OPTIMAL JOB DESIGN AND INFORMATION ELICITATION*

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Abstract. When managers rely on their subordinates for local information but cannot commit to how such information is used, the incentives for effort and information elicitation become intertwined. This incentive problem influences the firm's job design decision, i.e., whether to assign all tasks in a job to one worker ("individual assignment") or split those among a group ("team assignment"). Team assignment facilitates information elicitation but suffers from diseconomies of scope in incentive provision. The optimal job design is driven by the workers' likelihood of being informed (about local conditions) and the noise in the performance measure used to reward them.

1. Introduction

Managerial decision-making in a hierarchical organization often relies on local information that cannot be directly accessed by the headquarter but may be available to its lower-ranked employees. A host of key business decisions—such as launching new product lines, undertaking new business ventures, investing in new R&D initiatives—require detailed information on customer preferences, profitability prospects, and technological capabilities. This information is more likely to be available to the junior workers who are more familiar with the local market conditions and the firm's production process. Effective managerial decision-making, therefore, calls for timely provision of information that may be dispersed within an organization.

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However, the firm and the workers may have conflicting interests on how information may be used, and when relaying local information to their manager, the workers may manipulate information to steer the firm's decision towards their own interests. A worker may deem his information "unfavorable" if it leads the firm to take an action that may reduce the worker's future rents. Consequently, he may attempt to conceal such information, particularly when the firm cannot commit on how the information may be used in its decision process. Such conflict of interest creates a complex incentive problem as the incentives for effort and information elicitation get intricately entwined (Athey and Roberts, 2001).

Starting from the seminal work by Marschak (1955) and Marschak and Radner (1972) on team theory, a large literature has explored the limits on information provision in an organization and how these limits are influenced by the organization's structure (Aoki, 1986). However, this literature typically abstracts away from the problem of incentives as the employees' objective is assumed to be perfectly aligned with that of the employer. The goal of our article is to explore how the problem of intertwined incentives for effort and information elicitation shapes a critical part of the organizational structure, namely, job design.

An essential problem in organizational design is how to group different tasks into jobs that may be assigned to the workers. An organization may typically choose between two natural designs: it may opt for "individual assignment" where all tasks associated with a specific production process are assigned to the same worker who remains solely accountable for his job output. Alternatively, it may choose "team assignment" where different tasks of the production process are assigned to different workers who are held jointly accountable for their job performance. The broad prevalence of individual and team assignments in project management structures has been well-documented in the management literature (Chandler, 1962; Galbraith, 1971; Larson and Gobeli, 1989; Hobday, 2000; Lechler and Dvir, 2010). Firms often adopt a "project-based" structure where a manager is assigned to oversee all aspects of a project, or a "functional" structure where projects are divided into segments and different segments are overseen by different managers.

In exploring the relative merits of the two structures, this literature mostly focuses on the gains from task specialization vis-a-vis task coordination. But if decision-relevant information is accessible only to the workers who are directly involved in the production process, the two job designs have distinct implications on how information may be dispersed within the organization. Under individual assignment, all information pertaining to a production process can be observed only by the worker who has been assigned to it, whereas multiple workers may access this information when they are working as a team. We highlight that when the workers need to be incentivized for both effort provision and information elicitation, the choice between these two designs is shaped by a novel trade-off: Team assignment may facilitate information elicitation but it suffers from "diseconomies of scope" in incentive provision that may undermine the workers' effort level.

We explore this trade-off using a stylized model that attempts to capture the essential aspects of job design in a minimal setting. A principal can hire up to two agents to work on a project that consists of two tasks. At the beginning of the game the principal chooses a job design: under individual assignment, only one agent is hired, and he is responsible for both tasks of the project; but under team assignment, two agents are hired and each is assigned to exactly one of the two tasks. The project can either succeed or fail. The likelihood of success of the project depends on the level of effort exerted in its tasks and the underlying "state of the world" that may be observed only by the agent(s) who work on the project. While performing a task, an agent may observe the state of the world (associated with the project) with some probability and reports his observation to the principal. Although an agent cannot misrepresent the state (i.e., the observation on the state is "hard" information) he may conceal it by feigning ignorance. Upon receiving the agents' reports on the state, the principal decides whether to continue or cancel the project. The project output is not verifiable, but the agents' effort in the project is reflected by a contractible but noisy performance measure.

Incentives are provided through a wage contract that ties an agent's pay to the principal's cancellation decision and the realization of the performance measure (if the project is continued). The misalignment between the performance measure and the project outcome gives rise to a conflict of interest between the principal and the agent(s). If the observed state does not bode well for the project's success but is unlikely to affect the performance measure (if the project is implemented), an agent may attempt to conceal his information to let the project proceed whereas the principal would have been better off by canceling it.

We show that the optimal job design is driven by two salient informational frictions: the "availability" of agents' information (i.e., the likelihood that an agent gets to observe the state while performing his assigned tasks) and the "noise" in the agents' performance measure (i.e., the extent of misalignment between the measure and the output). Team assignment is strictly optimal when the agents are highly likely to observe the state but there is significant misalignment between the performance measure and the project output. In contrast, when the extent of misalignment is relatively small, individual assignment is strictly optimal regardless of the agents' likelihood of being informed about the state.

The intuition for this result can be gleaned from the aforementioned trade-off between information elicitation and "diseconomies of scope" in incentive provision. As the principal relies on the agents' report to make her decision, an agent may have an incentive to manipulate his report and effort levels so as to change the project outcome in his favor. Team assignment helps in information elicitation as an agent does not fully control the outcome of a project. An agent's attempt to conceal information may falter if his teammate happens to provide the same information to the principal; moreover, he could influence the effort only in a part of the project. Hobday (2000) offers a stark example of this issue in his case study of a large pan-European firm. He finds that a major problem with the individual assignment (project-based structure) relative to the team assignment (functional structure) was that the project managers had direct control over the resources and had "'gone their own way' [as it became] difficult for senior company managers at HQ to keep properly informed and maintain some degree of control and consistency across the activities…" (p. 886).

But under team assignment the principal needs to reward the two agents separately to induce effort on the two tasks of the project. And such "diseconomies of scope" in incentive provision can blunt incentives. As the principal cannot commit to how she may use the agents' report, in equilibrium, her decision on project continuation must be sequentially rational. On receiving the agents' reports, she proceeds with the project if and only if her expected payoff from proceeding with the project is larger than what she obtains from canceling it. Given a project continuation policy that the principal intends to follow, this requirement puts an upper limit on the reward she can pay to the agents for the project's success (as per the performance measure). Under team assignment, this limit is more binding as the principal needs to reward the two agents separately and the total reward payout is larger. Consequently, strong incentives may not be feasible. In contrast, under individual assignment, a single reward payment induces effort in both tasks, and such economies of scope in incentive provision make it easier to provide strong incentives without violating the upper limit on the reward payments.

