OPTIMAL JOB DESIGN AND INFORMATION ELICITATION*

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Abstract. Managers often rely on their subordinates for local information that aids decision-making but cannot commit to a decision rule. When the firm and the workers have conflicting interests on how such information gets used, incentives for effort and information elicitation become intertwined. We explore how this incentive problem may be solved through job design—the choice between "individual assignment" where all tasks in a given job are assigned to the same worker, and "team assignment" where the tasks are split among a group. Team assignment facilitates information elicitation but suffers from "diseconomies of scope" in incentive provision. This tradeoff drives the optimal job design, and it is shaped by two key parameters—the workers' ex-ante likelihood of being informed and the noise in the performance measure that is used to reward the worker. Individual assignment is optimal when the performance measure is well-aligned, but team is optimal when the measure is noisy and the workers are highly likely to be informed about the local conditions.

1. Introduction

Managerial decision-making in a hierarchical organization often relies on local information that cannot be directly accessed by the headquarter but may be available to its lower-ranked employees. A host of key business decisions, such as launching new product lines, undertaking new business ventures, investments in new R&D initiatives, all require detailed information on customer preferences, profitability prospects, and technological capabilities that is more likely to be available to the junior workers who are more familiar with the local market conditions and the firm's production process. Effective decision-making, therefore, calls for timely provision of information that may be dispersed within an organization.

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However, the firm and the workers may have conflicting interests on how information may be used, and when relaying local information to their manager, the workers may manipulate information to steer the firm's decision towards their own interests. A worker may deem his information "unfavorable" if the firm's expected action under such information could reduce the worker's future rents. Consequently, he may attempt to filter or conceal such information, particularly when the firm cannot commit on how the information may be used in its decision process. Such conflict of interest creates a complex incentive problem as the incentives for effort and information elicitation get intricately entwined (Athey and Roberts, 2001).

Starting from the seminal work by Marschak (1955) and Marschak and Radner (1972) on team theory, a large literature has explored the limits on information provision in an organization and how these limits are influenced by the organization's structure (Aoki, 1986). However, this literature typically abstracts away from the problem of incentives as the employees' objective is assumed to be perfectly aligned with that of the employer. The goal of our paper is to explore how the problem of intertwined incentives for effort and information elicitation shapes a critical part of the organizational structure, namely, job design.

An essential problem in organizational design is how to group different tasks into jobs that may be assigned to the workers. An organization may typically choose between two natural designs: it may opt for "individual assignment" where all tasks associated with a specific production process are assigned to the same worker who remains solely accountable for his job output. Alternatively, it may choose "team assignment" where different tasks of the production process are assigned to different workers who are held jointly accountable for their job performance. When decision-relevant information is accessible only to the workers who are directly involved in the production process, the two job designs have distinct implications on how information may be dispersed within the organization. Under individual assignment, all information pertaining to a production process can be observed only by the worker who has been assigned to it, whereas multiple workers may access this information when they are working as a team.

The broad prevalence of individual and team assignments in project management structures has been well-documented in the management literature (Galbraith, 1971; Larson and Gobeli, 1989; Hobday, 2000; Lechler and Dvir, 2010). Firms often adopt a "project-based" structure where a manager is assigned to oversee all aspects of a project, or a "functional" structure where projects are divided into segments and different segments are overseen by different managers. In exploring the relative merits of the two structures, this literature mostly focuses on the gains from task specialization vis-a-vis task coordiation. We highlight that when the workers need to be incentivized for both effort provision and information elicitation, the choice between these two designs is shaped by a novel trade-off: Team assignment may facilitate information elicitation as no worker can fully control the information or project outcome, but it suffers from "diseconomies of scope" in incentive provision and may undermine workers' effort.

We explore this trade-off in a stylized model of job design in a principal-agent environment. In our setup a principal hires two agents to work on two projects. Each project has two tasks, and a project can either succeed or fail. The likelihood of success depends on the level of effort exerted in its tasks and the underlying "state of the world" that may be observable only to the agent(s) who are assigned to that project. At the beginning of the game, the principal chooses a job design: under individual assignment, each agent is responsible for a given project and is expected to exert effort in both tasks that are associated with it. Under team, an agent is assigned exactly one task from each of the two projects. While performing a task, an agent may observe the state of the world (pertaining to its associated project) with some probability and reports it to the principal. While the agent cannot misrepresent the state (i.e., observation on the state is "hard" information) he may conceal it by feigning ignorance. Up on receiving the agents' report on the state, the principal decides whether to continue or cancel a project. The project output is not verifiable, but the agents' effort in a project is reflected by a contractible but noisy performance measure.

Incentives are provided through a wage contract that ties an agent's pay to the principal's cancellation decision and the realization of the performance measures (if the project is continued). The misalignment between the performance measure and the project outcome gives rise to a conflict of interest between the principal and the agents. If the observed state does not bode well for the project's success but is unlikely to affect the performance measure (if the project is implemented), the agent may conceal his information to let the project proceed whereas the principal would have been better off by canceling it.

We show that the optimal job design is driven by two salient informational frictions: the "availability" of agents' information (i.e., the likelihood that the agent gets to observe the state while performing his assigned tasks) and the "noise" in the agents' performance measure (i.e., the extent of misalignment between the measure and the output). Team assignment is strictly optimal when the agents are highly likely to observe the state but there is significant misalignment between the performance measure and the project output. In contrast, when the extent of misalignment is relatively small, individual assignment is strictly optimal regardless of the agents' likelihood of being informed about the state.

The intuition for this result can be gleaned from the aforementioned trade-off between information elicitation and "diseconomies of scope" in incentive provision. Since the principal relies on the agents' report but cannot commit on how this information may be used, the agent may attempt to control the projects' outcome by manipulating his information and effort levels. Team assignment helps in mitigating such incentive problem as the agent does not fully control the outcome of a project. His attempt to conceal information may falter if his teammate happens to provide the same information to the principal, and he influences the effort only in a part of the project.

But incentive provision under team assignment suffers from "diseconomies of scope": the principal needs to reward the two agents separately to induce effort on the two tasks that are associated with the project. And such diseconomies of scope can blunt incentives. As the principal cannot commit how she may use the agents' report, in equilibrium, her continuation policy must be sequentially rational. If the principal proceeds with the project under a certain information, it must be that her expected payoff from proceeding with the project (conditional on the agents' reports) is larger than what she might get from cancelling it. These requirements put an upper bound on the amount of reward the principal pays to the agents when the project is successful (as per the performance measure). Under team, the total reward payout is larger and such bounds are harder to meet as the principal needs to pay the reward for success twice (paying each of the two agents separately) to elicit effort in both tasks. Consequently, strong incentives may be infeasible.

In contrast, under individual assignment, a single reward payment would have induced effort in both tasks, and such economies of scope in incentive provision makes it easier to provide strong incentives without violating the bounds on reward payments. However, under individual assignment, information elicitation becomes harder as the agent fully controls the outcome of a project through his report and effort.

Thus, between the two forms of job design, team assignment faciliates information elicitation whereas individual assignment facilitates the provision of effort incentives.

When the performance measure is considerably misaligned and can indicate success even when the project fails, the agents have strong incentives to conceal unfavorable information to let the project continue. This is when the team's advantage in information elicitation is most useful: an agent's attempt to conceal information could be undone by his teammate, particularly when his teammate is very likely to have the same information. Due to such misalignment effort is also more sensitive to rewards and strong incentives may be feasible despite the scope diseconomies that arise under team. As a result, team assignment becomes optimal. In contrast, if the performance measure is relatively well-aligned with the project output, information elicitation is relatively easy as the agent has little to gain from concealing information from the principal. Thus, individual assignment becomes optimal—it allows the principal to exploit the economies of scope in incentive provision and offer strong incentives for effort without distorting the agent's reporting incentives.

Related literature: Our paper contributes to a growing literature on the interplay between incentives and communication of dispersed information within an organization. As mentioned earlier, the literature on team theory that followed from Marshak and Radner (1972) explores managerial decision-making when there are physical constraints on the flow of information (and the headquarters' ability to process it) but typically assumes that the workers are non-strategic in their communication (see, e.g., Cremer, 1980; Aoki, 1986; Genakoplos and Milgrom, 1991; Bolton and Dewatripont, 1994). Several authors have subsequently analyzed strategic communication by privately informed workers and how it shapes the allocation of decision rights within organization (Dessein, 2002; Alonso, Dessein, and Matouschek, 2008; Rantakari, 2008). These papers focus on the tradeoff between the production efficiencies from coordination of actions and adaptation to local information but abstract away from the incentive problems in effort provision.

Levitt and Snyder (1997) is one of the first papers to analyze the interaction between the incentives for effort and truthful communication. They highlight a tradeoff between efficiency in decision making and effort incentives when the agents may be privately informed about their projects' viability. In order to induce both effort and truthful reporting of "bad news" (i.e., information that lowers the likelihood of the project's success), the optimal contract calls for an inefficiently "lenient" continuation policy where some projects with negative expected value are allowed to continue. But in their model the organizational structure is exogenously given; in contrast, we analyze how the interaction between the effort and reporting incentives drives the allocation of tasks within the organization.

Our paper complements the works by Athey and Roberts (2001), Friebel and Raith (2010), and Dessein, Garicano, and Gertner (2010), who explore organizational forms in the presence of the tradeoff between incentives for effort, communication, and efficient decision making. Athey and Roberts show that the tradeoff between effort incentives and efficient decision making can be mitigated by creating an organizational hierarchy by hiring a top-level manager who can obtain all information at a cost and coordinates the actions of her subordinates. However, they assume exogenous task allocation and do not allow for communication between agents. Strategic communication across organizational hierarchy plays a key role in Friebel and Raith (2010). They analyze the optimal firm structure for allocation of resources across its different divisions where the divisional managers are privately informed about the best use of such resources. The firm can integrate the units under a CEO with authority on resource allocation for more efficient allocation of resources but must elicit truthful reporting from the divisional managers. The optimality of such integration decision is driven by a tradeoff between the benefit of more efficient resource allocation and the cost of a distortion in the effort incentives that may be necessary for information elicitation. A similar integration issue is studied by Dessein, Garicano, and Gertner (2010) where a firm decides on whether to organize into business units (i.e., divisions with considerably autonomy) or create functional units that centralizes certain tasks for all divisions. The functional unit manager can implement standardization to capture synergy benefits but inflicts a cost on business unit managers by impeding adaptation to local information. The organization responds to this tradeoff by creating an incentive conflict between the business and functional unit managers and it drives the optimal allocation of authority and tasks within the organization. However, none of these papers explore the role of job design in incentivizing truthful communication within organization which is a key focus of our analysis.

This article also relates to a few other strands in the organizational economics literature. There is a vast literature on incentives in teams (Groves, 1973; Holmstrom, 1982; Mookherjee, 1984; McAfee and McMillan, 1991; Che and Yoo, 2001; Marino and Zabojnik, 2004; Kvaløy and Olsen, 2006; Rayo, 2007; Blanes i Vidal and Moller, 2016; Friebel et. al, 2017) that takes the team structure as given and analyze how the underlying production and information environment drive the optimal provision of effort incentives. A notable exception is Gromb and Martimort (2007) who consider a setup where the decision-maker relies on experts to gather and report mulitple signals on a risky project's profiability. They analyze a case where the decision-maker can either ask a single expert to acquire all signals or employ multiple experts where each one is responsible for acquiring exactly one signal. While this setup bears some resemblance to our job design problem, Gromb and Martimort's model differs from ours along various key dimensions. In particular, in their setup the agents' effort is useful for information acquisition but not for the project's value, experts have "soft information" (hence, can lie in their report), and the focus of their analysis is on the optimal incentives for such delegated experties when the contracting parties may collude among themselves.

Job design has also been explored by several scholars, primarily as a possible remedy for the multitasking problem (Holmstrom and Milgrom, 1991; Dewatripont, Jewitt, and Tirole, 2000; Besanko, Regibeau, and Rockett, 2005; Schottner, 2008; Corts, 2007; Mukherjee and Vasconcelos, 2011; Ishihara, 2017). In contrast, we abstract away from the multitasking problem; in our setting the conflict of incentives for effort and information elicitation is the key driver of the optimal job design. Finally, our work is reminiscent of the literature on authority and delegation where the contracting parties may have misaligned preferences over the managerial actions (Aghion and Tirole, 1997; Dessien, 2002; Alonso and Matouschek, 2008; Alonso, et. al 2008; Deimen and Szalay, 2019). In this literature, the misalignment is assumed to stem from exogenous bias in the agents' preferences that may distort the communication within organization. However, in our setup the agents' possible gains from information manipulation arises endogenously due to the moral hazard problem in the agent's effort provision and the firm's lack of commitment power over its continuation policy.

This paper is structured as follows. Section 2 presents our model. A benchmark case with public signal is analyzed in Section 3. The optimal contracts under individual and team assignment is characterized in Section 4. In Section 5 we present our main result on the optimal job design and explore its comparative statics. A final section, Section 6, discusses a few extensions of our model and presents a conclusion. All proofs are given in the Appendix.

2. Model

Players: A principal $\mathcal{P}$ (she) hires two agents (he), $\mathcal{A}_1$ and $\mathcal{A}_2$ to work on two risky projects, A and B, and concurrently gather information on the projects' financial viability. Below we index the agents by $i \in \{1, 2\}$ and the projects by $j \in \{A, B\}$.

Technology: The production technology is reminiscent of the canonical setup of Dewatripont et. al (2000). Each project consists of two tasks; to fix ideas, one may consider a firm exploring the launch of a new product, and a successful launch requires effort on product development and marketing. We denote the set of tasks associated with project j as $\{T_{j1}, T_{j2}\}$. To streamline notations, we may refer to task T_{jk} simply as task k, $k \in \{1, 2\}$, whenever it is efficacious to do so.

Each agent can perform at most two tasks. At the beginning of the game, the principal commits to a task allocation or "job design." The principal can choose one of two options: (i) "individual assignment," where each worker is assigned to a different project, and he works on both tasks that are associated with his project, and (ii) "team assignment," where each worker performs exactly one task from each of the two projects. Without loss of generality, we assume that under individual assignment, agent $\mathcal{A}_1$ works on project A (and performs tasks $\{T_{A1}, T_{A2}\}$), and agent $\mathcal{A}_2$ works on project B (and performs tasks $\{T_{B1}, T_{B2}\}$); whereas under team assignment, $\mathcal{A}_1$ performs the first task in both jobs, $\{T_{A1}, T_{B1}\}$, and $\mathcal{A}_2$ performs the second task, $\{T_{A2}, T_{B2}\}$.

Let $e_{jk} \in [0, 1/2]$ denote the effort exerted in task T_{jk} (i.e., task $k \in \{1, 2\}$ of project $j \in \{A, B\}$). Effort is private, and it costs the agent (who has been assigned to this task) $c(e_{jk}) = e_{jk}^2/2$.

The outcome of project j, $Y_j \in \{0, y\}$, can be either a "success" ($Y_j = y$) or a "failure" ($Y_j = 0$). The project's outcome depends on the effort exerted in each of its two tasks and on its underlying "state of the world", $\omega_j \in \{G, B\}$ that can either be "good" ($\omega_j = G$) or "bad" ($\omega_j = B$). The production function is given as (denote $\mathbf{e}_j := (e_{j1}, e_{j2})$):

$$\Pr(Y_j = y \mid \mathbf{e}_j; \omega_j) = \begin{cases} e_{j1} + e_{j2} & if\ \omega_j = G \\ 0 & if\ \omega_j = B \end{cases}.$$

In a "bad" state, the project always fails regardless of the agents' effort, and yields $Y_j = 0$. In a "good" state failure can be averted as $Y_j \in \{0, y\}$, and effort is productive as it increases the chance of obtaining a high output of $Y_j = y$.

The project outcome is not verifiable, but the agent's performance is reflected by a metric $M_j \in \{0, 1\}$ that can be verified. However, the metric M_j is a noisy measure of the project outcome as:

$$\Pr(M_j = 1 \mid \mathbf{e}_j; \omega_j) = \begin{cases} e_{j1} + e_{j2} & if\ \omega_j = G \\ \mu(e_{j1} + e_{j2}) & if\ \omega_j = B \end{cases},$$

and $\mu \in [0, 1)$. In the context of the product launch example, one may consider Y_j to be the product's long-term value to the firm whereas M_j is a measure of the product's profitability in the short run. The extent of misalignment between the metric and the project output is reflected by the parameter μ; for $\mu = 0$ the distributions of Y_j and M_j are identical, but for $\mu > 0$, the metric may reflect a "success" ($M_j = 1$) even in a bad state when the project fails with certainty. And at the extreme, when $\mu \to 1$, the metric no longer depends on the underlying state.

Information structure: At the beginning of the production process, the underlying state of a project, ω_j, is unknown to all players but players hold a common prior belief given as $\Pr(\omega_j = G) = \frac{1}{2}$, where ω_A and ω_B are statistically independent. But an agent, up on completing an assigned task T_{jk}, *privately* observes the state ω_j with probability $\alpha \in [0, 1)$. Thus, under individual assignment, the agent assigned to project $j \in \{A, B\}$ learns the underlying state ω_j with probability $1 - (1 - \alpha)^2$. And, under team assignment, the probability at least one of the two agents assigned to project j learns the state ω_j is also

$1-(1-\alpha)^2$. Denote $\mathcal{A}_i$'s observation on the state ω_j as $x_i^j \in \{G, B, \emptyset\}$, where $x_i^j = \emptyset$ if $\mathcal{A}_i$ does not observe ω_j.

Reporting: The agents simultaneously report their information on the underlying states to the principal. The observation on the state is "hard information": an agent cannot misreport the state but can hide his observation by feigning ignorance. Under individual assignment, denote $\mathcal{A}_i$'s report as $r_i \in \{G, B, \emptyset\}$, where $r_i = \emptyset$ when the agent claims to have failed to observe the state associated with his project. And under team assignment, $\mathcal{A}_i$ reports $r_i = \left(r_i^A, r_i^B\right)$ where $r_i^j \in \{G, B, \emptyset\}$ is the report on state ω_j, $j \in \{A, B\}$. With a slight abuse of notation, we denote the collective report of the two agents on state ω_j as $r^j \in \{G, B, \emptyset\}$ (i.e., the information on ω_j that the principal obtains from the two reports).

