that $\bar{\pi} > \bar{\pi}_r$. Finally, it follows from the same argument as the proof of $U'_r(\pi) > -1$ (in Step 3 of the proof of Lemma 2 Part (i)) that $U'_-(\pi) > -1$. The details are omitted.

Part (iii): Since $\bar{\pi} > \bar{\pi}_r$, $(\bar{\pi}, U(\bar{\pi})) \notin \mathcal{E}_r$. Hence, it cannot be sustained by $a \in \{A, R, O\}$. Also, it cannot be sustained by $a = \mathcal{O}$ as the continuation payoff $\pi^{\mathcal{O}}(\bar{\pi}) = \bar{\pi}/\delta$ is not feasible. Hence, it must be sustained by $a = \mathcal{A}$; in other words, $a = \mathcal{A}$ is used on U. Next, we show that if $(\pi', U(\pi'))$ is supported by $\mathcal{A}$ for some π', then $u^{\mathcal{A}}(\pi) \geq U_r(\pi)$ for all $\pi > \pi'$. This implies that $U(\pi) = u^{\mathcal{A}}(\pi)$ for all $\pi > \pi'$. The proof is given by the following steps.

Step 1. Let π_l be the smallest π such that $U(\pi) = u^{\mathcal{A}}(\pi)$ (π_l exists as both U and $u^{\mathcal{A}}$ are continuous). Notice that $\pi_l \geq (1-\delta)y$ as $\pi_s^{\mathcal{A}}(\pi_l) \geq 0$. Now we first show that for all $\pi < \pi_l$, either $U(\pi) = U_r(\pi)$ or $U(\pi)$ is sustained by randomization. The argument is as follows. Since $(\pi, U(\pi))$ is not supported by $a = \mathcal{A}$, we have $U(\pi) > u^{\mathcal{A}}(\pi) \geq u_r^A(\pi)$. Hence, $(\pi, U(\pi))$ is either supported by a pure action $a \in \{\mathcal{O}, O, R\}$ or by a randomization. But if $(\pi, U(\pi))$ is supported by any of these pure actions, it must be that $U(\pi) = U_r(\pi)$. Recall that $\pi^O = \pi_r^O = (1-\delta)py$ and the actions $\mathcal{O}$ and O are never used for any $\pi > \delta\pi^O$.

Hence, for $\pi \in [0,(1-\delta)\,py]$, $U(\pi) = U_r(\pi)$ as both are straight lines between $(0,0)$ and $\left(\pi^O, u_r^R\left(\pi^O\right)\right)$. Moreover, if $(\pi, U(\pi))$ is supported by $a = R$ for some $\pi \in ((1-\delta)\,py, \pi_l]$, we trivially have $U(\pi) = U_r(\pi) = u^R(\pi)$.

Step 2. Next, we claim that for all $\pi < \pi_l$,

$$\frac{d}{d\pi}U(\pi) \geq \frac{d}{d\pi}U_r(\pi)\,. \tag{A6}$$

This is trivially the case if for all $\pi < \pi_l$, $(\pi, U(\pi))$ is supported by a pure action $a \in \{\mathcal{O}, O, R\}$, and hence, $U(\pi) = U_r(\pi)$. Now suppose that $(\pi, U(\pi))$ is sustained by a randomization. Denote the left point of the randomization be π_L; so, $U'(\pi) = U'(\pi_L)$. Moreover, we can argue that $U'(\pi_L) \geq U_r'(\pi_L)$. Since $(\pi_L, U(\pi_L))$ is supported by $a = R$, $u_r^R(\pi_L) = U(\pi_L)$. Also, $U(\pi_L) \geq U_r(\pi_L)$ (as the inequality holds for all π), and $U_r(\pi_L) \geq u_r^R(\pi_L)$ as U_r is the frontier when the rigid action is available. So, we obtain $U(\pi_L) = U_r(\pi_L)$, and the fact that $U(\pi) \geq U_r(\pi)$ for all π, implies $U'(\pi_L) \geq U_r'(\pi_L)$. But also note that $U_r'(\pi_L) \geq U_r'(\pi)$ as U_r is concave. Combining these observations we have $U'(\pi) = U'(\pi_L) \geq U_r'(\pi_L) \geq U_r'(\pi)$, as claimed in $(A6)$.