When the performance measure is considerably misaligned and can indicate success even when the project fails, the agents have strong incentives to conceal unfavorable information to let the project continue. This is when the team's advantage in information elicitation is most useful: an agent's attempt to conceal information could be undone by his teammate, particularly when his teammate is very likely to have the same information. Due to such misalignment, strong incentives also become feasible under team assignment despite the diseconomies of scope in incentive provision (we elaborate on this later). As a result, team assignment becomes optimal. In contrast, if the performance measure is relatively well-aligned with the project output, information elicitation is relatively easy as the agent has little to gain from concealing information from the principal. Thus, individual assignment becomes optimal—it allows the principal to exploit the economies of scope in incentive provision and offer strong incentives for effort without distorting the agent's reporting incentives.

A leading application: Although our analysis may relate to any production environment where elicitation of local information is important for managerial decision-making, it is particularly relevant in the context of complex project management. An important component of the management of such projects is periodic evaluation of its implementation and progress, especially at the earlier stages of the project. At these evaluations, information is collected, and corrective decisions are made, including cancellation of the project if it proves to be nonviable (Haji-Kazemi et al. 2013; Nikander and Eloranta, 2001; Williams et al. 2012).

A key component of these evaluation processes is the detection of "early warning signs" (EWS). "[An] early warning sign is an observation or signal, message, or some other form of communication that is or can be seen as... [a] sign of the existence of some future or incipient positive or negative issue" (Williams et al. 2012; p. 38). Management scholars and practitioners alike emphasize the need for identifying and acting on EWS as it could be cheaper to proactively avoid future problems than to build capabilities to manage crises when they occur. (A similar point has also been noted in the literature on Systems Dynamics; see, e.g., Black and Repenning, 2001, Repenning, 2001.) In the evaluation of complex projects, formal assessments of project viability that follow a set of prespecified procedures are often too limited in their scope in detecting the pertinent warning signs. Consequently, the managers tend to rely also on their workers' informal assessments as they may identify potential issues based on their own knowledge and experience with the project environment. Thus, communication between the workers and managers becomes of key importance (even though the informal nature of the assessment makes the workers' reports subjective).

The literature on EWS recognizes that such communication need not be perfect. The detection and communication of EWS may be impaired by overoptimistic assessments of benefits and underestimation of risks, but the workers may also strategically obfuscate information for their own gains at the expense of their employer. In particular, the workers can select areas of focus in their reports, and they can strategically exploit this discretion to hide signs of problems if they anticipate that the firm's response would lower their future payoffs (Haji-Kazemi et al. 2013). Our analysis provides an explanation as to why it may be difficult to dissuade the workers from obfuscating information about the project's future viability, and how job design could help in mitigating this problem. We return to this issue later in Section 6.3.

Related literature: Our article contributes to a growing literature on the interplay between incentives and communication of dispersed information within an organization. As mentioned earlier, the literature on team theory that followed from Marshak and Radner (1972) explores managerial decision-making when there are physical constraints on the flow of information (and the headquarters' ability to process it), but typically assumes that the workers are non-strategic in their communication (see, e.g., Cremer, 1980; Aoki, 1986; Genakoplos and Milgrom, 1991; Bolton and Dewatripont, 1994). Several authors have subsequently analyzed strategic communication by privately informed workers and how it shapes the allocation of decision rights within organization (Dessein, 2002; Alonso, Dessein, and Matouschek, 2008; Rantakari, 2008). These articles focus on the trade-off between the production efficiencies from coordination of actions and adaptation to local information, but abstract away from the incentive problems in effort provision.

Levitt and Snyder (1997) is one of the first articles to analyze the interaction between the incentives for effort and truthful communication. They highlight a trade-off between efficiency in decision-making and effort incentives when the agents may be privately informed about their projects' viability. In order to induce both effort and truthful reporting of "bad news" (i.e., information that lowers the likelihood of the project's success), the optimal contract calls for an inefficiently "lenient" continuation policy where some projects with negative expected value are allowed to continue. But in their model, the organizational structure is exogenously given; in contrast, we analyze how the interaction between the effort and reporting incentives drives the allocation of tasks within the organization.

Our article complements the works by Athey and Roberts (2001), Friebel and Raith (2010), and Dessein, Garicano, and Gertner (2010), who explore organizational forms in the presence of the trade-off between incentives for effort, communication, and efficient decision-making. Athey and Roberts show that the trade-off between effort incentives and efficient decision-making can be mitigated by creating an organizational hierarchy. The firm may hire a top-level manager who can obtain all information at a cost and coordinates the actions of her subordinates. However, they assume exogenous task allocation and do not allow for communication between agents.

Strategic communication plays an important role in Friebel and Raith (2010). They analyze the decision to integrate two units under a CEO with authority over resource allocation. The unit managers are assumed to be privately informed about the productivity of such resources in their respective divisions. The optimality of integration is driven by a trade-off between the benefit of more efficient resource allocation and the cost of a distortion in the effort incentives that arises under integration (due to the need for information elicitation). A similar integration issue is studied by Dessein, Garicano, and Gertner (2010). They consider a firm that decides whether to organize into business units (i.e., divisions with considerable autonomy) or create functional units that centralize certain tasks for all divisions. The functional unit manager can implement standardization to capture synergy benefits, but inflicts a cost on business unit managers by impeding adaptation to local information. To reduce distortion in the standardization decision and induce cooperation across units, the firm may tie the functional unit managers' compensation to the profits generated by the other business units.

In these articles, the trade-offs that drive a firm's organizational structure are considerably different from what we study. We abstract away from the issue of resource allocation across divisions or standardization and instead focus on the role of job design in incentivizing information transmission within the organization. Furthermore, in both Friebel and Raith (2010), and Dessein, Garicano, and Gertner (2010) the problem of information elicitation crops up only in an integrated firm; under non-integration, the problem does not arise because there is no need for coordination between the divisions. In our setup, information elicitation is essential regardless of the firm's job design decision, allowing us to compare how the two forms of job design affect information transmission within the firm.

This article also relates to a few other strands in the organizational economics literature. There is a vast literature on incentives in teams (Groves, 1973; Holmström, 1982; Mookherjee, 1984; McAfee and McMillan, 1991; Che and Yoo, 2001; Marino and Zabojnik, 2004; Kvaløy and Olsen, 2006; Rayo, 2007; Blanes i Vidal and Moller, 2016; Friebel et. al, 2017) that takes the team structure as given and analyzes how the underlying production and information environment drives the optimal provision of effort incentives. A notable exception is Gromb and Martimort (2007) who consider a setup where the decision-maker relies on experts to gather and report multiple signals on a risky project's profitability. They analyze a case where the decision-maker can either ask a single expert to acquire all signals or employ multiple experts where each one is responsible for acquiring exactly one signal. Although this setup bears some resemblance to our job design problem, Gromb and Martimort's model differs from ours along various key dimensions. In particular, in their setup the agents' efforts are useful for information acquisition but not for the project's value, experts have "soft information" (hence, can lie in their report), and the focus of their analysis is on the optimal incentives for such delegated expertise when the contracting parties may collude among themselves.