Given the agents' reports, the principal decides whether to implement a project or to cancel it. The project outcome (Y_j) and the associated performance measure M_j are realized only if the project is implemented. If a project is cancelled, the principal earns her outside option, as described later in this section. The agents' reports, like the project outcomes Y_j, are not verifiable.

Contract: As mentioned above, the principal commits to a job design $d \in \{\mathcal{I}, \mathcal{T}\}$ that specifies either individual assignment $(d = \mathcal{I})$ or team assignment $(d = \mathcal{T})$. As neither the projects' outcomes nor the agents' reports are verifiable, the principal cannot commit to a cancellation policy, and can only commit to a wage schedule that depends on (i) whether the project has been implemented, and (ii) in the event the project is implemented, on the realization of the associated performance measure $M_j \in \{0, 1\}$. To streamline notations, we set $M_j = \emptyset$ if project j gets cancelled. Thus, under individual assignment, agent $\mathcal{A}_1$'s contract is given by the wage schedule $w_1^I(M_A)$, $M_A \in \{0, 1, \emptyset\}$ as he is only responsible for project A (similarly, $w_2^I(M_B)$ for agent $\mathcal{A}_2$), and under team assignment, by the pair of schedules $\left\{w_{1A}^T(M_A); w_{1B}^T(M_B)\right\}$ as he works on parts of both projects (similarly, $\left\{w_{2A}^T(M_A); w_{2B}^T(M_B)\right\}$ for agent $\mathcal{A}_2$). Denote the wage schedule for $\mathcal{A}_i$ under the job design $d \in \{\mathcal{I}, \mathcal{T}\}$ as $\mathcal{W}_i^d$.

We denote a contract as $\phi := \left\{d, \mathcal{W}_1^d, \mathcal{W}_2^d\right\}$, and let Φ be the set of all such contracts.

Time line: The timeline of the game is summarized below:

- $\mathcal{P}$ chooses a job design $d \in \{\mathcal{I}, \mathcal{T}\}$, and publicly offers a wage schedule $\left\{\mathcal{W}_1^d, \mathcal{W}_2^d\right\}$.
- $\mathcal{A}_1$ and $\mathcal{A}_2$ (simultaneously) accept or reject the contract $\phi = \left\{d, \mathcal{W}_1^d, \mathcal{W}_2^d\right\}$. The game proceeds only if both accept.

- $\mathcal{A}_i$ exerts effort in the two tasks that have been assigned to him.
- $\mathcal{A}_i$ may observe the state(s) ω_j from his assigned tasks and reports to $\mathcal{P}$.
- $\mathcal{P}$ decides which project, if any, to cancel.
- The project outcomes, performance measures, and payoffs are realized; and the game ends.

PAYOFFS: All players are risk neutral. If the agents accept the contract offered by the principal, the payoff of an agent $\mathcal{A}_i$ is given by his expected wage net of his cost of effort. And the payoff of the principal is given by the expected output from the two projects (when implemented) net of the expected wage payment. If a project is cancelled, the principal can undertake an "outside option" that yields a payoff of $\underline{\pi}$ (> 0). With a slight abuse of notation, we set $Y_j = \underline{\pi}$ if project j gets cancelled. (Recall that in this case we also set the performance metric $M_j = \emptyset$.)

Thus, under individual assignment the agents' ex-post payoffs are:

$$\begin{aligned} u_1^I &:= w_1^I(M_A) - c(e_{A1}) - c(e_{A2}), \\ u_2^I &:= w_2^I(M_B) - c(e_{B1}) - c(e_{B2}); \end{aligned}$$

and the principal's ex-post payoff is $\pi^I := \pi_A^I + \pi_B^I$ where

$$\pi_A^I := Y_A - w_1^I(M_A), \text{ and } \pi_B^I := Y_B - w_2^I(M_B).$$

Analogously, the payoffs under team assignment are given as

$$\begin{aligned} u_1^T &:= w_{1A}^T(M_A) + w_{1B}^T(M_B) - c(e_{A1}) - c(e_{B1}), \\ u_2^T &:= w_{2A}^T(M_A) + w_{2B}^T(M_B) - c(e_{A2}) - c(e_{B2}), \end{aligned}$$

and $\pi^T := \pi_A^T + \pi_B^T$ where

$$\pi_j^T = Y_j - \left(w_{1j}^T(M_j) + w_{2j}^T(M_j)\right).$$

The expectations over project outcome and performance metric must account for the agents' reporting strategy and the principal's cancellation strategy (as we will elaborate below). We do not explicitly mention this dependence to economize on notations.

We assume that a priori the principal is indifferent between cancelling a project and implementing it without seeking any information from the agents, which implies the following restriction on the parameters.

Assumption 1. $\underline{\pi} = \max_{e_{j1},e_{j2}} \ \frac{1}{2}\left(e_{j1}+e_{j2}\right)y - c\left(e_{j1}\right) - c\left(e_{j2}\right) = \frac{1}{4}y^2$.

We also assume that the outside option of both agents is 0.

Strategies and Equilibrium concept: The strategy of the principal, $\sigma_{\mathcal{P}}$, has two components: (i) A contract $\phi \in \Phi$ offered at the beginning of the game that stipulates the job design $d \in \{\mathcal{I}, \mathcal{T}\}$, and the agents' wage schedules given the chosen design, $\mathcal{W}_1^d$ and $\mathcal{W}_2^d$. (ii) A *continuation policy*, $\mathcal{C}_j$, that stipulates the principal's continuation decision on project j, $j \in \{A, B\}$, as a function of the agents' reports r_1 and r_2. The strategy of the agent $\mathcal{A}_i$, $\sigma_{\mathcal{A}_i}$, has three components: (i) accept or reject the contract offered by the principal, (ii) an *effort policy* $\mathcal{E}_i$ that stipulates effort levels on the assigned tasks, and (iii) a *reporting policy* ρ_i that maps the agent's observed signals to his report r_i. We use *perfect Bayesian Equilibrium* (PBE) in pure strategies as a solution concept.

As the projects are independent and the players' payoffs are additively separable across projects, without loss of generality, we limit attention to the class of equilibria where players use symmetric strategies (i.e., $\mathcal{C}_A = \mathcal{C}_B$, $\rho_1 = \rho_2$, and $w_i^I\left(M_A\right) = w_i^I\left(M_B\right)$). We look for the PBE that yields the highest payoff to the principal in each of the two subgames that follows from a given job design $d \in \{\mathcal{I}, \mathcal{T}\}$. The optimal job design d is the one that yields the highest payoff to the principal.

3. A public information benchmark

We begin our analysis by considering a benchmark case where the agents' observations on the state(s) are *publicly verifiable* information. Thus, the principal does not need to elicit any information from the agents on the projects' viability, and she can also commit at the outset to a cancellation policy that depends on the observed state. This case serves as an useful benchmark for the exploration of the optimal job design in our model: it highlights how the principal's need for information elicitation and her lack of commitment power on continuation decisions drive the key trade-off between individual and team assignment.[1]

[1]The class of wage contracts in this benchmark case is assumed to be the same as the one defined in the main model. Even though the wage payments could be tied to the agents' observed state (when the observations are publicly verifiable), as we will explain below, the principal does not benefit from doing so.

As in our main model, denote $x^j \in \{G, B, \emptyset\}$ as the information on the state ω_j observed by the agent(s) assigned to project j ($x^j = \emptyset$ if neither of the two agents observes ω_j), but now assume that x^j is publicly observed. Suppose that the principal opts for individual assignment ($d = \mathcal{I}$), commits to proceed with project j if and only if $x^j \in X_P^j \subseteq \{G, B, \emptyset\}$, and offers the agents a wage schedule $\{\mathcal{W}_1, \mathcal{W}_2\}$.

In the subgame that follows, the agent $\mathcal{A}_i$'s expected payoff from exerting effort $\mathbf{e}'_j := \left(e'_{j1}, e'_{j2}\right)$ is:

(1)

$$U_i^I\left(\mathbf{e}'_j, X_P^j\right) := \Pr\left(x^j \in X_P^j\right) \sum_{M_j \in \{0,1\}} w_i^I\left(M_j\right) \left[\sum_{\omega_j \in \{G,B\}} \Pr\left(M_j \mid \mathbf{e}'_j, \omega_j\right) \Pr\left(\omega_j \mid x^j \in X_P^j\right)\right] + \Pr\left(x^j \notin X_P^j\right) w_i^I(\emptyset) - \sum_{k \in \{1,2\}} c\left(e'_{jk}\right).$$

That is, with probability $\Pr\left(x^j \in X_P^j\right)$ the project continues, and agent $\mathcal{A}_i$ earns his expected wage conditional on the event that the observation on the underlying state is in X_P^j. Otherwise, the project is cancelled, and the agent earns his "cancellation wage" $w_i^I(\emptyset)$. Notice that the agent incurs the cost of his effort regardless of the principal's decision on the project's implementation.

If the effort profile $\mathbf{e}_j$ is supported in equilibrium, it must satisfy $\mathcal{A}_i$'s incentive compatibility constraint:

$$(IC_I) \qquad \mathbf{e}_j = \arg\max_{e'_{j1}, e'_{j2}} U_i^I\left(\mathbf{e}'_j, X_P^j\right) \quad \forall\, j,$$

and his participation constraint:

$$(IR_I) \qquad U_i^I\left(\mathbf{e}_j, X_P^j\right) \geq 0.$$

Also, the principal's expected payoff under the effort profiles $\{\mathbf{e}_A, \mathbf{e}_B\}$ is (recall that we set $Y_j = \underline{\pi}$ if project j gets cancelled):

$$\Pi^I := \mathbb{E}\left[Y_A - w_1^I\left(M_A\right) \mid \mathbf{e}_A,\ X_P^A\right] + \mathbb{E}\left[Y_B - w_2^I\left(M_B\right) \mid \mathbf{e}_B,\ X_P^B\right].$$

The optimal contract stipulates the wage schedule and continuation policy (given by the sets X_P^j) that maximize Π^I subject to (IR_I) and (IC_I).

Next, consider the case where the principal opts for team assignment ($d = \mathcal{T}$) and offers a wage schedule $\{\mathcal{W}_1, \mathcal{W}_2\}$. In the subgame that follows, the agents' subsequent effort choices constitute a Nash Equilibrium. Thus, if the contract induces the agent $\mathcal{A}_i$ to exert an effort profit $\mathbf{e}_i := (e_{Ai}, e_{Bi})$, it must be a best response to the other agent $\mathcal{A}_{-i}$'s effort level $\mathbf{e}_{-i}$.

Analogous to $U_i^I\left(\mathbf{e}'_j, X_P^j\right)$, denote $\mathcal{A}_i$'s expected payoff under team assignment as $U_i^T\left(\mathbf{e}'_i, \mathbf{e}_{-i}, X_P^A, X_P^B\right)$. The agent's incentive compatibility constraint parallels its counterpart under individual assignment, and can be written as:

$$(IC_T) \qquad \mathbf{e}_i = \arg\max_{\mathbf{e}'_i} \ U_i^T\left(\mathbf{e}'_i, \mathbf{e}_{-i}, X_P^A, X_P^B\right) \quad \forall i.$$

Also, $\mathcal{A}_i$'s participation constraint requires:

$$(IR_T) \qquad U_i^T\left(\mathbf{e}_i, \mathbf{e}_{-i}, X_P^A, X_P^B\right) \geq 0 \ \ \forall i.$$

Thus, the optimal contract stipulates the wage scheme and continuation policy (given by the sets X^j) that maximize the principal's expected payoff

$$\Pi^T := \sum_{j\in\{A,B\}} \mathbb{E}\left[Y_j - \left(w_1^T\left(M_j\right) + w_2^T\left(M_j\right)\right) \ \mid \mathbf{e}_1, \mathbf{e}_2, X_P^A, X_P^B\right],$$

subject to (IR_T) and (IC_T).

Proposition 1. *Under both individual and team assignment, in the optimal contract the principal proceeds with project j if and only if the bad state is not observed (i.e., $x^j \in \{G, \emptyset\}$) and obtains a payoff*

$$S^* := \left(1 + \alpha - \frac{1}{2}\alpha^2\right)\underline{\pi}.$$

That is, in the benchmark case, job design does not affect the principal's payoff under the optimal contract.

The above finding shows that the choice of job design is irrelevant when information is public, and the principal can commit to her continuation policy. Regardless of job design, the principal can always commit to the optimal continuation policy and use the wage contract to induce first-best effort while extracting all surplus from the agent. Thus, the issue of job design becomes relevant only when the agents' observations on the projects' underlying state remain private (and consequently, the principal cannot commit to her continuation policy).

4. Optimal contract

In this section we explore how the principal's need for information elicitation while being unable to commit to her continuation policy shapes the choice between team and individual assignment. In contrast to the benchmark case, when the agents are privately informed, the wage contract not only affects the agents' effort but it also interferes with their incentives to reveal information as well as the principal's incentive to continue with the project. The analysis below highlights how the optimal job design is driven by such intertwined incentives.

4.1. **Optimal contract under individual assignment.** We begin our analysis with the case of individual assignment. That is, we assume that the principal chooses $d = \mathcal{I}$, and in the continuation game we solve for the PBE that yields the highest payoff to the principal. But before we present the formal analysis, it is instructive to describe our solution method. Since we are looking for symmetric equilibria, we only focus on agent $\mathcal{A}_1$ who performs all tasks that are associated with project A. Also, to streamline notations, we drop the agent and project indices.

Our goal is to find the PBE with the largest ex-ante payoff for the principal, and we proceed in two steps: First, we fix a reporting and continuation policy pair $(\rho, \mathcal{C})$, i.e., a "communication protocol," and search for the optimal wage contract $\mathcal{W}$ and effort policy $\mathcal{E}$ such that the tuple $(\mathcal{W}, \mathcal{E}, \rho, \mathcal{C})$ can be supported in a PBE. Next, we compare the payoffs of the principal obtained in the first step across all possible communication protocols.

Lemma 1. *Without loss of generality, we can restrict attention to the following two communication protocols: (i) if the state is observed to be G, report G, otherwise report $\emptyset$; proceed with the project if and only if $r = G$, and (ii) if the state is observed to be B, report B, otherwise report $\emptyset$; proceed with the project if and only if $r \neq B$.*

Lemma 1 implies that we only have to consider two classes of PBE: one where the project proceeds if and only if there is "good news", i.e., the agent's observation is $x \in X_P = \{G\}$, and another where the project proceeds if and only if there is "no bad news", i.e., the agent's observation $x \in X_P = \{G, \emptyset\}$.Thus, without loss of generality, the communication protocols that are relevant for our analysis can be summarized by the set $X_P \in \{\{G\}, \{G, \emptyset\}\}$. Also, for brevity of notation, we can denote $w_1^I(0) =: w_F$ (wage when the performance metric indicates "failure"), $w_1^I(\emptyset) - w_1^I(0) =: \Delta_C$ (wage premium for cancellation), and $w_1^I(1) - w_1^I(0) =: \Delta_S$ (wage premium for success).

Given a wage contract $\{w_F, \Delta_C, \Delta_S\}$, effort levels e_1 and e_2, and X_P (i.e., the set of agent's observation under which the project proceeds), the firm's ex-ante payoff is:

$$\Pi^I := \Pr(x \in X_P)\left[\Pr(\omega = G \mid x \in X_P)(y - \Delta_S) + \Pr(\omega = B \mid x \in X_P)(-\mu\Delta_S)\right]\sum_k e_k \\ + \Pr(x \notin X_P)\left[\underline{\pi} - \Delta_C\right] - w_F.$$

If the project proceeds, it yields a revenue $(y = Y)$ only when the state is good, but the wage premium for success may be paid even if the state is bad (as the performance measure is not perfectly aligned with the project's outcome). And if the project is cancelled, the principal gets her outside option and pays the wage premium for cancellation. The agent's ex-ante payoff can be written analogously as:

$$U^I := \Pr(x \in X_P)\left[\Pr(\omega = G \mid x \in X_P) + \mu\Pr(\omega = B \mid x \in X_P)\right]\Delta_S \sum_k e_k \\ + \Pr(x \notin X_P)\Delta_C + w_F - \tfrac{1}{2}\sum_k e_k^2.$$

Now, if the tuple $(w_F, \Delta_C, \Delta_S; e_1, e_2; X_P)$ is supported as a PBE, the following constraints must be met. First, for each of the two communication protocols given in Lemma 1, the principal's decision must be sequentially rational. In other words, if the principal believes that the agent's signal x is in X_P (given the agent's report), it must be more profitable for her to proceed with the project than to cancel it. Similarly, if the principal believes that the agent's signal is not in X_P, it must be more profitable for her to cancel the project than to proceed with it. Therefore, the principal's incentive compatibility constraints require:

$$(IC_P^I\text{-}1) \quad \left[\Pr(\omega = G \mid x \in X_P)(y - \Delta_S) - \mu\Pr(\omega = B \mid x \in X_P)\Delta_S\right]\sum_k e_k \geq \underline{\pi} - \Delta_C,$$

and

$$(IC_P^I\text{-}2) \quad \left[\Pr(\omega = G \mid x \notin X_P)(y - \Delta_S) - \mu\Pr(\omega = B \mid x \notin X_P)\Delta_S\right]\sum_k e_k \leq \underline{\pi} - \Delta_C.$$

Next, we have the agent's participation constraint:

$$(IR_I) \qquad U^I \geq 0.$$

Finally, consider the agent's incentive compatibility constraint. Let $U(e_1, e_2; \rho)$ be the agent's payoff given his efforts and reporting policy ρ (fixing the wage contract and the principal's continuation policy). The agent's on-path payoff U^I must be the largest payoff attainable for any feasible choice of effort profile and reporting policy. So, we require:

$$U^I = \max_{e_1', e_2', \rho'} U(e_1', e_2'; \rho'). \tag{2}$$

Stipulating (2) is equivalent to imposing the following two constraints: First, a standard incentive compatibility constraint that requires the effort levels to be optimal for the agent given his equilibrium reporting strategy (as per the communication protocol $\{\rho, \mathcal{C}\}$); i.e.,

$$(e_1, e_2) = \arg\max_{e_1', e_2'} U(e_1', e_2'; \rho). \tag{2a}$$

Second, the agent may not gain from a "double deviation" either where he simultaneously deviates on his effort levels and his reporting strategy. Now, given a communication protocol $\{\rho, \mathcal{C}\}$, if the agent can profitably deviate to some other reporting policy ρ' it must be that his report changes the principal's decision on whether to proceed with the project (under the continuation policy $\mathcal{C}$). Consider the two communication protocols as mentioned in Lemma 1. In the first case, the associated reporting policy is to report $x = G$ truthfully and report $\emptyset$ if $x \in \{\emptyset, B\}$, where in the second one the agent reports $x = B$ truthfully and reports $\emptyset$ if $x \in \{G, \emptyset\}$. So, in the first case the only relevant deviation for the agent is to conceal information when $x = G$, and in the second case it is to conceal the information when $x = B$. In other words, in both of these cases, it is sufficient to consider only one type of deviation: the agent reports $\emptyset$ regardless of his observation. We denote this reporting policy as $\rho_\emptyset$. Hence, we must have:

$$U^I \geq \max_{e_1', e_2'} U(e_1', e_2'; \rho_\emptyset). \tag{2b}$$

It is instructive to elaborate on the conditions $(2a)$ and $(2b)$ as they, along with the principal's incentive constraints, illustrate the key trade-offs associated with information elicitation.