Step 3. Now, suppose to the contrary that $u^{\mathcal{A}}(\pi) < U_r(\pi)$ for some $\pi > \pi_l$. It follows that there exists some $\hat{\pi} \in (\pi_l, \pi)$ such that

$$\frac{d}{d\pi}u^{\mathcal{A}}(\hat{\pi}) < \frac{d}{d\pi}U_r(\hat{\pi})\,,$$

where the derivative can be thought of as the right or left derivative (with the proper inequalities) when the derivative fails to exist. Let

$$D := \left\{\pi : \frac{d}{d\pi}u^{\mathcal{A}}(\pi) < \frac{d}{d\pi}U_r(\pi) \;\middle|\; \pi \geq \pi_l\right\}.$$

Note that for any $\tilde{\pi} \in [\pi_L, \inf D)$, we have $u^{\mathcal{A}}(\tilde{\pi}) = U(\tilde{\pi})$, and therefore,

$$\frac{d}{d\pi}u^{\mathcal{A}}(\tilde{\pi}) = \frac{d}{d\pi}U(\tilde{\pi})\,.$$

Step 4. Take a $\pi_m \in D$ such that $\pi_s^{\mathcal{A}}(\pi_m) < \inf D$. Since $\pi_m \in D$, we have

$$\frac{d}{d\pi}u^{\mathcal{A}}(\pi_m) < \frac{d}{d\pi}U_r(\pi_m)\,.$$

But since

$$\pi + u^{\mathcal{A}}(\pi) = (1-\delta)(y-C) + \delta\left[\theta\left(\pi_s^{\mathcal{A}}(\pi) + U\left(\pi_s^{\mathcal{A}}(\pi)\right)\right) + (1-\theta)\left(\bar{\pi} + U(\bar{\pi})\right)\right],$$

we have,

$$\frac{d}{d\pi}u^{\mathcal{A}}(\pi)=\theta\frac{d}{d\pi}U\left(\pi_s^{\mathcal{A}}(\pi)\right)-(1-\theta)\,. \tag{A7}$$

Similarly, for $a\in\{A,R\}$, we obtain

$$\frac{d}{d\pi}u_r^{a}(\pi)=\theta\frac{d}{d\pi}U_r\left(\pi_s^{a}(\pi)\right)-(1-\theta)\,. \tag{A8}$$

Using $(A7)$ and $(A8)$ along with the fact that $\pi_s^{\mathcal{A}}(\pi)=\pi_s^{A}(\pi)$, we obtain:

$$\begin{aligned}\frac{d}{d\pi}U_r(\pi_m) &\leq \max\left\{\frac{d}{d\pi}u_r^{A}(\pi_m),\frac{d}{d\pi}u_r^{R}(\pi_m)\right\}\\ &= \theta\max\left\{\frac{d}{d\pi}U_r\left(\pi_s^{A}(\pi_m)\right),\frac{d}{d\pi}U_r\left(\pi_s^{R}(\pi_m)\right)\right\}-(1-\theta)\\ &= \theta\frac{d}{d\pi}U_r\left(\pi_s^{\mathcal{A}}(\pi_m)\right)-(1-\theta)\,.\end{aligned}$$

The inequalities above then imply that

$$\frac{d}{d\pi}U\left(\pi_s^{\mathcal{A}}(\pi_m)\right)<\frac{d}{d\pi}U_r\left(\pi_s^{\mathcal{A}}(\pi_m)\right).$$

But this is a contradiction because if $\pi_s^{\mathcal{A}}(\pi_m)\in[\pi_l,\inf D)$, we have

$$\frac{d}{d\pi}U\left(\pi_s^{\mathcal{A}}(\pi_m)\right)=\frac{d}{d\pi}u^{\mathcal{A}}\left(\pi_s^{\mathcal{A}}(\pi_m)\right)\geq\frac{d}{d\pi}U_r\left(\pi_s^{\mathcal{A}}(\pi_m)\right),$$

by the definition of D. And if $\pi_s^{\mathcal{A}}(\pi_m)<\pi_L$, this contradicts $(A6)$. ■

Proof of Proposition 2. The proof closely follows its counterpart for Proposition 1. Part (i) directly follows from part (i) of Lemma 9.

Next consider part (iv). This claim directly follows from parts (ii) and (iii) of Lemma 9 where we relabel π_l (i.e., the lowest value of π for which $(\pi,U(\pi))$ is supported by $a=\mathcal{A}$) as π^A.