Job design has also been explored by several scholars, primarily as a possible remedy for the multitasking problem (Holmström and Milgrom, 1991; Dewatripont, Jewitt, and Tirole, 2000; Besanko, Regibeau, and Rockett, 2005; Schottner, 2008; Corts, 2007; Mukherjee and Vasconcelos, 2011; Ishihara, 2017). In contrast, we abstract away from the multitasking problem; in our setting, the interplay between the incentives for effort and information elicitation is the key driver of the optimal job design. Finally, our work is reminiscent of the literature on authority and delegation where the contracting parties may have misaligned preferences over the managerial actions (Aghion and Tirole, 1997; Dessien, 2002; Alonso and Matouschek, 2008; Alonso, et. al 2008; Deimen and Szalay, 2019). In this literature, the misalignment is assumed to stem from exogenous bias in the agents' preferences that may distort the communication within organizations. However, in our setup the agents' possible gains from information manipulation arise endogenously due to the moral hazard problem in the agents' effort provision and the firm's lack of commitment power over its continuation policy.

2. Model

Players: A principal $\mathcal{P}$ can hire up to two agents, $\mathcal{A}_1$ and $\mathcal{A}_2$, to work on a risky project and concurrently gather information on the project's financial viability.

Technology: The production technology is reminiscent of the canonical setup of Dewatripont et. al (2000). The project consists of two tasks: T_1 and T_2. To fix ideas, one may consider a firm exploring the launch of a new product, and a successful launch requires effort on product development and marketing.

At the beginning of the game, the principal commits to a task allocation or "job design." The principal can choose one of two options: (i) "individual assignment," where only one agent is hired, and he is put in charge of both tasks of the project, and (ii) "team assignment," where two agents are hired and each one performs exactly one of the two tasks of the project. Without loss of generality, we assume that under individual assignment, agent $\mathcal{A}_1$ is hired (and he performs tasks $\{T_1, T_2\}$), whereas under team assignment, $\mathcal{A}_1$ performs the first task, T_1, and $\mathcal{A}_2$ performs the second one, T_2.[1]

[1]One may note that in our model, the total number of workers hired by the firm changes with its job design. This feature of the model does not play any role in our analysis. The same results can be obtained in a model with two projects where under individual assignment each agent is assigned to exactly one project and is responsible for its two tasks, and under team assignment an agent is given exactly one task from each of the two projects (thus, the firm size is invariant to job design). Both setups capture the relevant distinction between individual and team assignment: under individual assignment one agent is entirely responsible for a given project whereas under team assignment different agents undertake different aspects of the project.

Performing a task requires effort. Let $e_k \in [0, 1/2]$ be the effort exerted in task T_k ($k \in \{1, 2\}$). Effort is private and costs the agent assigned to this task $c(e_k) = e_k^2/2$. The profile of effort exerted in the project is denoted as $\mathbf{e} := (e_1, e_2)$.

The outcome of the project, $Y \in \{0, y\}$, can be either a "success" ($Y = y$) or a "failure" ($Y = 0$), where $y \in (0, 1/2]$.[2] The project outcome depends on the effort exerted in each of its two tasks and on its underlying "state of the world" $\omega \in \{G, B\}$ that can either be "good" ($\omega = G$) or "bad" ($\omega = B$). More specifically, the production function is given as:

$$\Pr(Y = y \mid \mathbf{e}; \omega) = \begin{cases} e_1 + e_2 & \textit{if } \omega = G \\ 0 & \textit{if } \omega = B \end{cases}.$$

In a "bad" state, the project always fails regardless of the agents' effort, and yields $Y = 0$. In a "good" state failure can be averted, and effort is productive as it increases the chance of obtaining a high output of $Y = y$.

The project outcome is not verifiable, but the agent's performance is reflected by a metric $M \in \{0, 1\}$ that can be verified and signals "success" ($M = 1$) or "failure" ($M = 0$). However, the metric M is a noisy measure of the project outcome as:

$$\Pr(M = 1 \mid \mathbf{e}; \omega) = \begin{cases} e_1 + e_2 & \textit{if } \omega = G \\ \mu(e_1 + e_2) & \textit{if } \omega = B \end{cases},$$

where $\mu \in [0, 1)$. In the context of the product launch example, one may consider Y to be the product's long-term value to the firm, whereas M is a measure of the product's profitability in the short run. The extent of misalignment between the metric and the project output is reflected by the parameter μ: if $\mu > 0$, the metric may reflect a success even in a bad state when the project fails with certainty. This parameter will play an important role in our analysis as it captures the extent of the conflict of interest between the principal and the agents over the continuation of the project.[3]

Information structure: At the beginning of the production process the underlying state of the project, ω, is unknown to all players who hold a common prior belief $\Pr(\omega = G) = \frac{1}{2}$.

[2]The bound on y ensures that in the optimal contract problem we can ignore the possibility of corner solutions for the agents' effort choice.

[3]Although the project output always depends on the underlying state, the performance measure's sensitivity to the state depends on μ. In particular, for high μ the underlying state has little influence on the realization of the performance measure.

However, an agent who works on the project could learn about ω. In particular, upon completing an assigned task T_k, the agent *privately* observes the state ω with probability $\alpha \in [0, 1)$. The observability of the underlying state of the project in its two tasks is statistically independent. Thus, under individual assignment, agent $\mathcal{A}_1$ (who performs both tasks of the project) learns the underlying state ω with probability $1-(1-\alpha)^2$. And, under team assignment, the probability that at least one of the two agents (between $\mathcal{A}_1$ and $\mathcal{A}_2$) learns the state is also $1-(1-\alpha)^2$. Denote $\mathcal{A}_i$'s observation on the state ω as $x_i \in \{G, B, \emptyset\}$, where $x_i = \emptyset$ if $\mathcal{A}_i$ does not observe ω.

Reporting: Upon exerting effort on his assigned task(s), an agent reports his observation on the state to the principal. Under team assignment, the agents report simultaneously. The observation on the state is "hard information": an agent cannot misreport the state but can hide his observation by feigning ignorance. We denote agent $\mathcal{A}_i$'s report as $r_i \in \{G, B, \emptyset\}$, where $r_i = \emptyset$ when the agent claims to have failed to observe the state associated with the project. Furthermore, let the principal's received information on the state be $r \in \{G, B, \emptyset\}$. Trivially, under individual assignment, $r = r_1$ as $\mathcal{A}_1$ is fully responsible for the project. However, under team assignment $r = \emptyset$ if $r_1 = r_2 = \emptyset$, and $r = G$ if at least one of the two agents reports G, i.e., either $r_1 = G$ or $r_2 = G$. (The case for $r = B$ is analogous).

Given the agents' reports, the principal decides whether to continue the project or to cancel it. The project outcome Y and the associated performance measure M are realized only if the principal continues the project. If the project is cancelled, the principal earns her outside option, as described later in this section. The agents' reports, like the project outcome Y, are not verifiable.