Consider a communication protocol from those specified in Lemma 1, and suppose that the project proceeds if $x \in X_P$, $(X_P \in \{\{G\}, \{G, \emptyset\}\})$. Regarding condition $(2a)$, it is routine to check that U is concave in effort for any wage contract and communication protocol, and hence, the condition can be replaced by its associated first-order condition:

$$(IC_A^I\text{-}1) \qquad e_i = \Pr(x \in X_P)\left[\Pr(\omega = G \mid x \in X_P) + \mu \Pr(\omega = B \mid x \in X_P)\right]\Delta_S.$$

The condition $(2b)$, however, is slightly more intricate. In order to simplify this condition one need to account for the fact that when the agent deviates from his equilibrium reporting policy ρ to $\rho_\emptyset$ (i.e., reports $\emptyset$ regardless of his observation), it affects the project's continuation probability. And in case the project continues, the likelihood of a state ω conditional on the project being continued is same as its prior probability as the project would continue regardless of the agent's observed signal x.

Let $p_\emptyset^I$ be the probability that the project continues when the agent deviates to the reporting policy $\rho_\emptyset$, given the equilibrium communication protocol (i.e., $p_\emptyset^I = 1$ if $X_P = \{G, \emptyset\}$ and $p_\emptyset^I = 0$ if $X_P = \{G\}$.) Also, for brevity of notation, denote $p^I := \Pr(x \in X_P)$, and let

$$\begin{aligned} P^I &:= \Pr(\omega = G \mid x \in X_P) + \mu \Pr(\omega = B \mid x \in X_P), \\ P_\emptyset^I &:= \Pr(\omega = G) + \mu \Pr(\omega = B). \end{aligned}$$

Now, off-path, the agent's payoff can be derived as:

$$\begin{aligned} \max\nolimits_{e_1', e_2'} U(e_1', e_2'; \rho_\emptyset) = &\ \max\nolimits_{e_1', e_2'} p_\emptyset^I \left[\Pr(\omega = G) + \mu \Pr(\omega = B)\right]\Delta_S \sum_k e_k' - \tfrac{1}{2}\sum_k e_k'^2 \\ &\qquad + \left(1 - p_\emptyset^I\right)\Delta_C + w_F \\ = &\ \left(p_\emptyset^I P_\emptyset^I \Delta_S\right)^2 + \left(1 - p_\emptyset^I\right)\Delta_C + w_F. \end{aligned}$$

The agent's on-path payoff can be computed analogously, and $(2b)$ simplifies to:

$$(IC_A^I\text{-}2) \qquad \left[\left(p^I P^I\right)^2 - \left(p_\emptyset^I P_\emptyset^I\right)^2\right]\Delta_S^2 \geq \left(p^I - p_\emptyset^I\right)\Delta_C.$$

Thus, the optimal wage contract that supports a communication protocol given by $X_P \in \{\{G\}, \{G, \emptyset\}\}$ solves the following program:

$$\mathcal{P}^I : \max_{\substack{w_F, \Delta_C, \Delta_S, \\ e_1, e_2}} \Pi^I \ \ s.t.\ (IR^I),\ \left(IC_P^I\text{-}1\right),\ \left(IC_P^I\text{-}2\right),\ \left(IC_A^I\text{-}1\right), \text{ and } \left(IC_A^I\text{-}2\right).$$

Lemma 2. *The program $\mathcal{P}^I$ always admits a solution for $X_P = \{G, \emptyset\}$, and admits a solution for $X_P = \{G\}$ if and only if α is sufficiently large.*

The PBE that yields the highest payoff to the principal (under individual assignment) induces the communication protocol (given by $X_P \in \{\{G\}, \{G, \emptyset\}\}$) for which the value of the program $\mathcal{P}^I$ is the largest.

4.2. **Optimal contract under team assignment.** The analysis of team assignment resembles our above discussion on individual assignment, but the two forms of job design differ in two key aspects: First, under team assignment each agent gets exactly one signal from each job. In particular, both agents may observe the underlying state associated with a job. Thus, an agent cannot fully control the flow of information about a given project as his attempt to hide information would fail if the other agent happens to reveal it. Second, for each of the two projects, both agents must be (individually) incentivized for information elicitation and effort provision. (In contrast, under individual assignment the principal has to incentivize only one agent for each project; the agent is responsible for both tasks associated with the project and observes both signals on the project's underlying state). As we will explain later, these two distinctions give rise to the key trade-off between ease of information elicitation and economies of scope in incentive provision that drives the optimal job design.

Now, consider the principal's optimal contracting problem. As mentioned in the previous section, since the production environment and the wage schemes are both assumed to be additively separable across projects, without loss of generality, we may require $w_{iA}^T(M_A) = w_{iB}^T(M_B)$. Consequently, we can formulate the principal's optimal contracting problem as one where there is only one project (with two tasks) and the principal hires two agents: each agent performs exactly one of the two tasks and observes exactly one of the two signals on the project's state.

Analogous to the case of individual assignment, we seek to characterize the PBE of this continuation game with the largest ex-ante payoff for the principal. The analysis follows the same two-step process that we have described above: first, we fix a communication protocol and derive the optimal wage contract that support this protocol in equilibrium; and next, we compare the principal's payoff across all possible communication protocols that could be sustained in equilibrium.

With a slight abuse of notation, we continue to denote the strategies of the players in this game by the tuple $(\mathcal{W}_i^*, \mathcal{E}_i^*, \rho_i^*, \mathcal{C}^*)$. To streamline notation, we drop the project index and relabel $w_{ij}^T(0) =: w_{iF}$, $w_{ij}^T(\emptyset) - w_{ij}^T(0) =: \Delta_{iC}$ (wage premium for cancellation), and

$w_{ij}^{T}(1) - w_{ij}^{T}(0) =: \Delta_{iS}$ (wage premium for success). Also, we denote the team's collective observation on the state as x^T, where

$$x^T := \begin{cases} G & if\ x_i = G \text{ for some } i \\ B & if\ x_i = B \text{ for some } i \\ \emptyset & if\ x_1 = x_2 = \emptyset \end{cases}.$$

As in the case of individual assignment, we can again limit attention to only two communication protocols as stated in the lemma below. (We omit the proof of this lemma as it is follows the same argument as that of Lemma 1.)

Lemma 3. *Without loss of generality, we can restrict attention to the following two communication protocols: (i) reporting policy for agent $\mathcal{A}_i$ $(i = 1, 2)$: if the state is observed to be G, report G, otherwise report $\emptyset$; principal proceeds with the project only if $r_i = G$ for some i, and (ii) reporting policy for agent $\mathcal{A}_i$ $(i = 1, 2)$: if state is observed to be B, report B, otherwise report $\emptyset$; principal proceeds only if $r_i \neq B$ for all i.*

Thus, without loss of generality, as before, the communication protocols that are relevant for our analysis of team assignment can be summarized by the set $X_P \in \{\{G\}, \{G, \emptyset\}\}$. Given a wage contract $\{w_{iF}, \Delta_{iC}, \Delta_{iS}\}$, effort levels e_1 and e_2, and X_P, it is routine to check that the firm's ex-ante payoff is:

$$\begin{aligned} \Pi^T \quad := {} & \Pr\left(x^T \in X_P\right) \times \\ & \left[\Pr\left(\omega = G \mid x^T \in X_P\right)\left(y - \sum_i \Delta_{iS}\right) + \Pr\left(\omega = B \mid x^T \in X_P\right)\left(-\mu \sum_i \Delta_{iS}\right)\right] \sum_k e_k \\ & + \Pr\left(x^T \notin X_P\right)\left[\underline{\pi} - \sum_i \Delta_{iC}\right] - \sum_i w_{iF}. \end{aligned}$$

The agent i's participation constraint requires:

$$(IR_i^T) \qquad \begin{aligned} U_i^T := {} & \Pr\left(x^T \in X_P\right) \times \\ & \left[\Pr\left(\omega = G \mid x^T \in X_P\right) + \mu \Pr\left(\omega = B \mid x^T \in X_P\right)\right] \Delta_{iS} \sum_k e_k \\ & + \Pr\left(x^T \notin X_P\right) \Delta_{iC} + w_{iF} - \tfrac{1}{2} e_i^2 \quad \geq 0. \end{aligned}$$

The principal's incentive compatibility constraints ensure that is it optimal for the principal to proceed with the project if x^T is in X_P and to cancel it otherwise:

$$(IC_P^T\text{-1}) \qquad \Big[\Pr\left(\omega = G \mid x^T \in X_P\right)\Big(y - \sum_i \Delta_{iS}\Big) + \Pr\left(\omega = B \mid x^T \in X_P\right)\Big(-\mu \sum_i \Delta_{iS}\Big)\Big] \sum_k e_k \geq \underline{\pi} - \sum_i \Delta_{iC},$$

and

$$(IC_P^T\text{-2}) \qquad \Big[\Pr\left(\omega = G \mid x^T \notin X_P\right)\Big(y - \sum_i \Delta_{iS}\Big) + \Pr\left(\omega = B \mid x^T \notin X_P\right)\Big(-\mu \sum_i \Delta_{iS}\Big)\Big] \sum_k e_k \leq \underline{\pi} - \sum_i \Delta_{iC},$$

Notice that in contrast to its counterpart under individual assignment, the (IC_P) constraints highlight that the project's success and cancellation both would require the principal to pay the corresponding wage premium to both of the two agents. As we will see later, the need for such "double payment" captures diseconomies of scope in incentive provision under team assignment.

Finally, consider the agents' incentive compatibility constraints. As before, the constraint would require that neither of the two agents can gain by unilaterally deviating to a different effort choice and reporting policy. However, there is a salient distinction between the constraints under team and their counterpart under individual assignment. Under team assignment, an agent chooses the effort in only one of the two tasks, and reports only one of the two signals on the project's underlying state. Thus, an agent cannot fully influence the project's output and the associated performance measure, nor he can fully control the information on the underlying state that may be communicated to the principal.

Let $U_i\left(e_i, \rho_i; e_j, \rho_j\right)$ be the agent $\mathcal{A}_i$'s payoff given the two agents' efforts and reporting policies (fixing the wage contracts and the principal's continuation policy). The agent's on-path payoff U_i^T must be the largest payoff attainable for any feasible choice of effort profile and reporting policy (given the other agent's equilibrium effort and reporting policy). So, the constraint requires:

$$U_i^T = \max_{e_i', \rho_i'} U_i\left(e_i', \rho_i'; e_j, \rho_j\right). \tag{3}$$

As before, it is sufficient to consider only two types of deviation: (i) the agent follows his equilibrium reporting policy ρ_i but deviates on his effort level, (ii) the agent reports $\emptyset$ regardless of his observation, and chooses his effort level accordingly. Again, with a slight abuse of notation, we denote the latter reporting policy (given in (ii)) as $\rho_\emptyset$. Thus, the incentive compatibility constraint (3) for agent $\mathcal{A}_i$ $(i = 1, 2)$ is equivalent to the following two conditions:

$$e_i = \arg\max_{e_i'} U_i\left(e_i', \rho_i; e_j, \rho_j\right) \tag{3a}$$

and

$$U_i^T \geq \max_{e_i'} \; U_i\left(e_i', \rho_\emptyset; e_j, \rho_j\right). \tag{3b}$$

Now, $(3a)$ implies that e_i satisfies the following first-order condition $(i = 1, 2)$:

$$e_i = \Pr\left(x^T \in X_P\right)\left[\Pr\left(\omega = G \mid x^T \in X_P\right) + \mu \Pr\left(\omega = B \mid x^T \in X_P\right)\right]\Delta_{iS}. \tag{$IC_{A_i}^T$-1}$$

Also, $(3b)$ can be simplified in the same fashion in which we streamlined its counterpart under individual assignment. However, one needs to account for the fact that under team, an agent's attempt to conceal information may be undermined by the report of the other agent. In parallel to our analysis of individual assignment, let $p_\emptyset^T$ be the probability that the project continues when agent i deviates to the reporting policy $\rho_\emptyset$, given the equilibrium communication protocol. Also, denote $p^T := \Pr\left(x^T \in X_P\right)$, and

$$\begin{aligned} P^T &:= \Pr\left(\omega = G \mid x^T \in X_P\right) + \mu \Pr\left(\omega = B \mid x^T \in X_P\right), \\ P_\emptyset^T &:= \Pr\left(\omega = G \mid \rho_\emptyset, \rho_j, \mathcal{C}\right) + \mu \Pr\left(\omega = B \mid \rho_\emptyset, \rho_j, \mathcal{C}\right), \end{aligned}$$

where $\Pr\left(\omega \mid \rho_\emptyset, \rho_j, \mathcal{C}\right)$ denotes the probability of the state ω conditional on the event that the project proceeds under the communication protocol $\left\{\rho_\emptyset, \rho_j, \mathcal{C}\right\}$. Now, plugging in the agent's on- and off-path payoffs, condition $(3b)$ can be stated as:

$$\begin{array}{c} \frac{1}{2}\left[\left(p^T P^T\right)^2 - \left(p_\emptyset^T P_\emptyset^T\right)^2\right]\Delta_{iS}^2 + \left[\left(p^T P^T\right)^2 - \left(p_\emptyset^T P_\emptyset^T\right)\left(p^T P^T\right)\right]\Delta_{iS}\Delta_{jS} \\ \geq \left(p^T - p_\emptyset^T\right)\Delta_{iC}. \end{array} \tag{$IC_{A_i}^T$-2}$$

Thus, the optimal wage contract under team assignment that supports a communication protocol given by $X_P \in \{\{G\}, \{G, \emptyset\}\}$ solves the following program:

$$\mathcal{P}^T : \max_{\{w_{iF}, \Delta_{iC}, \Delta_{iS}\}, e_1, e_2} \Pi^T \;\; s.t. \;\left(IR_i^T\right),\; \left(IC_P^T\text{-}1\right),\; \left(IC_P^T\text{-}2\right),\; \left(IC_{A_i}^T\text{-}1\right), \text{ and } \left(IC_{A_i}^T\text{-}2\right).$$

Lemma 4. *(i) The program $\mathcal{P}^T$ always admits a solution for $X_P = \{G, \emptyset\}$ and admits a solution for $X_P = \{G\}$ if and only if both α and μ are sufficiently large.*

(ii) If $\mathcal{P}^T$ admits a solution, it also admits a symmetric solution where $w_{1F} = w_{2F} = w_F$, $\Delta_{1S} = \Delta_{2S} = \Delta_S$ and $\Delta_{1C} = \Delta_{2C} = \Delta_C$.

The PBE that yields the highest payoff to the principal (under team assignment) induces the communication protocol (given by $X_P \in \{\{G\}, \{G, \emptyset\}\}$) for which the value of the program $\mathcal{P}^T$ is the largest.

5. Optimal job design

By comparing the principal's payoffs associated with the optimal contracts under team and individual accountability, we can now characterize the optimal job design.

Proposition 2. ***(Optimal job design)*** *There exist two thresholds μ_0 and μ_1 (given α), $\mu_0 < \mu_1$, such that:*

(i) if $\mu < \mu_0$, it is optimal to choose individual assignment where the agent reports B only if he observes the state to be B, and reports $\emptyset$ otherwise; the principal proceeds with the project only if the report is not B. The associated optimal contract is efficient and the principal's payoff is S^* (as defined in Proposition 1).

(ii) If $\mu > \mu_1$, it is optimal to choose team assignment where the agent reports B only if he observes the state to be B, and reports $\emptyset$ otherwise; the principal proceeds with the project only if no agent reports B. The associated optimal contract is efficient and the principal's payoff is S^*.

(iii) Otherwise, ($\mu_0 \leq \mu \leq \mu_1$) the principal is indifferent between team and individual assignments: both designs, along with the corresponding communication protocol as stated in parts (i) and (ii) above, yield the same payoff of $S^*$for the principal.

Moreover, the parameter thresholds μ_0 and μ_1 vary with α in the following manner.