Finally, consider parts (ii) and (iii). We know that $\left(\pi^O,U\left(\pi^O\right)\right)$ is supported by $a=R$. For any $\pi\in\left(\pi^O,\pi^A\right)$ consider the payoff pair $(\pi,U(\pi))$. Note that $(\pi,U(\pi))$ cannot be supported by $a=A$ since $u^{\mathcal{A}}(\pi)>u^A(\pi)$ for all $\pi\geq(1-\delta)y$, and $a=A$ is not feasible when $\pi<(1-\delta)y$. Moreover, it also cannot be supported by $a=O$ or $a=\mathcal{O}$ as these actions can support payoffs on the frontier only if $\pi<\delta\pi^O$ (by Lemma 9, part (i)). Hence, $(\pi,U(\pi))$ must be supported either by $a=R$ or by randomization. Let π^R be the highest value of π such that $(\pi,U(\pi))$ is supported by $a=R$ (again, π^R exists as both U and u_r^R are continuous). So, $U\left(\pi^R\right)=u_r^R\left(\pi^R\right)$. Moreover, as $U\left(\pi^R\right)\geq U_r\left(\pi^R\right)\geq u_r^R\left(\pi^R\right)$, we have $U\left(\pi^R\right)=u_r^R\left(\pi^R\right)=U_r\left(\pi^R\right)$. But this implies $U(\pi)=U_r(\pi)$ for all $\pi\in\left[\pi^O,\pi^R\right]$ as

$U'(\pi) \geq U_r'(\pi)$ for all $\pi < \pi^A$ (by (A6)). Now, since $U_r(\pi^R) = u_r^R(\pi^R)$, i.e. $(\pi^R, U_r(\pi^R))$ is sustained by the rigid action, it follows directly from the characterization of U_r that $U_r(\pi) = u_r^R(\pi)$ for all $\pi \in [\pi^O, \pi^R]$. Hence, $U(\pi) = U_r(\pi) = u_r^R(\pi)$ for all $\pi \in [\pi^O, \pi^R]$.

Finally, if $\pi^R < \pi^A$, by definition of π^R and the argument given above, it directly follows that for any $\pi \in (\pi^R, \pi^A)$, $(\pi, U(\pi))$ must be sustained by randomization between two PPE payoffs, one sustained by R and the other by $\mathcal{A}$. Hence, $U(\pi)$ must be linear if $\pi \in (\pi^R, \pi^A)$, and without loss of generality, we can assume that the two end points are $(\pi^R, U(\pi^R))$ and $(\pi^A, U(\pi^A))$. ■

Proof of Lemma 5. We have already shown $\pi^O = \pi_r^O = (1-\delta)\,py$ and $\bar{\pi} > \bar{\pi}_r$ in Lemma 9.

To see why $\pi^R \leq \pi_r^R$, suppose on the contrary, $\pi^R > \pi_r^R$. Since $(\pi^R, U(\pi^R))$ is sustained by $a = R$, $U(\pi^R) = u_r^R(\pi^R)$. But $\pi^R > \pi_r^R$ implies $U_r(\pi^R) > u_r^R(\pi^R)$, and hence, we must have $U_r(\pi^R) > U(\pi^R)$, which is a contradiction (as $U_r(\pi) \leq U(\pi)$ for all π).

That $U_r(\pi) = U(\pi)$ *for all* $\pi \leq \pi^R$ follows from Proposition 1 and 2. Next, we show that $U(\pi) > U_r(\pi)$ for $\pi > \pi^R$.

Take some $\pi' > \pi^R$. The agent's payoff $U_r(\pi')$ can be supported by either the rigid action, or the adaptive action, or by randomization. If it is supported by the rigid action, $U_r(\pi') = u^R(\pi') < U(\pi')$, where the last inequality follows from the definition of π^R. If $(\pi', U_r(\pi'))$ is supported by the adaptive action $a = A$, then $U_r(\pi') = u^A(\pi') < u^{\mathcal{A}}(\pi') \leq U(\pi')$, where the first inequality has been proved above in Step 1 of the proof of Lemma 9, and the second one follows from the definition of U. Finally, if $(\pi', U_r(\pi'))$ is supported by randomization, it must be that $\pi' \in (\pi_r^R, \pi_r^A)$, and there exists a $\lambda \in (0,1)$ such that $U_r(\pi') = \lambda U_r(\pi_r^A) + (1-\lambda)\, U_r(\pi_r^R)$. Since $(\pi_r^A, U(\pi_r^A))$ is supported by $a = A$, we have $U(\pi_r^A) > U_r(\pi_r^A)$ (as argued above). But this implies that