Our assumption on the nature of the agents' information and reports (i.e., the agents' observation is "hard" information, but their reports are not verifiable by the court of law) alludes to a scenario where the agents cannot fabricate information to the principal because she is capable of evaluating the evidence in support of the agents' reports, and an unsubstantiated report may be treated as a lack of any information on the state (see, e.g., Tirole, 1986). However, the evaluation of the evidence must be done in the context of the production environment (e.g., as mentioned in our discussion of early warning signs in project management), requiring specialized knowledge that resides within the organization and cannot be easily transmitted to a third party. Consequently, it may be difficult for the court of law to properly interpret the reports, rendering them non-verifiable for the purpose of formal contracting.

CONTRACT: As mentioned above, at the beginning of the game, the principal commits to a job design $d \in \{\mathcal{I}, \mathcal{T}\}$ that specifies either individual assignment ($d = \mathcal{I}$) or team assignment ($d = \mathcal{T}$). As neither the agents' reports nor the project's outcome are verifiable, the principal cannot commit to a project cancellation policy, and can only commit to a wage schedule that depends on (i) whether the project has been implemented, and (ii) in the event the project is implemented, the realization of the performance measure $M \in \{0, 1\}$. To streamline the notation, we set $M = \emptyset$ if the project gets cancelled. Thus, under individual assignment, agent $\mathcal{A}_1$'s contract is given by the wage schedule $w_1^I(M)$, $M \in \{0, 1, \emptyset\}$. And under team assignment, the agent $\mathcal{A}_1$ and $\mathcal{A}_2$'s contracts are given by the schedules $w_1^T(M)$ and $w_2^T(M)$, respectively. Denote the wage schedule for $\mathcal{A}_i$ under the job design $d \in \{\mathcal{I}, \mathcal{T}\}$ as $\mathcal{W}_i^d$ (where we set $\mathcal{W}_2^{\mathcal{I}} = \emptyset$, as $\mathcal{A}_2$ does not receive an offer under individual assignment).

We denote a contract as $\phi := \left\{d, \left\{\mathcal{W}_1^d, \mathcal{W}_2^d\right\}\right\}$, and let Φ be the set of all such contracts.

PAYOFFS: All players are risk neutral. If the contract offered by the principal is accepted by the agent(s), an agent's ex-ante payoff is given by his expected wage net of his cost of effort. And the ex-ante payoff of the principal is given by the expected output from the project (when implemented) net of the expected wage payment. If the project is cancelled, the principal takes an "outside option" that yields a payoff of $\underline{\pi}$ (> 0) and pays the cancellation wage. With a slight abuse of notation, we set $Y = \underline{\pi}$ if the project gets cancelled (recall that we also set the performance metric $M = \emptyset$).

Thus, under individual assignment, agent $\mathcal{A}_1$'s ex-post payoff is (recall that in this case agent $\mathcal{A}_2$ is not hired) $u_1^I := w_1^I(M) - c(e_1) - c(e_2)$, and the principal's ex-post payoff is $\pi^I := Y - w_1^I(M)$. Analogously, under team assignment, the payoff of agent $\mathcal{A}_i$, $i \in \{1, 2\}$, is given as $u_i^T := w_i^T(M) - c(e_i)$, and the principal's ex-post payoff is $\pi^T = Y - w_1^T(M) - w_2^T(M)$.

If the principal's offer is rejected by the agent(s), the principal obtains her outside option $\underline{\pi}$ and the agents obtain their outside option 0.

We assume that a priori the principal is indifferent between canceling the project and implementing it without seeking any information from the agents, which implies the following restriction on the parameters.[4]

Assumption 1. $\underline{\pi} = \max_{e_1, e_2} \frac{1}{2}(e_1 + e_2) y - c(e_1) - c(e_2) = \frac{1}{4} y^2$.

[4]This indifference condition is not critical for our findings but it considerably improves the algebraic tractability of our analysis. The optimal job design remains qualitatively unaltered as long as the principal's outside option $\underline{\pi}$ is not too large or too small relative to the "no information surplus" of $\frac{1}{4}y^2$.

Time line: The timeline of the game is summarized below:

- $\mathcal{P}$ chooses a job design $d \in \{\mathcal{I}, \mathcal{T}\}$, and publicly offers a wage schedule $\{\mathcal{W}_1^d, \mathcal{W}_2^d\}$.
- The agent(s) accept(s) or reject(s) the contract ϕ. The game proceeds only if the agent(s) accept(s).
- $\mathcal{A}_i$ exerts effort in the tasks that have been assigned to him.
- $\mathcal{A}_i$ may observe the state ω from his assigned task(s) and reports to $\mathcal{P}$.
- $\mathcal{P}$ decides whether to proceed or cancel the project.
- The project outcome, performance measure, and payoffs are realized; and the game ends.

Strategies and Equilibrium concept: The strategy of the principal, $\sigma_{\mathcal{P}}$, has two components: (i) A contract $\phi \in \Phi$ offered at the beginning of the game that stipulates the job design $d \in \{\mathcal{I}, \mathcal{T}\}$ and the agents' wage schedules given the chosen design, $\{\mathcal{W}_1^d, \mathcal{W}_2^d\}$. (ii) Given ϕ, a *continuation policy*, $\mathcal{C}$, that stipulates the principal's continuation decision on the project as a function of the agents' reports r_1 and r_2. The strategy of the agent $\mathcal{A}_i$, $\sigma_{\mathcal{A}_i}$, has three components: given the principal's contract offer ϕ, (i) a decision on whether to accept or reject the contract offer, (ii) an *effort policy* $\mathcal{E}_i$ that stipulates effort levels on the assigned tasks, and (iii) a *reporting policy* ρ_i that maps the agent's observation on the state, x_i, to his report r_i. We use *perfect Bayesian Equilibrium* (PBE) in pure strategies as a solution concept.

For each job design $d \in \{\mathcal{I}, \mathcal{T}\}$, we look for the PBE that yields the highest payoff to the principal in the continuation game. The optimal job design d is the one that yields the highest equilibrium payoff to the principal.

3. Preliminary analyses

This section presents two preliminary analyses to highlight the key trade-off that drives job design in our setting. First, we explore a benchmark case to show that the choice of job design is irrelevant when the agents' information is public and the principal can commit to a project continuation policy. Next, we present a partial analysis of the model to illustrate how job design becomes a strategic choice for the principal when she has to rely on the agents for information and lacks commitment power over her continuation decision.

3.1. **A public information benchmark.** Suppose that the agents' observations on the state are *publicly verifiable* information. Thus, the principal does not need to elicit any information from the agents on the project's viability, and she can also commit to a cancellation policy that depends on the observed state.[5] Let $x \in \{G, B, \emptyset\}$ denote the information on the state ω collectively observed by the agent(s) assigned to the project, where $x = \emptyset$ if neither of the two agents observes ω. In contrast to our main model, here x is publicly observed.