Proposition 3. ***(Comparative statics)*** *The threshold μ_0 is increasing in α. Also, there exists a cutoff α^* such that $\mu_1 = 1$ for $\alpha \leq \alpha^*$ and μ_1 is decreasing in α for $\alpha \geq \alpha^*$.*

Propositions 2 and 3 (illustrated in Figure 1) show how the optimal job design is driven by the "availability" of the agents' signal (as captured by α) and the "alignment" of the performance measure with the project's output (as captured by μ). For low α (i.e., $\alpha \leq \alpha^*$), individual assignment is always optimal; for low μ (i.e., $\mu < \mu_0$) it strictly dominates team assignment but otherwise (i.e., $\mu \geq \mu_0$) both designs yield the same (optimal) payoff. In contrast, when α is large, team assignment is strictly optimal provided μ is large as well (i.e., $\mu > \mu_1$). However, as before, for moderate μ the two designs yield the same payoff, and for small μ individual assignment remains strictly optimal.

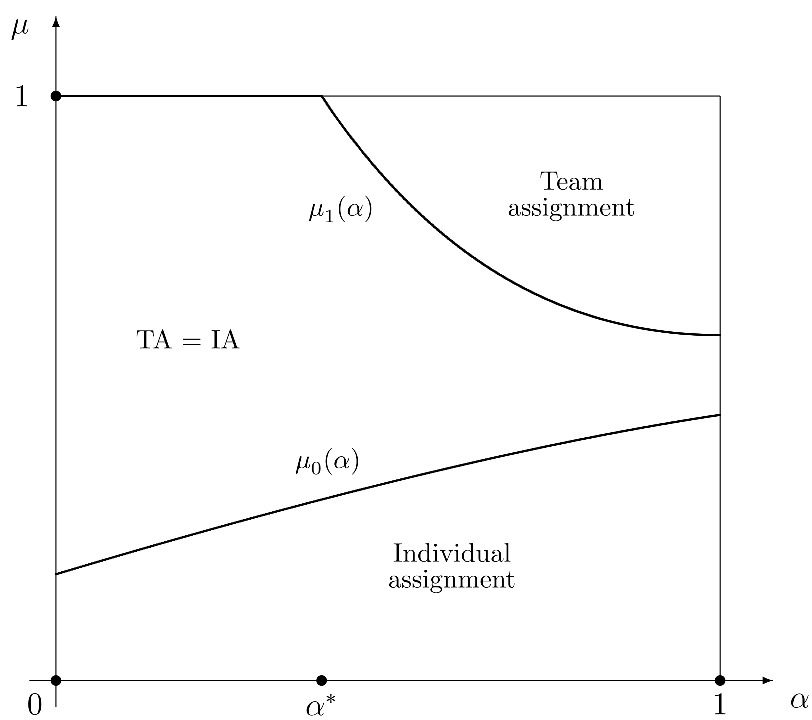

Figure 1: Optimal job design as a function of α and μ.

To see the intuition behind the above result, recall that our setup highlights two key frictions. First, the principal lacks information on the project's viability and must elicit it from the agents. Second, even though the principal's continuation decision depends on the agents' information, she cannot commit to any continuation policy ex-ante. These two frictions give rise to a trade-off that drives the optimal job design: relative to individual assignment, team facilitates information elicitation but suffers from diseconomies of scope in incentive provision.

Team assignment helps in information elicitation as an agent cannot fully control the outcome of the project (and the performance measure). Even if the agent attempts to suppress information and adjust his effort (in his assigned task) accordingly, his gains from such deviations are muted by the fact that his teammate may still reveal the information to the principal. Also, the agent cannot control the level of effort on the task that is performed by his teammate. But, under individual assignment such a "double deviation", i.e., concurrent manipulation of reporting and effort, may be more profitable for the agent: he fully controls what the principal gets to learn about the project's underlying state and how much effort is exerted on both tasks that are associated with the project. In fact, he stands to profit from it when both α and μ are large.

When α is large, the agent's control over the project's continuation is more valuable as he is now more likely to observe the state and, under individual assignment, he can hide any unfavorable information. In particular, the agent would have a strong incentive to conceal the bad state (and let the project continue) if he expects to earn a large payoff even if the project fails. This is indeed the case when μ is large, i.e., the performance measure is significantly misaligned with the project's outcome: in a bad state, the measure is more likely to indicate success (given the effort levels) even though the project is sure to fail. Moreover, should the agent deviate on his reporting policy and hide the bad state, he may also exert more effort (vis-a-vis the on-path effort levels) so as to further increase his gains from deviation. Thus, when α and μ are both large, deterring the agent from double-deviation gets harder under individual assignment, and team's advantage over individual assignment in information elicitation becomes stronger. This is why team assignment dominates individual assignment when α and μ are high.

However, team assignment lacks economies of scope in incentive provision: in order to induce effort on both tasks associated with the project, the principal needs to incentivize the two agents separately. Notice that under individual assignment a single wage payment (w_F, w_S, or w_C based on the project's outcome) incentivizes the agent to exert efforts on all tasks. In contrast, in a team, each of the two agents are assigned to exactly one of the two tasks. Hence, if the principal were to induce the same level of effort in both tasks of the project her wage bill doubles ($2w_F$, $2w_S$, or $2w_C$).

Such *dis*economies of scope may be costly to the principal. As the principal lacks commitment power over the continuation policy, her (IC_P) constraints must hold. That is, for any given job design with communication protocol given by X_P, (i) the principal's expected payoff from proceeding when the agents' observation is in X_P must be larger than her payoff from cancelling the project, and (ii) the payoff from cancelling must be larger than her expected payoff from proceeding with the project if the agents' observation is not in X_P. Thus, any feasible contract must ensure that the principal earns more from proceeding when the signal is in X_P than when it is not. For example, $\left(IC_P^I\text{-}1\right)$ and $\left(IC_P^I\text{-}2\right)$ implies:

$$[\Pr(\omega = G \mid x \in X_P)(Y - \Delta_S) + \Pr(\omega = B \mid x \in X_P)(-\mu\Delta_S)] \sum_k e_k \geq$$
$$[\Pr(\omega = G \mid x \notin X_P)(Y - \Delta_S) + \Pr(\omega = B \mid x \notin X_P)(-\mu\Delta_S)] \sum_k e_k.$$

This difference in earnings is given by the difference in the expected output of the project

$$\left[[\Pr(\omega = G \mid x \in X_P) - \Pr(\omega = G \mid x \notin X_P)] \sum_k e_k\right] Y,$$

and the difference in the expected wage payout

$$\left[(1-\mu)[\Pr(\omega = G \mid x \in X_P) - \Pr(\omega = G \mid x \notin X_P)] \sum_k e_k\right] \Delta_S.$$

Now, for any $X_P \in \{\{G\}, \{G, \emptyset\}\}$ the probabilities that the project and the performance measure indicate success (i.e., $y = Y$ and $M = 1$) are both larger (given the effort levels in the two tasks) when the agents' signal is in X_P than when it is not. Therefore, when the principal needs to pay the wage premium for success (Δ_S) twice in order to elicit the same amount of effort in both tasks—as is the case under team assignment—the difference in her expected wage payouts is larger. Consequently, the aforementioned feasibility constraint is harder to satisfy under team, and individual assignment becomes more favorable.

Also note that team's relative disadvantage (due to diseconomies of scope) becomes more acute when μ is small (i.e., the measure is well-aligned with the project's output). As the agent is unlikely to earn a reward for success when the state is bad, the wage premium for success needs to be sufficiently large so as to incentivize him to exert effort. And when the principal needs to pay such large premiums *twice*—as is the case under team assignment—her continuation policy is less likely to remain credible: proceeding with the project when the signal is in X_P may be less profitable than proceeding when it is not (i.e., (IC_P) gets violated). This explains why individual accountability dominates team when μ is low.

The above discussion may be summarized as follows: For low μ, provision of incentives under team assignment gets compromised due to acute diseconomies of scope, but incentives under individual assignment remain sharp as information elicitation is relatively easy ("double deviation" is less profitable as a successful performance is unlikely to arise when the state is bad). Thus, individual assignment strictly dominates team. However, for large μ, diseconomies of scope does not distort incentive provision under team: as the required success premium is smaller, it may be feasible for the principal to pay it to both agents separately. Thus, both designs yield the same payoff as long as information elicitation does not distort incentives under individual assignment. But information elicitation gets harder under individual assignment if α is also large (along with μ), and team assignment becomes strictly optimal.

Notice that at the optimal job design diseconomies of scope does not distort incentives, and neither does the need for information elicitation. Therefore, the associated contract yields the efficient level of surplus as obtained in the public information benchmark (in Section 3). However, this observation critically hinges on our modeling assumption that the agents' observation on the state does not contain any noise (conditional on observing it in the first place). As we discuss in the next section, when the agent's signal is noisy, the optimal job design may entail inefficiencies both in the principal's continuation policy and in the agents' effort levels.

6. Discussion and conclusion

While our model adopts a stylized information setup for analytical tractability, the key trade-off that we highlight here (between information elicitation and diseconomies of scope) may continue to shape the firm's job design decision in some related and more general settings. We consider two such extensions of our model. First, we relax the assumption that an informed agent observes the state without any noise, and assume that an agent's signal may be imprecise. Next, we relax the assumption that the observability of the underlying state of a project in each of its two tasks is statistically independent, and explore the case where they are mutually exclusive.

6.1. **Imprecise signals.** In our model, the agent, conditional on observing the state, always observes it without any noise. While this assumption improves the analytical tractability of the model, it is conceivable that the agents may not be able to directly observe the state but only acquire an imprecise signal on the same. How would our characterization of the optimal job design change if the agents' information were noisy?

In order to explore this issue, we consider the following modification to our model: Suppose that the state $\omega_j \in \{G, B\}$ associated with the project j ($j \in \{A, B\}$) is never directly observed, but the agents' may observe a signal $\sigma_j \in \{G, B\}$ that is informative of ω_j. Let

$$\Pr(\omega_j = G \mid \sigma_j = G) = \Pr(\omega_j = B \mid \sigma_j = B) = \theta,$$

where $\theta \in (1/2, 1)$ reflects the precision of the signal. In parallel with the information structure of our model, we assume that the agent assigned in task T_{jk} privately observes σ_j with probability α. And with a slight abuse of notation, we denote the agent $\mathcal{A}_i$'s observation on the signal σ_j as $x_i^j \in \{G, B, \emptyset\}$, where $x_i^j = \emptyset$ if $\mathcal{A}_i$ does not observe σ_j in any of his assigned tasks. We keep all other aspects of our model unaltered. Notice that our main model corresponds to the case where $\theta = 1$.

Though a complete characterization of the optimal job design for this case is analytically intractable, the following proposition suggests that our main result is robust to a small noise in the agents' signal.

Proposition 4. *There exists a threshold $\theta^* < 1$ such that for $\theta > \theta^*$, the qualitative characterization of the optimal job design is the same as its counterpart in our main model (as given in Proposition 2), and the optimal contract is always efficient.*

However, if the agents' signal becomes sufficiently noisy (i.e., when θ is sufficiently low) our main result may no longer hold. Recall that under the optimal contract (in our main model), the project proceeds even when the agents fail to reveal their signal, i.e., the project continues unless the agent(s) report(s) a bad state. But when the agents' signal is sufficiently noisy, information elicitation gets harder. An agent now has a stronger incentive to hide a bad signal and let the project pass, since with some probability, a bad signal may still be associated with a good state.

This effect may introduce two sources of inefficiencies. First, the principal may reduce the effort incentives so as to mitigate the agent's incentive to hide a bad signal. (Recall that as the agent's effort increases, the performance measure is more likely to indicate success. Thus when the efforts are high, the agent has stronger incentive to continue the project under a bad signal.) Second, if such distortions to the effort level is too costly, the principal may also distort her continuation policy: the project may proceed only if the signal is good. And at the extreme, i.e., when θ is low enough, it is optimal for the principal to proceed with all projects without soliciting any information from the agents (or, equivalently, to settle for the outside option). These inefficiencies are illustrated in Figure 2 above that presents a numerical solution for the optimal job design problem.

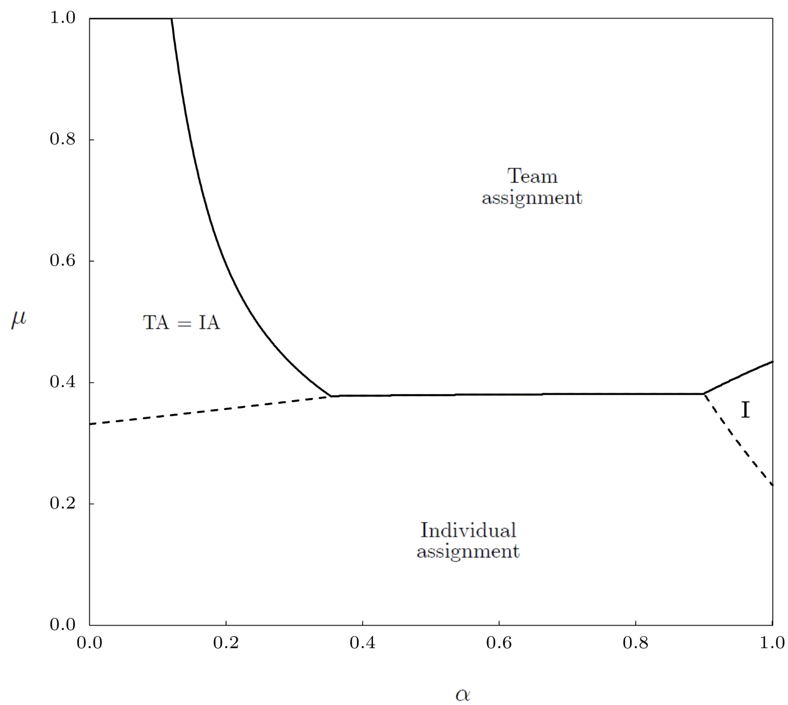

Figure 2: Optimal job design with imprecise signal ($\theta = 0.77$):
In region I individual assignment is optimal but continuation decision is inefficient; project continues only if the report is good.

6.2. **Exclusive signals.** So far, we have assumed that the observability of the underlying state of a project in each of its two tasks is statistically independent. Such a setup may reflect a scenario where each task T_{jk} (of project j) gives access to a different (and independent) source of information, each of which may reveal the state ω_j with probability α. But it is conceivable that the informativeness of these sources may not be independent. In this subsection, we focus on one such scenario: sources being mutually exclusive in terms of their informativeness. An exploration of this case further illustrates how the agents' ability to control the outcome of a project through their efforts may affect the optimal job design.

To formalize this idea, we make the following modification to our model. We assume that exactly one of the two tasks associated with a given project may yield information about the project's underlying state. In particular, with probability 1/2, only task T_{j1} can yield information: the agent performing task T_{j1} observes the state with probability α, whereas the agent performing T_{j2} never observes it. And with probability 1/2, only T_{j2} is informative: the agent performing task T_{j1} never observes the state whereas the agent performing T_{j2} observes it with probability α. We keep all other aspects of the model unchanged.

Notice that in this setup, under individual assignment, the probability that an agent observes the state of his assigned project is α. And this is also the probability that under team accountability at least one of the two agents observes the state. However, in this setting team assignment appears to lose its advantage in information elicitation: as the observability of the state is mutually exclusive between tasks, should an agent observe an unfavorable information he can completely suppress it as his teammate would necessarily be uninformed.

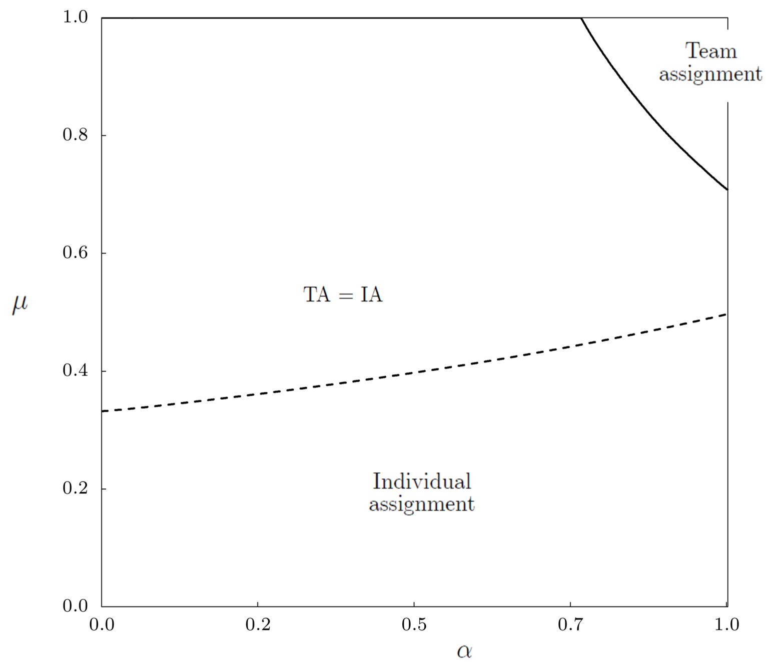

Figure 3: Optimal job design under mutually exclusive signals across tasks (within a project)

One may anticipate that such complete control over the information on the state may make team suboptimal to individual assignment as team still continues to suffer from diseconomies of scope in incentive provision. However, this intuition is incomplete. Notice that an agent controls the outcome of a project in two ways: through his reporting on the state that affects the project's continuation probability, and also through his effort(s) that affect(s) the project's output and the performance metric (should the project proceed). When the signals are mutually exclusive, the advantage of team in muting the former channel is indeed diminished. However, team assignment may still help information elicitation as the agent cannot control the effort in all tasks that are associated with the project. Numerical result suggests (see Figure 3) that team's advantage in information elicitation remains sufficiently strong even under mutually exclusive signals and, as in our main model, it may still dominate individual assignment when both α and μ are sufficiently large.

6.3. **Conclusion.** When effective decision-making requires local information, the incentive structure in an organization must meet two goals at once: induce the workers to exert costly effort and truthfully report their information even if the information may be detrimental to their own interest. This article explores how job design—allocation of tasks among workers—interacts with such intertwined incentives. We argue that the optimal job design is shaped by a novel tradeoff between the ease of information elicitation and diseconomies of scope in incentive provision. And this tradeoff, in turn, is driven by the interplay between the "availability" of the workers' information and the "alignment" of their performance measure with the firm's objective. In particular, team assignment may be optimal when the performance measure is considerably misaligned, but the workers are highly likely to be informed about the local condition. Our findings suggest a novel explanation of why team can offer better incentive even when measures of individual performance remain available.