$$U(\pi') \geq \lambda U(\pi_r^A) + (1-\lambda)\, U_r(\pi_r^R) > \lambda U_r(\pi_r^A) + (1-\lambda)\, U_r(\pi_r^R) = U_r(\pi'),$$

where the first inequality follows from the fact that both $(\pi_r^A, U(\pi_r^A))$ and $(\pi_r^R, U_r(\pi_r^R))$ are in $\mathcal{E}$. ■

Proof of Proposition 4. *Part* (i). First consider the case where the standardized procedure is already in place. Recall that in this case the PPE payoff set is $\mathcal{E}_r$ and the PPE payoff frontier is U_r.

Step 1A. Let $\widehat{\pi} := (1-\delta)y$. This is the lowest value of π for which the adaptive action is feasible. The proof consists of showing that for some parameters

$$U_r(\pi_r^O) + \left(\widehat{\pi} - \pi_r^O\right) \frac{d}{d\pi} u_r^A(\widehat{\pi}) > u_r^A(\widehat{\pi}), \tag{A9}$$

where the derivative can be thought of as the right or left derivative when the derivative fails to exist. (We also maintain this convention with the notation in the remainder of the proof.) Recall that $\pi_r^O = (1-\delta)py$. If this condition is satisfied, then the slope of $u_r^A(\pi)$ evaluated at $\pi = \widehat{\pi}$ is greater than the slope of the line that connects the points $\left(\pi_r^O, U_r(\pi_r^O)\right)$ and $\left(\widehat{\pi}, u_r^A(\widehat{\pi})\right)$, which implies that there exists $\pi' > \widehat{\pi}$ and $\lambda \in (0,1)$ such that $(1-\lambda)\pi_r^O + \lambda\pi' = \widehat{\pi}$ and $(1-\lambda)U_r(\pi_r^O) + \lambda u_r^A(\pi') > u_r^A(\widehat{\pi})$. In other words, there is a randomization between the points $\left(\pi_r^O, U_r(\pi_r^O)\right)$ and $\left(\pi', u_r^A(\pi')\right)$ that for $\pi = \widehat{\pi}$ yields a payoff to the agent strictly greater than $u_r^A(\widehat{\pi})$. But this implies that $U_r(\widehat{\pi}) > u_r^A(\widehat{\pi})$, which means that the point $(\widehat{\pi}, U_r(\widehat{\pi}))$ on the payoff frontier requires playing the rigid action in the current period with a positive probability. That is, the firm may ask the worker to perform the rigid action even thought the adaptive action is feasible.

Step 1B. We next show that there are parameter values for which (A9) is satisfied. Recall from $(A8)$ that

$$\frac{d}{d\pi} u_r^A(\pi) = \theta \frac{d}{d\pi} U_r\left(\pi_s^A(\pi)\right) - (1-\theta).$$

Since $\pi_s^A(\widehat{\pi}) = 0$,

$$\frac{d}{d\pi} u_r^A(\widehat{\pi}) = \theta \frac{d}{d\pi} U_r(0) - (1-\theta) = \theta \frac{U_r(\pi_r^O)}{(1-\delta)py} - (1-\theta).$$

(Recall that U_r is linear for $\pi < \pi_r^O$ and its slope is given by $U_r(\pi_r^O)/((1-\delta)py)$. Given this and that $\widehat{\pi} - \pi_r^O = (1-\delta)(1-p)y$, we can write (A9) as

$$U_r(\pi_r^O) + \left(\theta \frac{U_r(\pi_r^O)}{(1-\delta)py} - (1-\theta)\right)(1-\delta)(1-p)y > u_r^A(\widehat{\pi}). \tag{A10}$$

Step 1C. Now, since $(\pi_r^O, U_r(\pi_r^O))$ is sustained by the rigid action, by adding $\left(PK_A^R\right)$ and $\left(PK_P^R\right)$ and re-arranging we obtain that

$$U_r(\pi_r^O) = -(1-\delta)c + (1-\theta)\delta\left(\bar{\pi}_r + U_r(\bar{\pi}_r)\right). \tag{A11}$$