Consider first the case where the principal opts for individual assignment ($d = \mathcal{I}$). Suppose that she commits to proceeding with the project if and only if $x \in X_P \subseteq \{G, B, \emptyset\}$, and offers agent $\mathcal{A}_1$ a wage schedule $\mathcal{W}_1^{\mathcal{I}}$.

Agent $\mathcal{A}_1$'s expected payoff from accepting the offer and exerting effort $\mathbf{e} := (e_1, e_2)$ is:

$$U_1^I(\mathbf{e}, X_P) := \Pr(x \in X_P)\,\mathbb{E}\left[w_1^I(M) \mid \mathbf{e}, x \in X_P\right] + \Pr(x \notin X_P)\, w_1^I(\emptyset) - \sum_{k \in \{1,2\}} c(e_k).$$

That is, with probability $\Pr(x \in X_P)$ the project continues, and agent $\mathcal{A}_1$ earns his expected wage conditional on his effort and the event that the observation on the underlying state is in X_P. Otherwise, the project is cancelled, and the agent earns his "cancellation wage" $w_1^I(\emptyset)$. Regardless of the principal's decision on the project's continuation, the agent incurs the cost of his effort.

Similarly, the principal's expected payoff under effort profile $\mathbf{e}$ is given by:

$$\Pi^I := \Pr(x \in X_P)\,\mathbb{E}\left[Y - w_1^I(M) \mid \mathbf{e}, x \in X_P\right] + \Pr(x \notin X_P)\left[\underline{\pi} - w_1^I(\emptyset)\right].$$

Thus, the optimal contract under individual assignment stipulates the wage schedule and continuation policy given by the set X_P that maximize Π^I, subject to the agent's participation constraint $\left(IR^I\right)$ and incentive compatibility constraints $\left(IC^I\right)$ (i.e., the agent's payoff under the contract is at least as large as his outside option, and the stipulated effort level maximizes the agent's expected payoff under the given contract):

$$(IR^I) \qquad U_1^I(\mathbf{e}, X_P) \geq 0,$$

and

$$(IC^I) \qquad \mathbf{e} = \arg\max_{e_1', e_2'} U_1^I(\mathbf{e}', X_P).$$

[5]The class of wage contracts in this benchmark case is assumed to be the same as the one defined in the main model. Even though the wage payments could be tied to the agents' observed state (when the observations are publicly verifiable), as we will explain below, the principal does not benefit from doing so.

The optimal contract under team assignment ($d = \mathcal{T}$) may be obtained in a similar fashion. However, as both agents participate in the project, one needs to account for the fact that the payoff of each agent depends on the effort exerted by *both* agents. Suppose that the principal commits to proceeding with the project if and only if $x \in X_P \subseteq \{G, B, \emptyset\}$ and offers a wage schedule $\left\{\mathcal{W}_1^{\mathcal{T}}, \mathcal{W}_2^{\mathcal{T}}\right\}$. If the agents accept the offer, in the continuation game that follows, the agents' effort choices must constitute a Nash Equilibrium. Thus, if the contract induces agent $\mathcal{A}_i$ to exert effort e_i (in his assigned task), it must be a best response to the other agent $\mathcal{A}_{-i}$'s effort e_{-i} (in the other task).

Agent $\mathcal{A}_i$'s expected payoff under team assignment is

$$U_i^T\left(e_i, e_{-i}, X_P\right) := \Pr\left(x \in X_P\right) \mathbb{E}\left[w_i^T\left(M\right) \mid e_i, e_{-i}, x \in X_P\right] + \Pr\left(x \notin X_P\right) w_i^T\left(\emptyset\right) - c\left(e_i\right).$$

Thus, the optimal contract under team assignment stipulates the wage scheme and continuation policy given by the set X_P that maximize the principal's expected payoff

$$\begin{aligned} \Pi^T := \quad & \Pr\left(x \in X_P\right) \mathbb{E}\left[Y - w_1^T\left(M\right) - w_2^T\left(M\right) \mid \mathbf{e},\ x \in X_P\right] \\ & + \Pr\left(x \notin X_P\right)\left[\underline{\pi} - w_1^T\left(\emptyset\right) - w_2^T\left(\emptyset\right)\right], \end{aligned}$$

subject to the agents' participation and incentive compatibility constraints that parallel their counterparts under individual assignment:

$$(IR^T) \qquad U_i^T\left(e_i, e_{-i}, X_P\right) \geq 0 \quad \forall i,$$

and

$$(IC^T) \qquad e_i = \arg\max_{e_i'}\ U_i^T\left(e_i', e_{-i}, X_P\right) \quad \forall i.$$

Proposition 1. *Under both individual and team assignment, in the optimal contract the principal proceeds with the project if and only if the bad state is not observed (i.e., $x \in \{G, \emptyset\}$), and she obtains a payoff*

$$S^* := \left(1 + \alpha - \frac{1}{2}\alpha^2\right) \underline{\pi}.$$

That is, in the benchmark case, job design does not affect the principal's payoff under the optimal contract.

The proof of Proposition 1, along with all other proofs, are given in the Appendix.

The above finding shows that the choice of job design is irrelevant when the agents' information is public. Regardless of job design, the principal can commit to the optimal continuation policy and use the wage contract to induce first-best effort while extracting all surplus from the agents. Thus, the issue of job design becomes relevant only when the agents' observations on the project's underlying state remain private (and as the agents' reports are non-contractible, the principal cannot commit to a continuation policy).

3.2. **On the trade-off with job design.** Before we delve into our main analysis, it is instructive to illustrate how the principal's need for information elicitation, in conjunction with her lack of commitment power on continuation decisions, shapes the optimal job design. In what follows, we revert to our main model where the agents' information is private and the principal cannot commit to her continuation policy.

Suppose the principal wants to implement a given effort profile $\mathbf{e}^*$ and, as in the benchmark case, proceed with the project if and only if the bad state is not observed. One way of implementing this continuation policy is for the principal to ask an agent to report a bad state if he sees one and to report nothing otherwise, and the principal cancels the project if and only if she receives a bad report. But, in contrast to the benchmark analysis, here the wage contracts must satisfy two new incentive constraints: (i) The agents must find it optimal to report a bad state if they see one in addition to exerting effort $\mathbf{e}^*$; and (ii) the principal must find it optimal to cancel the project if and only if a bad state is reported. That is, as the wage contracts now affect the agents' reporting incentives and the principal's continuation decision, one needs to ensure that they induce the desired reporting and continuation policies. As we explain below, job design is important because it affects these additional constraints.

Let an agent's pay when the performance measure indicates success $(M = 1)$, failure $(M = 0)$ and when the project is cancelled be w_S, w_F, and w_C, respectively. Also, to streamline the exposition, in this subsection, we set $\alpha = 1$ (i.e., an agent who works on the project always observes the project's underlying state).