7. Appendix

This article contains the proofs omitted in the text.

Proof of Proposition 1. Under individual assignment, the optimal contracting program is given as:

$$\max\ \Pi^I := \mathbb{E}\left[Y_A - w_1^I(M_A)\ \mid \mathbf{e}_A,\ X_P^A\right] + \mathbb{E}\left[Y_B - w_2^I(M_B)\ \mid \mathbf{e}_B,\ X_P^B\right]$$
$$s.t.$$
$$\mathbf{e}_j = \arg\max_{e'_{j1}, e'_{j2}}\ U_i^I\left(\mathbf{e}'_j, X_P^j\right)\quad \forall\ j \qquad (IC_I)$$
$$U_i^I\left(\mathbf{e}_j, X_P^j\right) \geq 0 \qquad (IR_I)$$

By standard argument, (IR_I) must bind, and any effort profile can be implemented (i.e., made to satisfy the (IC_I)) by choosing the wage schedules $w_1^I(M_A)$ and $w_2^I(M_B)$ appropriately. Thus, the program boils down to:

$$\max_{\mathbf{e}_A, \mathbf{e}_B} \sum_{j \in \{A,B\}} \left[\mathbb{E}\left[Y_j \mid \mathbf{e}_j, X_P^j\right] - \frac{1}{2}e_{j1}^2 - \frac{1}{2}e_{j2}^2\right].$$

Denote $\pi(X_P^j) := \max_{\mathbf{e}_j}\ \mathbb{E}\left[Y_j \mid \mathbf{e}_j, X_P^j\right] - \frac{1}{2}e_{j1}^2 - \frac{1}{2}e_{j2}^2$, and it is routine to check:

$$\pi(X_P^j) := \begin{cases} (1 + (2\alpha - \alpha^2)(2\alpha - \alpha^2 - \frac{1}{2}))\underline{\pi} & if\ X_P^j = \{g\} \\ (1 + \alpha - \frac{1}{2}\alpha^2)\underline{\pi} & if\ X_P^j = \{g, \emptyset\} \\ \underline{\pi} & if\ X_P^j = \{g, \emptyset, b\} \end{cases}.$$

Comparing the values, we obtain that the optimal $X_P^j = \{g, \emptyset\}$ for all j. That is, under individual assignment, in the optimal contract the principal proceeds with project j if and only if the bad state is not observed, and obtains payoff $S^* = \left(1 + \alpha - \frac{1}{2}\alpha^2\right)\underline{\pi}$.

Similarly, under team assignment, the optimal contracting program is give as:

$$\max\ \Pi^T := \textstyle\sum_{j\in\{A,B\}} \mathbb{E}\left[Y_j - \left(w_1^T\left(M_j\right) + w_2^T\left(M_j\right)\right) \mid \mathbf{e}_1, \mathbf{e}_2, X_P^A, X_P^B\right]$$

$$s.t.$$

$$\mathbf{e}_i = \arg\max_{\mathbf{e}_i'}\ U_i^T\left(\mathbf{e}_i', \mathbf{e}_{-i}, X_P^A, X_P^B\right) \quad \forall i \qquad (IC_T)$$

$$U_i^T\left(\mathbf{e}_i, \mathbf{e}_{-i}, X_P^A, X_P^B\right) \geq 0 \quad \forall i \qquad (IR_T)$$

As in the case of individual assignment, we can plug (IR_T) in the objective function and ignore the (IC_T); the program boils down to:

$$\max_{e_{A1}, e_{A2};\ e_{B1}, e_{B2}} \sum_{j\in\{A,B\}} \mathbb{E}\left[Y_j - \frac{1}{2}e_{j1}^2 - \frac{1}{2}e_{j2}^2 \mid e_{j1}, e_{j2}, X_P^j\right].$$

Thus, given X_P^A and X_P^B, the principal's payoff is exactly the same as that in the case of individual assignment, and so claim follows. □

Proof of Lemma 1. Note that there are four possible reporting policies: for each $x \in \{G, B\}$, $r = \rho(x) = x$ or $\emptyset$ (and $\rho(\emptyset) = \emptyset$); and eight possible continuationpolicies: for each $r \in \{G, \emptyset, B\}$, $\mathcal{C}(r) = cancel$ or $proceed$.

Step 1. *Without loss of generality we can consider only two continuation policies.* Trivially, $\mathcal{C}(r) = cancel$ $\forall r$ yields a payoff of $\underline{\pi}$ (principal's outside option), and $\mathcal{C}(r) = proceed$ $\forall r$ also yields $\underline{\pi}$ (by Assumption 1). Also, as under any reporting policy,

$$\Pr(\omega = G \mid r = G) \geq \Pr(\omega = G \mid r = \emptyset) \geq \Pr(\omega = G \mid r = B),$$

in equilibrium, if $\mathcal{C}(B) = proceed$ then $\mathcal{C}(r) = proceed$ for all r, and if $\mathcal{C}(\emptyset) = proceed$ then $\mathcal{C}(G) = proceed$. Thus, without loss of generality, we can focus on equilibria that supports only one of the following two continuation policies: (i) $\mathcal{C}(r) = proceed$ only if $r = G$, and (ii) $\mathcal{C}(r) = proceed$ only if $r \in \{G, \emptyset\}$.

Step 2. *For each of the two continuation policies stated in Step 1, only one reporting policy may be played in equilibrium.*

Step 2a. Suppose, in the optimal contract, the principal's continuation policy (i), i.e., $\mathcal{C}(r) = proceed$ if and only if $r = G$. The two reporting policies of the agent where $\rho(G) = \emptyset$

(and $\rho(B) = B$ or $\emptyset$) yield the same payoff as the project gets cancelled under both policies. Also, the two reporting policies, $\rho(G) = G$ and $\rho(B) = B$ or $\emptyset$, yield the same payoff. But the policy $\rho(G) = G$ and $\rho(x) = \emptyset$ if $x \in \{\emptyset, B\}$ relaxes the principal's incentive constraints relative to the policy $\rho(x) = x$ for all $x \in \{G, B\}$ as

$$\Pr(\omega = G \mid x = \emptyset) \geq \Pr(\omega = G \mid x \in \{\emptyset, B\}) \geq \Pr(\omega = G \mid x = B).$$

Thus, if in the optimal contract continuation policy (i) is played, then without loss of generality, we assume that the associated reporting policy is $\rho(G) = G$ and $\rho(x) = \emptyset$ if $x \in \{\emptyset, B\}$.

Step 2b. Now suppose in the optimal contract continuation policy (ii), i.e., $\mathcal{C}(r) = proceed$ if and only if $r \in \{G, \emptyset\}$, is played. The two reporting policies of the agent where $\rho(B) = \emptyset$ (and $\rho(G) = G$ or $\emptyset$) yield the same payoff as the project always proceeds. Also, the two reporting policies, $\rho(B) = B$ and $\rho(G) = G$ or $\emptyset$, yield the same payoff. But the policy $\rho(B) = B$ and $\rho(x) = \emptyset$ if $x \in \{G, \emptyset\}$ relaxes the incentive constraints relative to the policy $\rho(x) = x$ for all $x \in \{G, B\}$. Thus, if in the optimal contract continuation policy (ii) is played, then without loss of generality, we assume that the associated reporting strategy is $\rho(G) = G$ and $\rho(x) = \emptyset$ if $x \in \{\emptyset, B\}$.

Together, the observations in Steps 1 and 2 imply that, without loss of generality, we can limit attention to two communication protocols: (i) If the state is observed to be G, report G, otherwise report $\emptyset$; proceed with the project if and only if $r = G$. (ii) If the state is observed to be B, report B, otherwise report $\emptyset$; proceed with the project if and only if $r \neq B$. □

Proof of Lemma 2. For brevity, we rewrite the objective function and all constraints by using the notations p^I, $p^I_\emptyset$, P^I and $P^I_\emptyset$ (as defined in Section 4.1), and the program $\mathcal{P}^I$ boils down to:

$$\max_{\substack{w_F, \Delta_C, \Delta_S, \\ e_1, e_2}} \quad \Pi^I := p^I\left[\left[\Pr(\omega = G \mid x \in X_P)y - P^I \Delta_S\right] \sum_k e_k + (1 - p^I)[\underline{\pi} - \Delta_C] - w_F\right]$$

$$s.t.$$

$$p^I P^I \Delta_S \sum_k e_k + (1 - p^I)\Delta_C + w_F - \tfrac{1}{2} \sum_k e_k^2 \geq 0 \qquad (IR^I)$$

$$[\Pr(\omega = G \mid x \in X_P)y - P^I \Delta_S] \sum_k e_k \geq \underline{\pi} - \Delta_C \qquad (IC^I_P\text{-}1)$$

$$[\Pr(\omega = G \mid x \notin X_P)y - P^I_C \Delta_S] \sum_k e_k \leq \underline{\pi} - \Delta_C \qquad (IC^I_P\text{-}2)$$

$$e_k = p^I P^I \Delta_S, \quad k = 1, 2 \qquad (IC^I_A\text{-}1)$$

$$\left[\left(p^I P^I\right)^2 - \left(p^I_\emptyset P^I_\emptyset\right)^2\right] \Delta_S^2 \geq \left(p^I - p^I_\emptyset\right) \Delta_C. \qquad (IC^I_A\text{-}2)$$

By standard argument, we claim that $\left(IR^I\right)$ binds. Using (IR) and $(IC_A^I\text{-}1)$ we can eliminate w_F and e_is and the program can be further simplifies to:

$$\max_{\Delta_C,\Delta_S} \; 2\left(p^I\right)^2 P^I \Pr(\omega=G \mid x\in X_P)y\Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I \Delta_S\right)^2$$
$$s.t.$$
$$[\Pr(\omega=G \mid x\in X_P)y - P^I\Delta_S]\left(2p^I P^I\Delta_S\right) \geq \underline{\pi} - \Delta_C \qquad (IC_P^I\text{-}1)$$
$$[\Pr(\omega=G \mid x\notin X_P)y - P_C^I\Delta_S]\left(2p^I P^I\Delta_S\right) \leq \underline{\pi} - \Delta_C \qquad (IC_P^I\text{-}2)$$
$$\left[\left(p^I P^I\right)^2 - \left(p_\emptyset^I P_\emptyset^I\right)^2\right]\Delta_S^2 \geq \left(p^I - p_\emptyset^I\right)\Delta_C \qquad (IC_A^I\text{-}2)$$

Case 1: $X_P=\{G,\emptyset\}$. Here $p_\emptyset^I=1$, and the program becomes:

$$\mathcal{P}^I_{\{g,\emptyset\}}:\begin{cases} \max_{\Delta_C,\Delta_S} \; 2\left(p^I\right)^2 P^I \Pr(\omega=G|x\in X_P)y\Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I\Delta_S\right)^2 \\ s.t. \\ \Delta_C \geq l_P := \underline{\pi} - [\Pr(\omega=G|x\in X_P)y - P^I\Delta_S]\left(2p^I P^I\Delta_S\right) & (IC_P^I\text{-}1) \\ \Delta_C \leq u_P := \underline{\pi} - [\Pr(\omega=G|x\notin X_P)y - P_C^I\Delta_S]\left(2p^I P^I\Delta_S\right) & (IC_P^I\text{-}2) \\ \Delta_C \geq l_A := \left[\left(P_\emptyset^I\right)^2 - \left(p^I P^I\right)^2\right]\frac{\Delta_S^2}{1-p^I} & (IC_A^I\text{-}2) \end{cases}$$

As Δ_C does not appear in the objective function, we can rewrite the program as:

$$\begin{cases} \max_{\Delta_S} \; 2\left(p^I\right)^2 P^I \Pr(\omega=G \mid x\in X_P)y\Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I\Delta_S\right)^2 \\ s.t. \\ u_P \geq l_P \Leftrightarrow \underline{\pi} \geq \left[2p^I \Pr(\omega=G \mid x\notin X_P)P^I y\right]\Delta_S - \left[2p^I P^I P_C^I - \frac{\left(P_\emptyset^I\right)^2 - \left(p^I P^I\right)^2}{1-p^I}\right]\Delta_S^2 \\ u_P \geq l_A \Leftrightarrow \Delta_S \leq \frac{y}{1-\mu} \end{cases}.$$

By routine calculation one obtains $\Pr(\omega=G|x\in X_P)=1/\left(2-\alpha'\right)$, $\Pr(\omega=G|x\notin X_P)=0$, and

$$p^I = 1-\frac{1}{2}\alpha',\; P^I = \frac{1}{2-\alpha'} + \mu\left(1-\frac{1}{2-\alpha'}\right),\; p_C^I=\mu,$$

where $\alpha' := 1-(1-\alpha)^2$. Also, to streamline notation, without loss of generality, we set $y=1$ (thus, by Assumption 1, $\underline{\pi}=\frac{1}{4}$). So, the program $\mathcal{P}^I_{\{g,\emptyset\}}$ boils down to:

$$\begin{cases} \max_{\Delta_S} \; -\left[\frac{1}{2}(1+\mu(1-\alpha'))\Delta_S - \frac{1}{2}\right]^2 + \frac{1}{4} + \frac{1}{8}\alpha' \\ s.t. \\ \frac{1}{4} \geq \frac{1}{2}\left(\alpha'\Delta_S\right)^2 \; and \; \frac{1}{1-\mu} \geq \Delta_S \end{cases}.$$

Notice that the objective function is strictly concave with peak at $\frac{1}{1+\mu(1-\alpha')}$ and the feasible set is always non-empty. Thus the solution always exists and is given as:

$$\Delta_S^* = \begin{cases} \frac{1}{1+\mu(1-\alpha')} & if \; \frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \frac{1}{2} \\ \frac{1}{\mu\sqrt{2\alpha'}} & otherwise \end{cases}.$$

The associated value is:

$$V^I_{\{g,\emptyset\}} = \begin{cases} \frac{1}{4} + \frac{1}{8}\alpha' & \textit{if } \frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \frac{1}{2} \\ \frac{1}{4} + \frac{1}{8}\alpha' - \frac{1}{2}\left[\frac{1}{\mu\sqrt{2\alpha'}}(1+\mu(1-\alpha')) - 1\right]^2 & \textit{otherwise} \end{cases} . \tag{4}$$

Case 2: $X_P = \{G\}$. Here $p^I_\emptyset = 0$, and the program becomes:

$$\mathcal{P}^I_{\{g\}} : \begin{cases} \max_{\Delta_C,\Delta_S} \quad 2\left(p^I\right)^2 P^I \Pr(\omega = G \mid x \in X_P) y\Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I \Delta_S\right)^2 \\ s.t. \\ \Delta_C \geq l_P := \underline{\pi} - \left[\Pr(\omega = G \mid x \in X_P)y - P^I\Delta_S\right]\left(2p^I P^I \Delta_S\right) & (IC^I_P\text{-}1) \\ \Delta_C \leq u_P := \underline{\pi} - \left[\Pr(\omega = G \mid x \notin X_P)y - P^I_C\Delta_S\right]\left(2p^I P^I \Delta_S\right) & (IC^I_P\text{-}2) \\ \Delta_C \leq u_A := p^I\left(P^I\right)^2\Delta_S^2 & (IC^I_A\text{-}2) \end{cases} .$$

As in Case 1, Δ_C does not enter into the objective function, and we can further simplify the program as:

$$\begin{cases} \max_{\Delta_S} \quad 2\left(p^I\right)^2 P^I \Pr(\omega = G \mid x \in X_P) y\Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I \Delta_S\right)^2 \\ s.t. \\ l_P \leq u_A \Leftrightarrow \underline{\pi} \leq \left[2p^I \Pr(\omega = G \mid x \in X_P)P^I y\right]\Delta_S - \left[p^I\left(P^I\right)^2\right]\Delta_S^2 \\ l_P \leq u_P \Leftrightarrow \Delta_S \leq \frac{y}{1-\mu} \end{cases} ,$$

and plugging the values for the probablities and setting $y = 1$ we obtain:

$$\begin{cases} \max_{\Delta_S} \quad \frac{1}{2}\alpha'^2\Delta_S(1-\Delta_S) + \frac{1}{4}(1-\frac{1}{2}\alpha') \\ s.t. \\ \alpha'\Delta_S\left(1-\frac{1}{2}\Delta_S\right) \leq \frac{1}{4} \textit{ and } \Delta_S \leq \frac{1}{1-\mu} \end{cases} .$$

The feasible set is non-empty if and only if $\alpha' \geq 1/2$ (equivalently, $\alpha \geq 1 - 1/\sqrt{2}$), and the objective function is concave with peak at 1. Thus, the solution of the program and the value would be:

$$\Delta^*_S = 1 \text{ and } V^I_{\{g\}} = \frac{1}{4} + \frac{1}{4}\alpha'\left(\alpha' - \frac{1}{2}\right) \text{ if } \alpha' \geq \frac{1}{2} \tag{5}$$

and no solution otherwise. □

Proof of Lemma 4. The proof is similar to that of Lemma 2. For brevity, we rewrite the objective function and all constraints by using the notations p^T, $p^T_\emptyset$, P^T and $P^T_\emptyset$ (as defined in Section 4.2), and the program $\mathcal{P}^T$ boils down to:

$$\max_{\substack{\Delta_{iC},\Delta_{iS}\\ w_{iF},e_i}} \Pi^T = p^T\left[\Pr(\omega = G \mid x^T \in P)y - P^T\sum_i \Delta_{iS}\right]\sum_k e_k + (1-p^T)\left[\underline{\pi} - \sum_i \Delta_{iC}\right] - \sum_i w_{iF}$$

$$s.t. \quad \forall i \in \{1,2\}$$

$$p^T P^T \Delta_{iS}\sum_k e_k + (1-p^T)\Delta_{iC} + w_{iF} - \tfrac{1}{2}e_i^2 \geq 0 \qquad (IR_i^T)$$

$$\left[\Pr(\omega = G \mid x^T \in X_P)y - P^T\sum_i \Delta_{iS}\right]\sum_k e_k \geq \underline{\pi} - \sum_i \Delta_{iC} \qquad (IC_P^T\text{-}1)$$

$$\left[\Pr(\omega = G \mid x^T \notin X_P)y - P_C^T\sum_i \Delta_{iS}\right]\sum_k e_k \leq \underline{\pi} - \sum_i \Delta_{iC} \qquad (IC_P^T\text{-}2)$$

$$e_i = p^T P^T \Delta_{iS} \qquad (IC_{A_i}^T\text{-}1)$$

$$\tfrac{1}{2}\left[\left(p^T P^T\right)^2 - \left(p_\emptyset^T P_\emptyset^T\right)^2\right]\Delta_{iS}^2 + \left[\left(p^T P^T\right)^2 - \left(p_\emptyset^T P_\emptyset^T\right)\left(p^T P^T\right)\right]\Delta_{iS}\Delta_{jS} \geq (p^T - p_\emptyset^T)\Delta_{iC} \qquad (IC_{A_i}^T\text{-}2)$$