Similarly, we can write

$$u_r^A(\widehat{\pi}) = -(1-\delta)C + (1-\theta)\delta\left(\bar{\pi}_r + U_r(\bar{\pi}_r)\right).$$

Hence, $u_r^A(\widehat{\pi}) = U_r(\pi_r^O) - (1-\delta)(C-c)$, and we can write (A10) as

$$\left(\theta \frac{U_r(\pi_r^O)}{(1-\delta)py} - (1-\theta)\right)(1-\delta)(1-p)y > -(1-\delta)(C-c).$$

Rearranging terms, we obtain that this inequality is equivalent to

$$\left(\theta U_r(\pi_r^O) - (1-\theta)(1-\delta)py\right)\frac{1-p}{p} > -(1-\delta)(C-c). \tag{A12}$$

Step 1D. Now, from (A11) and the fact that $\bar{\pi}_r + U_r(\bar{\pi}_r) \geq (1-\delta)(y-C)/(1-(1-\theta)\delta)$ (see Step R3 of the proof of Lemma 7) we obtain that

$$U_r(\pi_r^O) \geq -(1-\delta)c + (1-\theta)\delta \frac{(1-\delta)(y-C)}{1-(1-\theta)\delta}.$$

Thus, a sufficient condition for (A9) is that

$$\theta\left(c - (1-\theta)\delta \frac{y-C}{1-(1-\theta)\delta}\right) + (1-\theta)py < \frac{p(C-c)}{1-p}. \tag{A13}$$

There are parameters values that satisfy this condition as well as all the other assumptions of the model. For example, if $\theta\delta(1-p) - p(1-\delta) > 0$ (a condition compatible with Assumption 1 (ii)), the left-hand side of the above inequality increases with y and is satisfied when y sufficiently large.

Now consider the case where the standardized procedure has not been in place. In this case the PPE payoff set is $\mathcal{E}$ and the PPE payoff frontier is U. The proof is analogous to that above.

Step 2A. We need to show that there are parameter values for which

$$U(\pi^O) + \left(\widehat{\pi} - \pi^O\right)\frac{d}{d\pi}u^A(\widehat{\pi}) > u^A(\widehat{\pi}). \tag{A14}$$

Recall that $\pi^O = \pi_{r.}^O$ and that $U(\pi^O) = U_r(\pi_r^O)$. Hence, the only difference relative to the proof above is that we have $\frac{d}{d\pi}u^A(\widehat{\pi})$ instead of $\frac{d}{d\pi}u_r^A(\widehat{\pi})$ and $u^A(\widehat{\pi})$ instead of $u_r^A(\widehat{\pi})$.

We first show that $\frac{d}{d\pi}u^A(\widehat{\pi}) = \frac{d}{d\pi}u_r^A(\widehat{\pi})$. Recall that

$$\frac{d}{d\pi}u^A(\pi) = \theta \frac{d}{d\pi}U\left(\pi_s^A(\pi)\right) - (1-\theta).$$

Since $\pi_s^A(\widehat{\pi}) = 0$,

$$\frac{d}{d\pi}u^A(\widehat{\pi}) = \theta \frac{d}{d\pi}U(0) - (1-\theta) = \theta \frac{U(\pi^O)}{(1-\delta)py} - (1-\theta) = \theta \frac{U_r(\pi_r^O)}{(1-\delta)py} - (1-\theta),$$

where the second equality follows from the fact that $U(\pi) = U_r(\pi)$ for $\pi \leq \pi^O$ and $\pi^O = \pi_r^O$. Thus, $\frac{d}{d\pi}u^A(\widehat{\pi}) = \frac{d}{d\pi}u_r^A(\widehat{\pi})$.