Consider the agents' incentives first. Under individual assignment, the agent performs both tasks of the project, and if he follows a strategy where he reports the bad state when he sees one, the project will be cancelled with probability $\Pr(\omega = B) = \frac{1}{2}$. Now, a possible deviation for the agent is to follow a strategy where he suppresses information on the bad state (hence the project always continues) and exerts higher effort, $\mathbf{e}'$, say, so as to increase his chances of getting a favorable performance rating $(M = 1)$. (Recall that when $\mu > 0$ the performance measure can show a success even when the state is bad.) So, the agent's incentive constraint for reporting a bad state requires that his "on path" payoff must be at least as large as his deviation payoff, which simplifies to:

$$(IC_A^I\text{-}r) \qquad \begin{aligned} &\tfrac{1}{2}\left[\left(e_1^* + e_2^*\right)\left(w_S - w_F\right) + w_F\right] + \tfrac{1}{2}w_C - \tfrac{1}{2}\left(e_1^*\right)^2 - \tfrac{1}{2}\left(e_2^*\right)^2 \geq \\ &\qquad \tfrac{1}{2}\left(1+\mu\right)\left(e_1' + e_2'\right)\left(w_S - w_F\right) + w_F - \tfrac{1}{2}\left(e_1'\right)^2 - \tfrac{1}{2}\left(e_2'\right)^2. \end{aligned}$$

But under team assignment, an agent is never successful in suppressing information from the principal as the other agent would reveal it. Thus, if agent $\mathcal{A}_1$, say, attempts to deviate, he only ends up exerting a higher effort without changing the project's continuation probability. Consequently, his incentive constraint boils down to:

$$(IC_A^T\text{-}r) \qquad \begin{aligned} &\tfrac{1}{2}\left[\left(e_1^* + e_2^*\right)\left(w_S - w_F\right) + w_F\right] + \tfrac{1}{2}w_C - \tfrac{1}{2}\left(e_1^*\right)^2 \geq \\ &\qquad \tfrac{1}{2}\left[\left(e_1' + e_2^*\right)\left(w_S - w_F\right) + w_F\right] + \tfrac{1}{2}w_C - \tfrac{1}{2}\left(e_1'\right)^2. \end{aligned}$$

Comparing $\left(IC_A^I\text{-}r\right)$ and $\left(IC_A^T\text{-}r\right)$ we can readily observe the benefit of team assignment. The agent's incentive constraint is easier to satisfy under team assignment as it facilitates information elicitation: An agent cannot effectively suppress information, and thus, $\left(IC_A^T\text{-}r\right)$ is implied by the agent's incentive constraint for effort $\left(IC^T\right)$, imposing no additional restrictions on the wage payments. In contrast, under individual assignment, $\left(IC_A^I\text{-}r\right)$ imposes an additional restriction on the wage payments as the agent can manipulate the principal's continuation decision when the state is bad.

Next, consider the principal's incentives for project continuation. Take the case of individual assignment first. In order for the principal to cancel the project when the state is bad and continue when the state is good, her payoff from cancellation, $\underline{\pi} - w_C$, must be more than her continuation payoff under the bad state, and less than her continuation payoff under the good state. Under individual assignment, these conditions boil down to

$$-\mu\left(e_1^* + e_2^*\right)\left(w_S - w_F\right) - w_F \leq \underline{\pi} - w_C \leq \left(e_1^* + e_2^*\right)\left(y - \left(w_S - w_F\right)\right) - w_F,$$

and they imply the following constraint on the wages:

$$(IC_P^I) \qquad y \geq \left(1 - \mu\right)\left(w_S - w_F\right).$$

Under team assignment this condition remains the same except for one important change. Under team assignment, the principal has to pay two wage premia $\left(w_S - w_F\right)$ instead of one, as the two agents must be incentivized separately to exert effort in their respective tasks. Such diseconomies of scope in incentive provision imply the following constraint:

$$(IC_P^T) \qquad y \geq 2\left(1-\mu\right)\left(w_S - w_F\right).$$

Now, we can readily observe the disadvantage of team assignment relative to individual assignment. Compared to $\left(IC_P^I\right)$, $\left(IC_P^T\right)$ imposes a tighter restriction on the success premium $(w_S - w_F)$, thus making the principal's incentives harder to satisfy. Thus, team assignment facilitates information elicitation by relaxing the agents' incentive constraint, but suffers from diseconomies of scope in incentive provision that makes the principal's incentive constraints harder to satisfy.

We conclude this section with two observations. First, if one assumes that the agents' information is public (but the principal cannot commit to her continuation policy), team assignment's advantage in information elicitation becomes irrelevant: team assignment no longer helps in relaxing the agent's (IC) constraint but tightens the principal's (IC) constraint. Hence, individual assignment would be optimal regardless of the underlying parameters of the model. Similarly, if the principal can commit to her continuation policy contingent on the agents' report (but the agent's information remains private), the (IC_P) constraints are redundant. Therefore, the diseconomies of scope under team assignment do not affect the optimal contract even though team assignment continues to facilitate information elicitation. Consequently, in this case team assignment is always (weakly) optimal. In other words, the optimal job design becomes a strategic choice (driven by the aforementioned trade-off) only when the agent's information is private and the principal lacks commitment power regarding project continuation.

Second, the agent's incentive constraint under individual assignment $\left(IC_A^I\text{-}r\right)$ becomes harder to satisfy as μ increases, whereas the principal's incentive constraints indicate that the impact of the diseconomies of scope effect (under team assignment) decreases with μ. In particular, the impact of the diseconomies of scope effect disappears completely when $\mu = 1$, i.e., when the performance measure is completely insensitive to the underlying state. Therefore, one may expect team assignment to be optimal when μ is high and individual assignment to be optimal when μ is low. As we will see later, this observation continues to hold in our main analysis where we derive the optimal contract and the agents are assumed to be potentially uninformed (i.e., where $\alpha < 1$).

4. Optimal contract

To analyze the principal's choice between team and individual assignment, we first need to characterize the optimal contract under each of the two job designs. As mentioned above, when the agents are privately informed, the wage contract not only affects the agents' effort, but it also interferes with their incentive to reveal information as well as the principal's incentive to continue with the project. The optimal wage contract must take all these effects into account. Later in Section 5, we characterize the optimal job design by comparing the principal's maximal payoff under team and individual assignments.

4.1. **Optimal contract under individual assignment.** We begin our analysis with the case of individual assignment. That is, we assume that the principal chooses $d = \mathcal{I}$, and in the continuation game we solve for the PBE that yields the highest payoff to the principal. We proceed in two steps (also, to streamline notations, we drop the subscripts for agent's identity). First, we fix a reporting and continuation policy pair $(\rho, \mathcal{C})$, i.e., a "communication protocol," and solve for the optimal wage contract $\mathcal{W}$ and effort policy $\mathcal{E}$ such that the tuple $(\mathcal{W}, \mathcal{E}, \rho, \mathcal{C})$ can be supported in a PBE. Next, we compare the payoffs of the principal obtained in the first step across all possible communication protocols.