We can eliminate w_{iF} and e_is using (IR_i^T) (that must bind) and $(IC_{A_i}^T\text{-}1)$, and the program further simplifies to:

$$\max_{\Delta_{iC},\Delta_{iS}} \left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P)y\sum_i \Delta_{iS} + (1-p^T)\underline{\pi} - \tfrac{1}{2}\left(p^T P^T\right)^2 \sum_i \Delta_{iS}^2$$

$$s.t.$$

$$\left[\Pr(\omega = G \mid x^T \in X_P)y - P^T\sum_i \Delta_{iS}\right] p^T P^T \sum_i \Delta_{iS} \geq \underline{\pi} - \sum_i \Delta_{iC} \qquad (IC_P^T\text{-}1)$$

$$\left[\Pr(\omega = G \mid x^T \notin X_P)y - P_C^T\sum_i \Delta_{iS}\right] p^T P^T \sum_i \Delta_{iS} \leq \underline{\pi} - \sum_i \Delta_{iC} \qquad (IC_P^T\text{-}2)$$

$$\tfrac{1}{2}\left[\left(p^T P^T\right)^2 - \left(p_\emptyset^T P_\emptyset^T\right)^2\right](\Delta_{1S})^2 + \left[\left(p^T P^T\right)^2 - p_\emptyset^T P_\emptyset^T p^T P^T\right]\Delta_{1S}\Delta_{2S} \geq (p^T - p_\emptyset^T)\Delta_{1C} \qquad (IC_{A_1}^T\text{-}2)$$

$$\tfrac{1}{2}\left[\left(p^T P^T\right)^2 - \left(p_\emptyset^T P_\emptyset^T\right)^2\right](\Delta_{2S})^2 + \left[\left(p^T P^T\right)^2 - p_\emptyset^T P_\emptyset^T p^T P^T\right]\Delta_{1S}\Delta_{2S} \geq (p^T - p_\emptyset^T)\Delta_{2C} \qquad (IC_{A_2}^T\text{-}2)$$

Part (i). We now prove that if $\mathcal{P}^T$ admits a solution, it also admits a symmetric solution where $\Delta_{1S} = \Delta_{2S} = \Delta_S$ and $\Delta_{1C} = \Delta_{2C} = \Delta_C$. The proof is given in the following five steps.

Step 1: Suppose $\Delta^* := (\Delta^*_{1S}, \Delta^*_{2S}, \Delta^*_{1C}, \Delta^*_{2C})$ is a solution to $\mathcal{P}_T$. If $\Delta^*_{1S} = \Delta^*_{2S} = \Delta^*_G$ (say), we argue that there also exists a symmetric solution $(\Delta^*_S, \Delta^*_S, \Delta^*_C, \Delta^*_C)$ where

$$\Delta^*_C = \frac{1}{2}\sum_i \Delta^*_{iC}.$$

To see this, notice that under Δ^*, $(IC^T_{A_i}\text{-}2)$s imply:

$$\begin{aligned}\left[\tfrac{3}{2}\left(p^T P^T\right)^2 - \tfrac{1}{2}\left(p^T_\emptyset P^T_\emptyset\right)^2 - p^T_\emptyset P^T_\emptyset p^T P^T\right](\Delta^*_S)^2 &\geq \max\{(p^T - P^T_\emptyset)\Delta^*_{1C}, (p^T - P^T_\emptyset)\Delta^*_{2C}\} \\ &\geq \tfrac{1}{2}(p^T - P^T_\emptyset)(\Delta^*_{1C} + \Delta^*_{2C}) \\ &= (p^T - P^T_\emptyset)\Delta^*_C.\end{aligned}$$

Thus, $(\Delta^*_S, \Delta^*_S, \Delta^*_C, \Delta^*_C)$ is also a solution as it satisfies $(IC^T_{A_i}\text{-}2)$ and does not affect $(IC^T_P\text{-}1)$ and $(IC^T_P\text{-}2)$.

Step 2: Denote

$$\begin{aligned}\Pi^T(\Delta_{1S}, \Delta_{2S}) \;&:= \left(p^T\right)^2 P^T \Pr(\omega = G | x^T \in X_P) y \sum_i \Delta_{iS} + (1 - p^T)\underline{\pi} \\ &\quad - \tfrac{1}{2}\left(p^T P^T\right)^2 \sum_i \Delta^2_{iS}.\end{aligned}$$

Suppose $\Delta^* := (\Delta^*_{1S}, \Delta^*_{2S}, \Delta^*_{1C}, \Delta^*_{2C})$ is a solution to $\mathcal{P}_T$ but $\Delta^*_{1S} \neq \Delta^*_{2S}$. Without loss of generality, assume $\Delta^*_{1S} > \Delta^*_{2S}$. We argue that then Δ^* cannot be a solution. In particular, there exists $\varepsilon > 0$ and cancellation premiums Δ'_{iC}s such that $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta'_{1C}, \Delta'_{2C})$ is feasible and

$$\Pi^T(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon) > \Pi^T(\Delta^*_{1S}, \Delta^*_{2S}).$$

Observe that $\Pi_T(\Delta_{1S}, \Delta_{2S})$ is symmetric and concave in $(\Delta_{1S}, \Delta_{2S})$ with peak at

$$\Delta_{1S} = \Delta_{2S} = \frac{y}{P^T}\Pr\left(\omega = G | x^T \in X_P\right).$$

Also, the following holds: take any $(\Delta_{1S}, \Delta_{2S})$ such that $\Delta_{1S} \neq \Delta_{2S}$, $\Delta_{1S} > \Delta_{2S}$, say. Then, there exists $\varepsilon > 0$ such that

$$\Pi^T\left(\Delta_{1S} - \varepsilon, \Delta'_{2S} + \varepsilon\right) > \Pi^T\left(\Delta_{1G}, \Delta_{2G}\right).$$

So, we only need to show that there exists an $\varepsilon > 0$, and $\Delta'_{1C}, \Delta'_{2C}$ values such that $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta'_{1C}, \Delta'_{2C})$ is feasible. In order to prove this claim, it is worthwhile to first establish a few properties of the $(IC^T_{A_i}\text{-}2)$ constraints, as given in the next step.

Step 3: Denote

$$L_i(\Delta_{1S}, \Delta_{2S}) := A\left(\Delta_{iS}\right)^2 + B\Delta_{1S}\Delta_{2S},$$

where

$$A := \frac{\left(p^T P^T\right)^2 - \left(p_\emptyset^T P_\emptyset^T\right)^2}{2\left(p^T - p_\emptyset^T\right)} \text{ and } B := \frac{\left(p^T P^T - p_\emptyset^T P_\emptyset^T\right) p^T P^T}{p^T - p_\emptyset^T}.$$

Note that the ($IC^T_{A_i}$-2) constraints can be written as:

$$L_i\left(\Delta_{1S}, \Delta_{2S}\right) \geq \Delta_{iC} \text{ if } p^T - p_\emptyset^T > 0, \text{ and } L_i\left(\Delta_{1S}, \Delta_{2S}\right) \leq \Delta_{iC} \text{ otherwise.}$$

Also,

$$(B - A) = \frac{\left(p^T P^T - p_\emptyset^T P_\emptyset^T\right)^2}{2\left(p^T - p_\emptyset^T\right)},$$

and hence,

$$\text{sign}\,(B - A) = \text{sign}\left(p^T - p_\emptyset^T\right)$$

It is routine to check that for $X_P = \{g\}$, $p^T - p_\emptyset^T > 0$ and $p^T P^T - p_\emptyset^T P_\emptyset^T > 0$, whereas for $X_P = \{g, \emptyset\}$, $p^T - p_\emptyset^T < 0$ and $p^T P^T - p_\emptyset^T P_\emptyset^T < 0$. Thus,

$$A > 0, B > 0.$$

In the next two steps, we consider the two cases $p^T - p_\emptyset^T > 0$ and < 0, and show that the claim in Step 2 above holds in both cases.

Step 4: Suppose $p^T - p_\emptyset^T > 0$. So, ($IC^T_{A_i}$-2)s are given as:

$$L_i(\Delta_{1S}, \Delta_{2S}) \geq \Delta_{iC}.$$

There are three possibilities:

Case 1: Both ($IC^T_{A_i}$-2)s are slack at $(\Delta^*_{1S}, \Delta^*_{2S}, \Delta^*_{1C}, \Delta^*_{2C})$. Consider the solution

$$(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta^*_{1C}, \Delta^*_{2C})$$

where $\varepsilon > 0$. This solution leaves $\left(IC^T_P\right)$s unaffected, for sufficiently small ε, both ($IC^T_{A_i}$-2)s remain slack, and yields a higher value of Π^T(from Step 2).

Case 2: Exactly one of the two ($IC^T_{A_i}$-2)s is slack at $(\Delta^*_{1S}, \Delta^*_{2S}, \Delta^*_{1C}, \Delta^*_{2C})$. Suppose only ($IC^T_{A_1}$-2) is slack, say, (hence, (IC_{A_2}-2) is binding). Set

$$\Delta'_{1C} = \Delta^*_{1C} + \delta,\ \ \Delta'_{2C} = \Delta^*_{2C} - \delta$$

where $\delta > 0$. For δ sufficiently small, at $(\Delta^*_{1S}, \Delta^*_{2S}, \Delta'_{1C}, \Delta'_{2C})$, both ($IC^T_{A_i}$-2) become slack and $\left(IC^T_P\right)$s are unaffected, and hence, it is feasible. But then, as argued in Case 1, the solution $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta'_{1C}, \Delta'_{2C})$ is also feasible for $\varepsilon > 0$ sufficiently small, and attains a higher value of Π^T.

Case 3: Both $(IC^T_{A_i}\text{-}2)$*s are binding at* $(\Delta^*_{1S},\Delta^*_{2S},\Delta^*_{1C},\Delta^*_{2C})$. Consider changing $(\Delta^*_{1S},\Delta^*_{2S})$ to $(\Delta^*_{1S}-\varepsilon,\Delta^*_{2S}+\varepsilon)$. The left-hand side of $\left(IC^T_{A_i}\text{-}2\right)$ changes by

$$\delta_i := L_i\left(\Delta^*_{1S}-\varepsilon,\Delta^*_{2S}+\varepsilon\right) - L_i\left(\Delta^*_{1S},\Delta^*_{2S}\right)$$

where

$$\begin{aligned}\delta_1 &= -\varepsilon\left(2A\left(\Delta^*_{1S}-\frac{1}{2}\varepsilon\right) - B\left(\Delta^*_{1S}-\left(\Delta^*_{2S}+\varepsilon\right)\right)\right),\\ \delta_2 &= \varepsilon\left(2A\left(\Delta^*_{2S}+\frac{1}{2}\varepsilon\right) + B\left(\Delta^*_{1S}-\left(\Delta^*_{2S}+\varepsilon\right)\right)\right).\end{aligned}$$

Note that by $A>0$, $B>0$ and ε small enough, $\delta_2>0$.

So, if $\delta_1>0$, the perturbation relaxes both $\left(IC^T_{A_i}\text{-}2\right)$s and by argument given in Case 1, $(\Delta^*_{1S}-\varepsilon,\Delta^*_{2S}+\varepsilon,\Delta^*_{1C},\Delta^*_{2C})$ is an improvement.

If $\delta_1<0$, $\left(IC^T_{A_1}\text{-}2\right)$ is now violated, but $(IC_{A_2}\text{-}2)$ has become slack. Also note that by $B-A>0$,

$$\delta_2+\delta_1 = 2\varepsilon\left(B-A\right)\left(\Delta^*_{1S}-\left(\Delta^*_{2S}+\varepsilon\right)\right)>0.$$

Now, set

$$\Delta'_{1C}=\Delta^*_{1C}+\delta_1,\ \Delta'_{2C}=\Delta^*_{2C}-\delta_1.$$

Note that

$$L_1\left(\Delta^*_{1S}-\varepsilon,\Delta^*_{2S}+\varepsilon\right)=L_1\left(\Delta^*_{1S},\Delta^*_{2S}\right)+\delta_1=\Delta^*_{1C}+\delta_1=\Delta'_{1C},$$

and

$$\begin{aligned}L_2\left(\Delta^*_{1S}-\varepsilon,\Delta^*_{2S}+\varepsilon\right) &= L_2\left(\Delta^*_{1S},\Delta^*_{2S}\right)+\delta_2\\ &= L_2\left(\Delta^*_{1S},\Delta^*_{2S}\right)-\delta_1+\left(\delta_2+\delta_1\right)\\ &> L_2\left(\Delta^*_{1S},\Delta^*_{2S}\right)-\delta_1\\ &= \Delta^*_{2C}-\delta_1=\Delta'_{2C}.\end{aligned}$$

Hence, $(\Delta^*_{1S}-\varepsilon,\Delta^*_{2S}+\varepsilon,\Delta'_{1C},\Delta'_{2C})$ is feasible (note that $\left(IC^T_P\right)$s are unaltered by construction), and for $\varepsilon>0$ sufficiently small, attains a higher value of Π^T.

Step 5: Suppose $p^T-p^T_\emptyset<0$. Thus, $(IC^T_{A_i}\text{-}2)$s are

$$L_i\left(\Delta_{1S},\Delta_{2S}\right)\le\Delta_{iC}.$$

As before, there are three possibilities:

Case 1: Both $(IC^T_{A_i}\text{-}2)$*s are slack at* $(\Delta^*_{1S},\Delta^*_{2S},\Delta^*_{1C},\Delta^*_{2C})$. By argument in case 1 in Step 4, this solution can be improved up on.

Case 2: Exactly one of the two $(IC^T_{A_i}\text{-}2)$*s is slack at* $(\Delta^*_{1S}, \Delta^*_{2S}, \Delta^*_{1C}, \Delta^*_{2C})$. Suppose only $(IC^T_{A_1}\text{-}2)$ is slack, say, (hence, $(IC^T_{A_2}\text{-}2)$ is binding). Set

$$\Delta'_{1C} = \Delta^*_{1C} - \delta,\ \Delta'_{2C} = \Delta^*_{2C} + \delta$$

where $\delta > 0$. As in case 2 in Step 4, the solution $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta'_{1C}, \Delta'_{2C})$ is also feasible for $\varepsilon > 0$ sufficiently small, and attains a higher value of Π^T.

Case 3: Both $(IC^T_{A_i}\text{-}2)$*s are binding at* $(\Delta^*_{1S}, \Delta^*_{2S}, \Delta^*_{1C}, \Delta^*_{2C})$. Consider changing $(\Delta^*_{1S}, \Delta^*_{2S})$ to $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon)$. As in case 3 in Step 4, the left-hand side of $\left(IC^T_{A_i}\text{-}2\right)$ changes by

$$\delta_i := L_i\left(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon\right) - L_i\left(\Delta^*_{1S}, \Delta^*_{2S}\right)$$

where $\delta_2 > 0$ and

$$\delta_2 + \delta_1 = 2\varepsilon\left(B - A\right)\left(\Delta^*_{1S} - \left(\Delta^*_{2S} + \varepsilon\right)\right) < 0.$$

for ε small enough.

So, if $\delta_1 > 0$, the perturbation relaxes both $\left(IC^T_{A_i}\text{-}2\right)$s and by argument given in Case 1, $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta^*_{1C}, \Delta^*_{2C})$ is an improvement.

If $\delta_1 < 0$, $\left(IC^T_{A_2}\text{-}2\right)$ is now violated, but $\left(IC^T_{A_1}\text{-}2\right)$ has become slack. Now, set

$$\Delta'_{1C} = \Delta^*_{1C} - \delta_2,\ \Delta'_{2C} = \Delta^*_{2C} + \delta_2.$$

Note that

$$\begin{aligned} L_1\left(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon\right) &= L_1\left(\Delta^*_{1S}, \Delta^*_{2S}\right) + \delta_1 \\ &= L_1\left(\Delta^*_{1S}, \Delta^*_{2S}\right) - \delta_2 + \left(\delta_2 + \delta_1\right) \\ &< L_1\left(\Delta^*_{1S}, \Delta^*_{2S}\right) - \delta_2 \\ &= \Delta^*_{1C} - \delta_2 = \Delta'_{1C}. \end{aligned}$$

and

$$L_2\left(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon\right) = L_2\left(\Delta^*_{1S}, \Delta^*_{2S}\right) + \delta_2 = \Delta^*_{2C} + \delta_2 = \Delta'_{2C},$$

Hence, $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta'_{1C}, \Delta'_{2C})$ is feasible (note that $\left(IC^T_P\right)$s are unaltered by construction), and for $\varepsilon > 0$ sufficiently small, attains a higher value of Π^T.

Combining all cases stated above, we obtain that without loss of generality, we can focus on the solution where $\Delta_{1S} = \Delta_{2S} = \Delta_S$, $\Delta_{1C} = \Delta_{2C} = \Delta_C$. And from (IR^T_i), we obtain that under such a solution, we must have $w_{1F} = w_{2F} = w_F$. This observation completes the proof of part (i) of this lemma.