Step 2B. We now analyze $u^A(\widehat{\pi})$ and $U(\pi^O)$. As mentioned above,

$$U(\pi^O) = U_r(\pi_r^O) = -(1-\delta)\,c + (1-\theta)\,\delta\left(\bar{\pi}_r + U_r\left(\bar{\pi}_r\right)\right).$$

Similarly, we have that

$$u^A(\widehat{\pi}) = -(1-\delta)\,C + (1-\theta)\,\delta\left(\bar{\pi} + U\left(\bar{\pi}\right)\right).$$

Thus,

$$\begin{aligned} u^A(\widehat{\pi}) - U(\pi^O) &= -(1-\delta)\,(C-c) + (1-\theta)\,\delta[\bar{\pi} + U(\bar{\pi}) - (\bar{\pi}_r + U_r\left(\bar{\pi}_r\right))] \\ &\leq -(1-\delta)\,(C-c) + (1-\theta)\,\delta\left((y-C) - \tfrac{(1-\delta)(y-C)}{1-(1-\theta)\delta}\right) \\ &= -(1-\delta)\,(C-c) + \delta^2\tfrac{\theta(1-\theta)(y-C)}{1-(1-\theta)\delta}, \end{aligned} \tag{A15}$$

where the inequality follows from the fact that $\bar{\pi} + U(\bar{\pi}) \leq y - C$ (recall that $y - C$ is the maximum surplus possible) and that $\bar{\pi}_r + U_r\left(\bar{\pi}_r\right) \geq (1-\delta)\,(y-C)\,/(1-(1-\theta)\,\delta)$ (from Step R3 of the proof of Lemma 7).

Step 2C. Now, rearrange the condition (A14) as follows:

$$\left(\widehat{\pi} - \pi^O\right)\frac{d}{d\pi}u^A(\widehat{\pi}) > u^A(\widehat{\pi}) - U(\pi^O).$$

A lower bound for the left-hand side of this inequality is given by the left-hand side of the inequality (A13). And from (A15) we obtain an upper bound for the right-hand side. Plugging these bounds we obtain a sufficient condition for (A14) as:

$$\theta\left(c - (1-\theta)\,\delta\frac{(1-p-\delta)\,(y-C)}{(1-\delta)\,(1-p)\,(1-(1-\theta)\,\delta)}\right) + (1-\theta)py < \frac{p\,(C-c)}{1-p}. \tag{A16}$$

There are parameter values for which this condition is satisfied along all the other assumptions of the model. In particular, if

$$\theta\delta\,(1-p-\delta) > p\,(1-p)\,(1-\delta)(1-(1-\theta)\,\delta),$$

then the left-hand side of the condition is increasing in y and for sufficient high values of y it is satisfied. Also note that whenever $(A16)$ is satisfied, so is $(A13)$. So, the precautionary use of the rigid action is more likely to occur when the standard work process has already been put in place.

Part (ii). **Step 1.** Define $T_1(\pi) = \pi_s^A(\pi)$, $T_2(\pi) = \pi_s^A\left(\pi_s^A(\pi)\right)$, and $T_n(\pi)$ is defined accordingly. Also, let N be the number of consecutive shocks that guarantees that the rigid action is used when the relationship starts in Phase 1 (i.e., when the standardized procedure is yet to be established and the relationship starts with payoffs $(\bar{\pi}, 0)$). Similarly, let N_r be its counterpart when the relationship starts in Phase 2 (i.e., when the standardized procedure has been established and the relationship starts with payoffs $(\bar{\pi}_r, u^*)$). Recall that $\pi_s^A(\pi) = \pi_s^{\mathcal{A}}(\pi)$. Hence, $N = \min\left\{n \mid T_n(\bar{\pi}) \leq \pi^R\right\}$ and $N_r = \min\left\{n \mid T_n(\bar{\pi}_r) \leq \pi_r^R\right\}$.

Step 2. Also note that $\pi_s^A\left(\pi^A\right) \leq \pi^R$, as otherwise one can move both π_s^A and π_n^A to the left by $\varepsilon > 0$ and increase the payoff of the agent. (That is, when the relationship starts in Phase 1 and a series of consecutive shocks calls for randomization between $a = \mathcal{A}$ and $a = R$ for the very first time, even if $a = \mathcal{A}$ is realized in the current period, another shock in the current period surely move the relationship to Phase 2.) So, as $\pi^R \leq \pi_r^R$ and $\bar{\pi} > \bar{\pi}_r$, we have $N \geq N_r$. (Also note that even if $N = N_r$, the principal's payoff when the rigid action is used is lower if started out from $\bar{\pi}_r$ than if we start from $\bar{\pi}$. That is, at π^A the continuation payoff of the agent following a shock π_s^A is weakly smaller than π^R. If not, then one can move both π_s^A and π_n^A to the left by $\varepsilon > 0$ and increase the payoff of the agent.) ■

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