The following lemma greatly simplifies our analysis.

Lemma 1. *If there exists a PBE where the principal's payoff exceeds her outside option $\underline{\pi}$, then there also exists a payoff equivalent PBE where the associated communication protocol is one of the following:*

(i) If the state is observed to be G, the agent reports G, otherwise he reports $\emptyset$; the principal proceeds with the project if and only if the report $r = G$.

(ii) If the state is observed to be B, the agent reports B, otherwise he reports $\emptyset$; the principal proceeds with the project if and only if the report $r \neq B$.

Lemma 1 implies that without loss of generality we can limit attention to only two classes of PBE: one where the project proceeds if and only if there is "good news", i.e., if and only if the agent's observation $x \in X_P = \{G\}$; and another where the project proceeds if and only if there is "no bad news", i.e., if and only if the agent's observation $x \in X_P = \{G, \emptyset\}$. Thus, in parallel to our public information benchmark case, we can undertake our analysis by focusing on the set of signals X_P under which the project proceeds (however, here we must ensure that the corresponding communication protocols $(\rho, \mathcal{C})$ are supported in a PBE).

For brevity of notation, when expedient, we denote the first communication protocol in Lemma 1 as $X_P = \{G\}$ and the second one as $X_P = \{G, \emptyset\}$. Also, we denote $w^I(0) =:$

w_F (wage when the performance metric indicates "failure"), $w^I(\emptyset) - w^I(0) =: \Delta_C$ (wage premium for cancellation), and $w^I(1) - w^I(0) =: \Delta_S$ (wage premium for success).

Given a wage contract $\{w_F, \Delta_C, \Delta_S\}$, effort levels e_1 and e_2 in tasks 1 and 2 of the project, and communication protocol $X_P \in \{\{G\}, \{G, \emptyset\}\}$, the firm's ex-ante payoff is:

$$\begin{aligned}\Pi^I := \Pr(x \in X_P)\left[\Pr(\omega = G \mid x \in X_P)(y - \Delta_S) + \Pr(\omega = B \mid x \in X_P)(-\mu\Delta_S)\right]\sum_k e_k \\ + \Pr(x \notin X_P)\left[\underline{\pi} - \Delta_C\right] - w_F.\end{aligned}$$

If the project proceeds, which occurs when $x \in X_P$, it yields an output y only when the state is good, but the wage premium for success may be paid even if the state is bad (as the performance measure is not perfectly aligned with the project's outcome and may indicate success with probability μ). If the project is cancelled, which occurs when $x \notin X_P$, the principal obtains her outside option and pays the wage premium for cancellation. The agent's ex-ante payoff can be written analogously as:

$$\begin{aligned}U^I := \Pr(x \in X_P)\left[\Pr(\omega = G \mid x \in X_P) + \mu \Pr(\omega = B \mid x \in X_P)\right]\Delta_S \sum_k e_k \\ + \Pr(x \notin X_P)\Delta_C + w_F - \tfrac{1}{2}\sum_k e_k^2.\end{aligned}$$

Now, if the tuple $(w_F, \Delta_C, \Delta_S; e_1, e_2; X_P)$ is supported by a PBE, the following constraints must be met. First, for each of the two communication protocols given in Lemma 1, the principal's decision must be sequentially rational so that she has no incentive to deviate from the continuation policy $(\mathcal{C})$. In other words, if the principal believes that the agent's signal x is in X_P (given the agent's report), it must be more profitable for her to proceed with the project than to cancel it. Similarly, if the principal believes that the agent's signal is not in X_P, it must be more profitable for her to cancel the project than to proceed with it. Therefore, the principal's incentive compatibility constraints require:

$$(IC_P^I\text{-}1) \qquad \left[\Pr(\omega = G \mid x \in X_P)(y - \Delta_S) - \mu \Pr(\omega = B \mid x \in X_P)\Delta_S\right]\sum_k e_k \geq \underline{\pi} - \Delta_C,$$

and

$$(IC_P^I\text{-}2) \qquad \left[\Pr(\omega = G \mid x \notin X_P)(y - \Delta_S) - \mu \Pr(\omega = B \mid x \notin X_P)\Delta_S\right]\sum_k e_k \leq \underline{\pi} - \Delta_C.$$

Next, we have the agent's participation constraint:

$$(IR_I) \qquad U^I \geq 0.$$

Finally, consider the agent's incentive compatibility constraint. Let $U(e_1', e_2'; \rho')$ be the agent's payoff given his efforts e_1', e_2', and reporting policy ρ' (fixing the wage contract and the principal's continuation policy). The agent's on-path payoff U^I must be the largest payoff attainable for any feasible choice of effort profile and reporting policy. So, we require:

$$U^I = \max_{e_1', e_2', \rho'} U(e_1', e_2'; \rho'). \tag{1}$$

Stipulating (1) is equivalent to imposing the following two constraints: First, a standard incentive compatibility constraint that requires the effort levels to be optimal for the agent given his equilibrium reporting strategy (as per the communication protocol $(\rho, \mathcal{C})$); i.e.,

$$(e_1, e_2) = \arg\max_{e_1', e_2'} U(e_1', e_2'; \rho). \tag{1a}$$

Second, the agent may not gain from a "double deviation" either, where he simultaneously deviates on his effort levels and his reporting strategy. Now, given a communication protocol $(\rho, \mathcal{C})$, if the agent can profitably deviate to some other reporting policy ρ' it must be that his report changes the principal's decision on whether to proceed with the project (under the continuation policy $\mathcal{C}$). Consider the two communication protocols mentioned in Lemma 1. In the first one the associated reporting policy is to report G when $x = G$ and report $\emptyset$ if $x \in \{\emptyset, B\}$; in the second one the agent reports B when $x = B$ and reports $\emptyset$ if $x \in \{G, \emptyset\}$. So, in the first case the only relevant deviation for the agent is to conceal information when $x = G$, and in the second case it is to conceal the information when $x = B$. Thus, in both cases, it is sufficient to consider only one type of deviation: the agent reports $\emptyset$ regardless of his observation. We denote this reporting policy as $\rho_\emptyset$. Hence, we must have:

$$U^I \geq \max_{e_1', e_2'} U(e_1', e_2'; \rho_\emptyset). \tag{1b}$$

It is instructive to elaborate on the conditions $(1a)$ and $(1b)$ as they, along with the principal's incentive constraints, illustrate the key trade-offs associated with information elicitation.