Part (ii). Since we focus on $\Delta_{1S} = \Delta_{2S} = \Delta_S$ and $\Delta_{1C} = \Delta_{2C} = \Delta_C$, the program can be simplified as:

$$\mathcal{P}^T \begin{cases} \max\limits_{\Delta_C, \Delta_S} & 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1 - p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2 \\ s.t. & \left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p_\emptyset^T P_\emptyset^T\right)^2 - p_\emptyset^T P_\emptyset^T p^T P^T\right](\Delta_S)^2 \geq (p^T - p_\emptyset^T)\Delta_C \quad (IC_A^T\text{-}2) \\ & 2\left[\Pr(\omega = G \mid x^T \in X_P) y - 2P^T \Delta_S\right] p^T P^T \Delta_S \geq \underline{\pi} - 2\Delta_C \quad (IC_P^T\text{-}1) \\ & 2\left[\Pr(\omega = G \mid x^T \notin X_P) y - 2P_C^T \Delta_S\right] p^T P^T \Delta_S \leq \underline{\pi} - 2\Delta_C \quad (IC_P^T\text{-}2) \end{cases}$$

As in the case of individual assignment, we have two cases: $X_P = \{g, \emptyset\}$ and $X_P = \{g\}$.

Case 1: $X_P = \{G, \emptyset\}$. Here, $p^T - p_\emptyset^T < 0$; so we have:

$$\mathcal{P}^T_{\{g,\emptyset\}} \begin{cases} \max\limits_{\Delta_C, \Delta_S} & 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1 - p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2 \\ s.t. & \Delta_C \geq l_A := \left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p_\emptyset^T P_\emptyset^T\right)^2 - p_\emptyset^T P_\emptyset^T p^T P^T\right]\frac{\Delta_S^2}{p^T - p_\emptyset^T} \quad (IC_A^T\text{-}2) \\ & \Delta_C \geq l_P := \frac{1}{2}\underline{\pi} - \left[\Pr(\omega = G \mid x^T \in X_P) y - 2P^T \Delta_S\right] p^T P^T \Delta_S \quad (IC_P^T\text{-}1) \\ & \Delta_C \leq u_P := \frac{1}{2}\underline{\pi} - \left[\Pr(\omega = G \mid x^T \notin X_P) y - 2P_C^T \Delta_S\right] p^T P^T \Delta_S \quad (IC_P^T\text{-}2) \end{cases} .$$

Notice that Δ_C is not in the objective function, we can further simplify the program as:

$$\begin{cases} \max\limits_{\Delta_S} \; 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1 - p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2 \\ s.t. \\ u_P \geq l_A \Leftrightarrow \frac{1}{2}\underline{\pi} \geq \left[\Pr(\omega = G \mid x^T \notin X_P) y - 2P_C^T \Delta_S\right] p^T P^T \Delta_S \\ \quad + \frac{\Delta_S^2}{p^T - p_\emptyset^T}\left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p_\emptyset^T P_\emptyset^T\right)^2 - p_\emptyset^T P_\emptyset^T p^T P^T\right] \\ u_P \geq l_P \Leftrightarrow \Delta_S \leq \frac{y}{2(1-\mu)}. \end{cases}$$

By routine calculation, one obtains $\Pr(\omega = G | x^T \in X_P) = \frac{1}{2-\alpha'}$, $\Pr(\omega = G | x^T \notin X_P) = 0$, and

$$p^T = 1 - \tfrac{1}{2}\alpha'; \quad P^T = \mu + (1-\mu)\tfrac{1}{2-\alpha'}; \quad p_C^T = \mu;$$
$$p_\emptyset^T = 1 - \tfrac{1}{2}\alpha; \quad P_\emptyset^T = \mu + (1-\mu)\tfrac{1}{2-\alpha},$$

where $\alpha' := 1 - (1-\alpha)^2$. Also, as in the proof of Lemma 2, to streamline notation, without loss of generality, we set $y = 1$ (and hence, $\underline{\pi} = 1/4$). Plugging the values, the program becomes:

$$\begin{cases} \max\limits_{\Delta_S} & -\frac{1}{4}\left(1 + \mu\left(1 - \alpha'\right)\right)^2 \left(\Delta_S - \frac{1}{1+\mu(1-\alpha')}\right)^2 + \frac{1}{4}(1 + \frac{1}{2}\alpha') \\ s.t. & \Delta_S^2 \leq \frac{1}{2\mu^2\alpha(1-\alpha)} \text{ and } \Delta_S \leq \frac{1}{2(1-\mu)} \end{cases}$$

The solution is given as:

$$\Delta_S^* = \begin{cases} \frac{1}{1+\mu(1-\alpha)^2} & if\ \frac{1}{1+\mu(1-\alpha)^2} \leq \frac{1}{2(1-\mu)} \\ \frac{1}{2(1-\mu)} & otherwise \end{cases},$$

and the associated value is:

$$V_{\{g,\emptyset\}}^T = \begin{cases} \frac{1}{4} + \frac{1}{8}\alpha(2-\alpha) & if\ \frac{1}{1+\mu(1-\alpha)^2} \leq \frac{1}{2(1-\mu)} \\ \frac{1+\mu(1-\alpha)^2}{4(1-\mu)}\left[1 - \frac{1+\mu(1-\alpha)^2}{4(1-\mu)}\right] + \frac{1}{8}\alpha(2-\alpha) & otherwise \end{cases} \tag{6}$$

Case 2: $X_P = \{G\}$. Here, $p^T - p_\emptyset^T > 0$, so we have:

$$\mathcal{P}_{\{g\}}^T \begin{cases} \max_{\Delta_C,\Delta_S} & 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1-p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2 \\ s.t. & \Delta_C \leq u_A := \left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p_\emptyset^T P_\emptyset^T\right)^2 - p_\emptyset^T P_\emptyset^T p^T P^T\right] \frac{\Delta_S^2}{p^T - p_\emptyset^T} \quad (IC_A^T\text{-}2) \\ & \Delta_C \geq l_P := \frac{1}{2}\underline{\pi} - \left[\Pr(\omega = G \mid x^T \in X_P) y - 2P^T \Delta_S\right] p^T P^T \Delta_S \quad (IC_P^T\text{-}1) \\ & \Delta_C \leq u_P := \frac{1}{2}\underline{\pi} - \left[\Pr(\omega = G \mid x^T \notin X_P) y - 2P_C^T \Delta_S\right] p^T P^T \Delta_S \quad (IC_P^T\text{-}2) \end{cases}.$$

As Δ_C does not appear in the objective function, we can replace the constraints by requiring $l_P \leq u_A$ and $l_P \leq u_P$, and the program simplifies to:

$$\begin{cases} \max_{\Delta_S} \ 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1-p^T)\underline{\pi} - \left(p^T P^T\right)^2 (\Delta_S)^2 \\ s.t. \\ \frac{1}{2}\underline{\pi} \leq \left[\Pr(\omega = G \mid x^T \in X_P) y - 2P^T \Delta_S\right] p^T P^T \Delta_S \\ + \left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p_\emptyset^T P_\emptyset^T\right)^2 - p_\emptyset^T P_\emptyset^T p^T P^T\right] \frac{\Delta_S^2}{p^T - p_\emptyset^T} \\ \Delta_S \leq \frac{y}{2(1-\mu)} \end{cases}.$$

Plugging the values for the probabilities (and parameters), we obtain:

$$\begin{cases} \max_{\Delta_S} \ \Pi_{\{g\}}^T(\Delta_S) := -\frac{1}{4}\left(\alpha'(\Delta_S - 1)\right)^2 + \frac{1}{4}\left(1 - \alpha'\left(\frac{1}{2} - \alpha'\right)\right) \\ s.t. \\ \alpha(2-\alpha)\Delta_S - \frac{1}{2}\alpha(1-\alpha)\Delta_S^2 \geq \frac{1}{4} \\ \Delta_S \leq \frac{1}{2(1-\mu)} \end{cases}$$

Let $\hat{\alpha} := 0.12445$ and $K(\alpha) := \frac{1}{1-\alpha}\left(2 - \alpha - \sqrt{(2-\alpha)^2 - \frac{1-\alpha}{2\alpha}}\right)$. It is routine to check that the program does not admit a solution if $\alpha < \hat{\alpha}$ or $K(\alpha) > 1/2\,(1-\mu)$. Otherwise, the solution is as follows:

$$\Delta_S^* = \begin{cases} 1 & if\ \alpha \geq \hat{\alpha}\ and\ K(\alpha) \leq 1 \leq \frac{1}{2(1-\mu)} \\ \frac{1}{2(1-\mu)} & if\ \alpha \geq \hat{\alpha}\ and\ K(\alpha) \leq \frac{1}{2(1-\mu)} < 1 \\ \tilde{\alpha} & if\ \alpha \geq \hat{\alpha}\ and\ 1 < K(\alpha) \leq \frac{1}{2(1-\mu)} \end{cases},$$

and the associated value function is

$$V^T_{\{g\}} = \begin{cases} \Pi^T_{\{g\}}(1) & if\ \alpha \geq \hat{\alpha}\ and\ K(\alpha) \leq 1 \leq \frac{1}{2(1-\mu)} \\ \Pi^T_{\{g\}}\left(\frac{1}{2(1-\mu)}\right) & if\ \alpha \geq \hat{\alpha}\ and\ K(\alpha) \leq \frac{1}{2(1-\mu)} < 1 \\ \Pi^T_{\{g\}}(\tilde{\alpha}) & if\ \alpha \geq \hat{\alpha}\ and\ 1 < K(\alpha) \leq \frac{1}{2(1-\mu)} \end{cases} \tag{7}$$

Thus, we conclude that the program $\mathcal{P}^T$ always admits a solution for $X_P = \{g, \emptyset\}$ and admits a solution for $X_P = \{g\}$ if and only if α and μ are sufficiently large. □

Proof of Proposition 2. **Step 1.** Notice that program $\mathcal{P}^I_{\{g,\emptyset\}}$ and $\mathcal{P}^T_{\{g,\emptyset\}}$ have the objective function. Denote the the unconstrained maximum of that objective function as

$$V_{\{g,\emptyset\}} = \frac{1}{4} + \frac{1}{8}\alpha'.$$

Similarly, $\mathcal{P}^I_{\{g\}}$ and $\mathcal{P}^T_{\{g\}}$ have the same objective function, and we denote the unconstrained maximum as

$$V_{\{g\}} = \frac{1}{4}\left[1 - \alpha'\left(\frac{1}{2} - \alpha'\right)\right].$$

Since unconstrained maximum must be (weakly) larger than the value under a constrained maximization, we have

$$V^I_{\{g,\emptyset\}} \leq V_{\{g,\emptyset\}}, V^T_{\{g,\emptyset\}} \leq V_{\{g,\emptyset\}}, V^I_{\{g\}} \leq V_{\{g\}} \text{ and } V^T_{\{g\}} \leq V_{\{g\}}.$$

Further, we notice that $V_{\{g,\emptyset\}} - V_{\{g\}} = \frac{1}{4}\alpha'(1-\alpha') \geq 0$, so we have

$$V_{\{g\}} \leq V_{\{g,\emptyset\}}$$

and equality holds if and only if $\alpha' = 0$ or 1.

Step 2. Recall that the solutions for the programs $\mathcal{P}^I_{\{g,\emptyset\}}$ and $\mathcal{P}^T_{\{g,\emptyset\}}$ (see (4) and (6); we maintain $y = 1$ to streamline notation) stipulate

$$V^I_{\{g,\emptyset\}} = \frac{1}{4}\left(1 + \frac{1}{2}\alpha'\right) = S^*\left(= \frac{1}{4}\left(1 + \alpha - \frac{1}{2}\alpha^2\right)\right)$$

when $\frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \frac{1}{2}$, and

$$V^T_{\{g,\emptyset\}} = S^*$$

when $\frac{1}{1+\mu(1-\alpha)^2} \leq \frac{1}{2(1-\mu)}$.

Let μ_0 be the solution to the equation

$$\frac{1}{1+\mu(1-\alpha)^2} = \frac{1}{2(1-\mu)};$$

that is,

$$\mu_0 = \frac{1}{2+(1-\alpha)^2} = \frac{1}{3-\alpha'}. \tag{8}$$

Note that, for $\mu \in [0, \mu_0)$, $\frac{1}{1+\mu(1-\alpha)^2} > \frac{1}{2(1-\mu)}$; and for $\mu \in [\mu_0, 1)$, $\frac{1}{1+\mu(1-\alpha)^2} \leq \frac{1}{2(1-\mu)}$.

Next, define μ_1 as follows:

$$\mu_1 = \begin{cases} 1 & \textit{if } \frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} < \frac{1}{2}\ \forall \mu \in [0,1] \\ \mu^*(\alpha') & \textit{otherwise} \end{cases}, \tag{9}$$

where

$$\mu^*(\alpha') = \frac{1-\alpha'+\sqrt{2\alpha'}}{2\alpha' - (1-\alpha')^2}$$

is the unique solution to

$$\frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} = \frac{1}{2}$$

in $[0, 1]$.

Note that $\frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \frac{1}{2}$ for $\mu \in [0, \mu_1]$ and $\frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} > \frac{1}{2}$ for $\mu \in (\mu_1, 1]$.

Step 3. Notice that $\mu_0 < \mu_1\ \forall \alpha \in [0, 1]$ as using (8), one obtains

$$\frac{\alpha'\mu_0^2}{(1+\mu_0(1-\alpha'))^2} = \frac{\alpha'}{4(2-\alpha')^2} < \frac{1}{2}.$$

Combining above observations we obtain: (i) if $\mu < \mu_0$, $S^* = V^I_{\{g,\emptyset\}} > \max\{V^T_{\{g,\emptyset\}}, V^I_{\{g\}}, V^T_{\{g\}}\}$; that is, individual assignment with $X_P = \{G, \emptyset\}$ is optimal; (ii) if $\mu > \mu_1$, $S^* = V^T_{\{g,\emptyset\}} > \max\{V^I_{\{g,\emptyset\}}, V^I_{\{g\}}, V^T_{\{g\}}\}$; that is, team assignment with $X_P = \{G, \emptyset\}$ is optimal; (iii) if $\mu_0 \leq \mu \leq \mu_1$, $S^* = V^I_{\{g,\emptyset\}} = V^T_{\{g,\emptyset\}} > \max\{V^I_{\{g\}}, V^T_{\{g\}}\}$; that is, both team and individual assignment with $X_P = \{G, \emptyset\}$ are optimal. □

Proof of Proposition 3. **Step 1.** From (8) it directly follows that μ_0 is increasing in α.

Step 2. Now, consider the definition for μ_1 as given in (9). Note that when $\alpha' < \frac{1}{2}$, $\frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \alpha'\mu^2 \leq \alpha' < \frac{1}{2}$; so, $\mu_1 = 1$. And for $\alpha' \geq \frac{1}{2}$, we have

$$\mu_1 = \min\{1, \mu^*(\alpha')\}.$$

Note that

$$\frac{d}{d\alpha'}\mu^*(\alpha') = -\frac{\left(1-\frac{1}{\sqrt{2\alpha'}}\right)\left(2\alpha'-(1-\alpha')^2\right) + 2(2-\alpha')\left(1-\alpha'+\sqrt{2\alpha'}\right)}{\left(2\alpha'-(1-\alpha')^2\right)^2}.$$

For $\alpha' \in [\frac{1}{2}, 1)$ it is routine to check that $1 - \frac{1}{\sqrt{2\alpha'}} \geq 0$ and all other three terms in the numerator are strictly positive (denominator is positive by virtue of being a sqaure term).

So, $\frac{d}{d\alpha'}\mu^*(\alpha') < 0$. Hence, $\mu^*(\alpha')$ is also strictly decreasing in α when $\alpha \in \left[1 - 1/\sqrt{2}, 1\right]$ (recall $\alpha' := 1 - (1-\alpha)^2$).

Step 3. Finally, note that when $\alpha = 1 - \frac{1}{\sqrt{2}}$, $\mu^*(\alpha') = 2$; and when $\alpha = 1$, $\mu^*(\alpha') = \frac{1}{\sqrt{2}}$. As $\mu^*(\alpha')$ is decreasing in α, by Intermediate Value Theorem, there exists an α^* such that $\mu^*(\alpha^*) = 1$. Also, when $\alpha < \alpha^*$, $\mu^*(\alpha') > 1$; when $\alpha > \alpha^*$, $\mu^*(\alpha') < 1$.

Thus, for $1 - \frac{1}{\sqrt{2}} \leq \alpha \leq \alpha^*$, $\mu_1 = \min\{1, \mu^*(\alpha')\} = 1$ and for $\alpha \geq \alpha^*$, μ_1 is decreasing in α. □

Proof of Proposition 4. **Step 1:** Since Lemma 1 and 3 hold for any $\theta \in (\frac{1}{2}, 1)$ (note that the proofs of these lemmas presented above do not rely on any specific value of θ), we may continue to limit attention to the set of four programs $\mathcal{P}^I_{\{g,\emptyset\}}$, $\mathcal{P}^I_{\{g\}}$, $\mathcal{P}^T_{\{g,\emptyset\}}$, and $\mathcal{P}^T_{\{g\}}$ as defined in the proofs of Lemma 2 and 4. In this step, we compute the unconstrained maximum of these four programs. That is, for $\mathcal{P}^I_{X_P}$, $X_P \in \{\{g,\emptyset\},\{g\}\}$, we solve for

$$\overline{V}^d_{X_P} := \max_{\Delta_S} \;\; 2\left(p^I\right)^2 P^I \Pr(\omega = G \mid x \in X_P) y \Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I\right)^2 \Delta_S^2,$$

and for $\mathcal{P}^T_{X_P}$, $X_P \in \{\{g,\emptyset\},\{g\}\}$, we solve for

$$\overline{V}^T_{X_P} := \max_{\Delta_S} \;\; 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1-p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2.$$

Plugging in the values for all the probabilities, and solving for the optimization problem (notice that all objective functions are quadratic in Δ_S; hence solution exists and is unique) we obtain (recall that $\alpha' = 1 - (1-\alpha)^2$):

$$\overline{V}^I_{\{g,\emptyset\}} = \overline{V}^T_{\{g,\emptyset\}} = \frac{1}{4}\left[(1 - \alpha'(1-\theta))^2 + \frac{1}{2}\alpha'\right] =: \overline{V},$$

and

$$\overline{V}^I_{\{g\}} = \overline{V}^T_{\{g\}} = \frac{1}{4}\left[(\alpha'\theta)^2 + (1 - \frac{1}{2}\alpha')\right].$$

Note that

$$\overline{V} > \overline{V}^I_{\{g\}} = \overline{V}^T_{\{g\}}.$$

In what follows, we focus our attention on programs $\mathcal{P}^I_{\{g,\emptyset\}}$ and $\mathcal{P}^T_{\{g,\emptyset\}}$, as we show that for any given set of parameters, at least one of them achieves the value $\overline{V}$.

Step 2: We show that for θ sufficiently large, there exists a cutoff $\mu_0(\alpha;\theta)$ such that $V^T_{\{g,\emptyset\}} < \overline{V}$ if $\mu < \mu_0(\alpha;\theta)$; and $V^T_{\{g,\emptyset\}} = \overline{V}$ otherwise. Plugging the values of the probabilities, the program $\mathcal{P}^T_{\{g,\emptyset\}}$ can be written as:

$$\mathcal{P}^T_{\{g,\emptyset\}}: \begin{cases} \max_{\Delta_S} \;\; \overline{V} - \frac{1}{4}\left[(1-\alpha'(1-\theta)+\mu(1-\alpha'\theta))\,\Delta_S - (1-\alpha'(1-\theta))\right]^2 \\ s.t. \\ [1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)](1-\theta)\Delta_S + \frac{1}{2}\alpha(1-\alpha)(1-\theta\,(1-\mu))^2\Delta_S^2 \leq \frac{1}{4} \quad \left(C_1^T\right) \\ \Delta_S \leq \frac{1}{2(1-\mu)} \quad \left(C_2^T\right) \end{cases}.$$

The objective function achieves its peak at $\Delta_S^* = \frac{1-\alpha'(1-\theta)}{1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)}$. If Δ_S^* is feasible under constraints $\left(C_1^T\right)$ and $\left(C_2^T\right)$, $V^T_{\{g,\emptyset\}} = \overline{V}$; and $V^T_{\{g,\emptyset\}} < \overline{V}$ otherwise. Next, we analyze conditions under which this solution may be feasible.

Plugging Δ_S^* into $\left(C_2^T\right)$ and simplifying, we have:

$$\mu \geq \frac{1-\alpha'(1-\theta)}{3-\alpha'\theta-2\alpha'(1-\theta)} =: \mu_0(\alpha;\theta).$$

Thus, Δ_S^* satisfies constraint $\left(C_2^T\right)$ if and only if $\mu \geq \mu_0(\alpha;\theta)$.

We also claim that Δ_S^* satisfies $\left(C_1^T\right)$ if $\theta > 0.85$. To see this, plug Δ_S^* into the left-hand side of $\left(C_1^T\right)$, and we obtain:

$$\begin{aligned} &(1-\theta)(1-\alpha'(1-\theta)) + \tfrac{1}{2}\alpha(1-\alpha)(1-\alpha'(1-\theta))^2\left[\tfrac{1-\theta(1-\mu)}{1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)}\right]^2 \\ &\leq (1-\theta)(1-\alpha'(1-\theta)) + \tfrac{1}{2}\alpha(1-\alpha)(1-\alpha'(1-\theta))^2\left(\tfrac{1}{2-\alpha'}\right)^2 \\ &\leq (1-\theta) + \tfrac{\alpha(1-\alpha)}{2(2-\alpha')^2} \\ &\leq \tfrac{1}{4}. \end{aligned}$$

The first inequality follows as the expression is increasing in μ, the second one follows as $1-\alpha'(1-\theta) \in [0,1]$, and the final one holds since $\frac{\alpha(1-\alpha)}{2(2-\alpha')^2} < 0.1$ (for $\alpha \in [0,1]$).

Hence, for $\theta > 0.85$, $V^T_{\{g,\emptyset\}} < \overline{V}$ when $\mu < \mu_0(\alpha;\theta)$, and $V^T_{\{g,\emptyset\}} = \overline{V}$ otherwise.

Step 3: We show that for θ sufficiently large, there exists a cutoff $\mu_1(\alpha;\theta)$ such that $V^T_{\{g,\emptyset\}} = \overline{V}$ when $\mu \leq \mu_1(\alpha;\theta)$ and $V^T_{\{g,\emptyset\}} < \overline{V}$ otherwise. The proof is analogous to the one given in Step 2 above.

Plugging the values of the probabilities, the program $\mathcal{P}^I_{\{g,\emptyset\}}$ can be written as:

$$\mathcal{P}^I_{\{g,\emptyset\}}: \begin{cases} \max_{\Delta_S} \;\; \overline{V} - \frac{1}{4}\left[(1-\alpha'(1-\theta)+\mu(1-\alpha'\theta))\,\Delta_S - (1-\alpha'(1-\theta))\right]^2 \\ s.t. \\ (1-\theta)[1+\mu-\alpha'(1-\theta+\mu\theta)]\Delta_S + \frac{1}{2}\alpha'\left(1-\theta\,(1-\mu)\right)^2\Delta_S^2 \leq \frac{1}{4} \quad \left(C_1^I\right) \\ \Delta_S \leq \frac{1}{1-\mu} \quad \left(C_2^I\right) \end{cases}.$$

The objective function achieves its peak at $\Delta_S^* = \frac{1-\alpha'(1-\theta)}{1-\alpha'(1-\theta)+\mu(1-\alpha'\mu)}$. If Δ_S^* is feasible under constraints $\left(C_1^I\right)$ and $\left(C_2^I\right)$, $V_{\{g,\emptyset\}}^I = \overline{V}$; and $V_{\{g,\emptyset\}}^I < \overline{V}$ otherwise. Next, we analyze conditions under which this solution may be feasible.

It is routinely to check that Δ_S^* is always feasible under $\left(C_2^I\right)$:

$$\Delta_S^* = \frac{1-\alpha'(1-\theta)}{1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)} \leq 1 \leq \frac{1}{1-\mu}.$$

Now, plugging Δ_S^* in the left-hand side of $\left(C_1^I\right)$ we get:

$$L(\mu;\alpha,\theta) := (1-\theta)(1-\alpha'(1-\theta)) + \frac{1}{2}\alpha'\left(1-\alpha'(1-\theta)\right)^2\left(\frac{1-\theta+\mu\theta}{1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)}\right)^2.$$

Note that $L(\mu;\alpha,\theta)$ is increasing in $\mu \in [0,1]$, so it achieves its maximum at $\mu = 1$, where:

$$L(1;\alpha,\theta) = (1-\theta)(1-\alpha'(1-\theta)) + \frac{1}{2}\alpha'\left(\frac{1-\alpha'(1-\theta)}{2-\alpha'}\right)^2.$$

Now, if $\theta > 0.85$, we have $L(1;0,\theta) = 1-\theta < \frac{1}{4}$ and $L(1;1,\theta) = \theta - \frac{1}{2}\theta^2 > \frac{1}{4}$; also

$$\frac{d}{d\alpha}L\left(1;\alpha,\theta\right) = \frac{2-2\alpha}{\left(2-\alpha'\right)^3}\left[R_1+R_2+R_3+R_4\right],$$

where

$$\begin{aligned} R_1 &= \tfrac{1}{2}(1-\theta)^2\left(2\alpha'+\alpha'^3\right), & R_2 &= 3(1-\theta)^2(\alpha'-\alpha'^2),\\ R_3 &= 8\left(\tfrac{3}{4}-\theta\right)^2\alpha', & R_4 &= 1-8(1-\theta)^2. \end{aligned}$$

As $R_i \geq 0$ for $i = 1,...,4$, we have $\frac{d}{d\alpha}L\left(1;\alpha,\theta\right) \geq 0$. So by Intermediate Value Theorem, there exists a unique $\alpha^*(\theta) \in (0,1)$ such that $L(1;\alpha^*(\theta),\theta) = \frac{1}{4}$.

Next, define $\mu_1(\alpha;\theta)$ as follows: for $\alpha \leq \alpha^*(\theta)$, let $\mu_1(\alpha;\theta) = 1$; and for $\alpha > \alpha^*(\theta)$, let $\mu_1(\alpha;\theta)$ be the solution to $L(\mu;\alpha,\theta) = \frac{1}{4}$. That is:

$$\mu_1(\alpha;\theta) := \begin{cases} 1 & if\ \alpha \leq \alpha^*(\theta) \\ \frac{(1-\alpha'(1-\theta))\left(\sqrt{K}-(1-\theta)\sqrt{\alpha'}\right)}{(1-\alpha'(1-\theta))\theta\sqrt{\alpha'}-(1-\alpha'\theta)\sqrt{K}} & otherwise \end{cases},$$

where $K := \frac{1}{2} - 2(1-\theta)(1-\alpha'(1-\theta))$.

Notice that when $\alpha \leq \alpha^*(\theta)$, for all $\mu \leq 1 = \mu_1(\alpha;\theta)$, $L(\mu;\alpha,\theta) \leq \frac{1}{4}$, i.e., Δ_S^* satisfies $\left(C_1^I\right)$; when $\alpha > \alpha^*(\theta)$, for all $\mu \leq \mu_1(\alpha;\theta)$, $L(\mu;\alpha,\theta) \leq \frac{1}{4}$, i.e., Δ_S^* satisfies $\left(C_1^I\right)$, and for all $\mu > \mu_1(\alpha;\theta)$, $L(\mu;\alpha,\theta) > \frac{1}{4}$, i.e., Δ_S^* always violate $\left(C_1^I\right)$. As Δ_S^* always satisfies $\left(C_2^I\right)$ we conclude: for $\theta > 0.85$, $V_{\{g,\emptyset\}}^T = \overline{V}$ when $\mu \leq \mu_1(\alpha;\theta)$ and $V_{\{g,\emptyset\}}^T < \overline{V}$ otherwise.

Step 4: Define θ^* as the largest solution in $[0,1]$ to the equation $\mu_0(1;\theta) = \mu_1(1;\theta)$; i.e.,

$$\theta^* := \frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right).$$

As $\theta^* > 0.85$, the definition of μ_0 and μ_1 are valid for $\theta > \theta^*$.

Step 5: Note that $\mu_0(\alpha;\theta)$ is increasing in both α and θ for $\theta \in (\theta^*, 1]$:

$$\frac{d}{d\alpha}\mu_0(\alpha;\theta) = \frac{(2\theta-1)(2-2\alpha)}{\left[3-\alpha'\theta-2\alpha'(1-\theta)\right]^2} \geq 0,$$

and

$$\frac{d}{d\theta}\mu_0(\alpha;\theta) = \frac{\alpha'(2-\alpha')}{\left[3-\alpha'\theta-2\alpha'(1-\theta)\right]^2} \geq 0.$$

Step 6: Next, we claim that $\mu_1(\alpha;\theta)$ is decreasing in α and increasing in θ for $\theta \in (\theta^*, 1]$.

Recall that for $\alpha \leq \alpha^*(\theta)$, $\mu_1(\alpha;\theta) = 1$; for $\alpha > \alpha^*(\theta)$, taking the derivative of $\mu_1(\alpha;\theta)$ with respect to α we obtain:

$$\frac{d}{d\alpha}\mu_1(\alpha;\theta) = -\left(S_1S_2 + S_3S_4\right)S_5,$$

where

$$\begin{aligned}
S_1 &:= (1-\theta)\left[(1-\alpha'(1-\theta))\theta\sqrt{\alpha'} - (1-\alpha'\theta)\sqrt{K}\right],\\
S_2 &:= \tfrac{1}{2\sqrt{K}}\left(1-6(1-\theta)(1-\alpha'(1-\theta))\right) + \tfrac{1}{2\sqrt{\alpha'}}\left(1-3\alpha'(1-\theta)\right),\\
S_3 &:= (1-\alpha'(1-\theta))\left[\sqrt{K}-(1-\theta)\sqrt{\alpha'}\right],\\
S_4 &:= \tfrac{1}{2\sqrt{\alpha'}}\theta(1-3\alpha'(1-\theta)) + \tfrac{1}{2\sqrt{K}}\left(\theta+2\theta^2+6\alpha'^2-2\right),\\
S_5 &:= (2-2\alpha)\Big/\left[(1-\alpha'(1-\theta))\theta\sqrt{\alpha'}-(1-\alpha'\theta)\sqrt{K}\right]^2.
\end{aligned}$$

It is routine to check that $S_i \geq 0$ for all $i = 1, ..., 5$. Hence, $\frac{d}{d\alpha}\mu_1(\alpha;\theta) \leq 0$.

Next, consider the derivative of μ_1 with respect to θ:

$$\frac{d}{d\theta}\mu_1(\alpha;\theta) = \frac{\frac{\sqrt{\alpha'}}{\sqrt{K}}T_1 + T_2}{\left[1-\alpha'(1-\theta)\theta\sqrt{\alpha'}-(1-\alpha'\theta)\sqrt{K}\right]^2},$$

where

$$\begin{aligned}
T_1 &:= \tfrac{5}{2}(1-\theta) + \tfrac{1}{2}(-19+33\theta-13\theta^2)\alpha' + (1-\theta)\left(11-17\theta+4\theta^2\right)\alpha'^2 - 4(1-\theta^3)\alpha'^3,\\
T_2 &:= -\tfrac{1}{2}\alpha'(2-\alpha') + (1-\alpha'(1-\theta))\left[2-(3-2\theta)\alpha' + (1-\theta)(8\theta-3)\alpha'^2\right].
\end{aligned}$$

Below, we show that $T_1 > 0$ and $T_2 \geq 0$ that implies $\mu_1(\alpha;\theta)$ is increasing in θ.

Step 6a: To show $T_1 > 0$, we consider two cases: $A > 0$ and $A \leq 0$, where $A := -19 + 33\theta - 13\theta^2$.

Case 1: When $A > 0$, we have $11 - 17\theta + 4\theta^2 < 0$ and $13 - 17\theta + 4\theta^2 \geq 0$. Now,

$$\begin{aligned} T_1 \geq\ & \tfrac{5}{2}(1-\theta) + \tfrac{1}{2}(-19 + 33\theta - 13\theta^2)\alpha' + (1-\theta)\left(11 - 17\theta + 4\theta^2\right) - 4(1-\theta^3) \\ =\ & \tfrac{1}{2}(-19 + 33\theta - 13\theta^2)\alpha' + (1-\theta)\left(13 - 17\theta + 4\theta^2\right) + (1-\theta)\left[\tfrac{1}{2} - 4(1-\theta)^2\right] \\ >\ & 0. \end{aligned}$$

Case 2: When $A \leq 0$ and $\theta > \theta^* > 0.85$, we have $11 - 17\theta + 4\theta^2 < 0$. Now,

$$\begin{aligned} T_1 \geq\ & \tfrac{5}{2}(1-\theta) + \tfrac{1}{2}(-19 + 33\theta - 13\theta^2) + (1-\theta)(11 - 17\theta + 4\theta^2) - 4\left(1-\theta^3\right) \\ =\ & \tfrac{1}{2}\theta(5\theta - 4) > 0. \end{aligned}$$

Step 6b: To show $T_2 \geq 0$, we first define

$$\begin{aligned} T_3 &: = 2 - (3 - 2\theta)\alpha' + (1-\theta)(8\theta - 3)\alpha'^2, \\ T_4 &: = 2(1-\theta)(2-\alpha') + \alpha'(1 - \alpha'(1-\theta)). \end{aligned}$$

Now,

$$\begin{aligned} T_2 =\ & -\tfrac{1}{2}\alpha'(2-\alpha') + (1 - \alpha'(1-\theta))T_3 \\ \geq\ & -\tfrac{1}{2}\alpha'(2-\alpha') + (1 - \alpha'(1-\theta))T_4 \\ \geq\ & -\tfrac{1}{2}\alpha'(2-\alpha') + \tfrac{1}{2}\left(2-\alpha'\right)^2 \\ =\ & (2-\alpha')(1-\alpha') \geq 0. \end{aligned}$$

The first inequality follows as $T_3 \geq T_4$ (routine to check). The second inequality follows from the fact that as we have $\alpha > \alpha^*(\theta)$, we have $L\left(1;\alpha,\theta\right) > \frac{1}{4}$. And,

$$L\left(1;\alpha,\theta\right) > \frac{1}{4} \Leftrightarrow (1 - \alpha'(1-\theta))T_4 > \frac{1}{2}\left(2-\alpha'\right)^2.$$

Step 7: It is routine to check $\mu_1(1;\theta) > \mu_0(1;\theta)$. So, for any $\theta \in (\theta^*, 1]$, $\mu_1(\alpha;\theta) > \mu_0(\alpha;\theta)$ for all $\alpha \in [0,1]$ (as μ_0 is strictly increasing, and μ_1 is decreasing in α). Thus, from Step 2 and 3, we find for $\mu < \mu_0(\alpha;\theta)$, $V^I_{\{g,\emptyset\}} = \overline{V} > \max\left\{V^I_{\{g\}}, V^T_{\{g,\emptyset\}}, V^T_{\{g\}}\right\}$; for $\mu > \mu_1(\alpha;\theta)$, $V^T_{\{g,\emptyset\}} = \overline{V} > \max\left\{V^I_{\{g\}}, V^I_{\{g,\emptyset\}}, V^T_{\{g\}}\right\}$; otherwise, $V^I_{\{g,\emptyset\}} = V^T_{\{g,\emptyset\}} = \overline{V}$. Thus, the characterization of optimal job design is qualitatively identical to that in Proposition 2. □

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