Regarding condition $(1a)$, it is routine to check that U is concave in effort for any wage contract and any of the two communication protocols. Moreover, at the optimal contract, under our parameter specifications, the agent's effort-choice problem always admits an interior solution. Hence, the condition can be replaced by its associated first-order conditions: for $i = 1, 2$,

$$(IC_A^I\text{-}1) \qquad e_i = \Pr(x \in X_P)\left[\Pr(\omega = G \mid x \in X_P) + \mu \Pr(\omega = B \mid x \in X_P)\right]\Delta_S.$$

The condition $(1b)$, however, is slightly more intricate. To simplify it one needs to account for the fact that if the agent deviates from his equilibrium reporting policy ρ to $\rho_\emptyset$ (in which he reports $\emptyset$ regardless of his observation), it affects the project's continuation probability.[6]

Let $p_\emptyset^I$ be the probability that the project continues if the agent deviates to the reporting policy $\rho_\emptyset$ given the equilibrium communication protocol. Thus, $p_\emptyset^I = 0$ for the communication protocol given by $X_P = \{G\}$ as here the project is cancelled unless the agent reports a good state. Similarly, $p_\emptyset^I = 1$ for the protocol given by $X_P = \{G, \emptyset\}$ as under this protocol the project is cancelled only if the agent reports a bad state. Also, for brevity of notation, denote

$$p^I := \Pr(x \in X_P),$$

and

$$\begin{aligned} P^I :=& \quad \Pr(\omega = G \mid x \in X_P) + \mu \Pr(\omega = B \mid x \in X_P), \\ P_\emptyset^I :=& \quad \Pr(\omega = G) + \mu \Pr(\omega = B). \end{aligned}$$

Now, off-path, the agent's maximum payoff from using the reporting policy $\rho_\emptyset$ can be derived as (again, we can verify that under optimal contract we have an interior solution):

$$\begin{aligned} \max_{e_1', e_2'} U(e_1', e_2'; \rho_\emptyset) =& \quad \max_{e_1', e_2'} \; p_\emptyset^I \left[\Pr(\omega = G) + \mu \Pr(\omega = B)\right]\Delta_S \sum_k e_k' - \tfrac{1}{2}\sum_k e_k'^2 \\ & \qquad\qquad + \left(1 - p_\emptyset^I\right)\Delta_C + w_F \\ =& \quad \left(p_\emptyset^I P_\emptyset^I \Delta_S\right)^2 + \left(1 - p_\emptyset^I\right)\Delta_C + w_F. \end{aligned}$$

The agent's on-path payoff can be computed analogously, and $(1b)$ simplifies to:

$$(IC_A^I\text{-}2) \qquad \left[\left(p^I P^I\right)^2 - \left(p_\emptyset^I P_\emptyset^I\right)^2\right]\Delta_S^2 \geq \left(p^I - p_\emptyset^I\right)\Delta_C.$$

Thus, under individual assignment, the optimal wage contract that supports a communication protocol given by $X_P \in \{\{G\}, \{G, \emptyset\}\}$ solves the following program:

[6] In case the project continues, the likelihood of a state ω conditional on the project being continued is the same as its prior probability as the project would continue regardless of the agent's observed signal x.

$$\mathcal{P}^I : \max_{\substack{w_F, \Delta_C, \Delta_S, \\ e_1, e_2}} \Pi^I \;\; s.t. \; (IR^I), \; \left(IC_P^I\text{-}1\right), \; \left(IC_P^I\text{-}2\right), \; \left(IC_A^I\text{-}1\right), \text{ and } \left(IC_A^I\text{-}2\right).$$

Lemma 2. *The program $\mathcal{P}^I$ always admits a solution for $X_P = \{G, \emptyset\}$, and admits a solution for $X_P = \{G\}$ if and only if α is sufficiently large.*

If $\mathcal{P}^I$ admits a solution for both $X_P = \{G\}$ and $X_P = \{G, \emptyset\}$, the optimal contract induces the communication protocol under which the value of the program $\mathcal{P}^I$ is larger. In the case where $\mathcal{P}^I$ admits no solution for $X_P = \{G\}$, such communication protocol cannot be sustained in a PBE, and the optimal contract induces communication protocol $X_P = \{G, \emptyset\}$.[7]

4.2. **Optimal contract under team assignment.** The analysis of team assignment resembles our above discussion on individual assignment, but the two forms of job design differ in two key aspects. First, under team assignment, since both agents work on the project, both agents may observe the underlying state associated with it. Thus, an agent cannot fully control the flow of information about the project as his attempt to hide information would fail if the other agent also obtains the information and reveals it to the principal. Second, both agents must be (individually) incentivized for information elicitation and effort provision. (In contrast, under individual assignment the principal has to incentivize only one agent; the agent is responsible for both tasks associated with the project and observes both signals on the project's underlying state). As illustrated earlier and as we will explain in more detail later, these two distinctions give rise to the key trade-off between ease of information elicitation and economies of scope in incentive provision that drives the optimal job design.

Now, consider the principal's optimal contracting problem. Analogous to the case of individual assignment, we seek to characterize the PBE of the continuation game with the largest ex-ante payoff for the principal. The analysis follows the same two-step process described above: first, we fix a communication protocol and derive the optimal wage contract that supports it in equilibrium; and next, we compare the principal's payoff across all possible communication protocols that could be sustained in equilibrium.

[7] The reason why a solution need not always exist for $X_P = \{G\}$ is as follows. When α is small, the ex-ante probability of the project continuing is also small. Hence, in order to incentivize the agent to both exert effort and report the state G when he sees it, the principal must offer a high success premium (Δ_S) coupled with a low cancellation premium (Δ_C). But such a contract makes continuing the project too expensive for the principal relative to cancelling it, and therefore, there is no contract under which both the agent and the principal's incentive constraints are jointly satisfied.

To streamline notation, we again relabel $w_i^T(0) =: w_{iF}$, $w_i^T(\emptyset) - w_i^T(0) =: \Delta_{iC}$ (wage premium for cancellation), and $w_i^T(1) - w_i^T(0) =: \Delta_{iS}$ (wage premium for success). Also, we denote the team's collective observation on the state as x^T, where

$$x^T := \begin{cases} G & if\ x_i = G \text{ for some } i \\ B & if\ x_i = B \text{ for some } i \\ \emptyset & if\ x_1 = x_2 = \emptyset \end{cases}.$$

As in the case of individual assignment, we can limit attention to only two communication protocols as stated in Lemma 3 below. We omit the proof of this lemma as it follows the same argument as that of Lemma 1.

Lemma 3. *If there exists a PBE where the principal's payoff exceeds her outside option $\underline{\pi}$, then there also exists a payoff equivalent PBE where the associated communication protocol is one of the following:*

(i) Reporting policy for agent $\mathcal{A}_i$ $(i = 1,2)$: if the state is observed to be G, report G, otherwise report $\emptyset$; principal proceeds with the project if and only if $r_i = G$ for some i.

(ii) Reporting policy for agent $\mathcal{A}_i$ $(i = 1,2)$: if state is observed to be B, report B, otherwise report $\emptyset$; principal proceeds with the project if and only if $r_i \neq B$ for all i.

As before, the communication protocols that are relevant for our analysis of team assignment can be summarized by the set of the team's observation x^T for which the project proceeds. Thus, the first protocol is again labeled as $X_P = \{G\}$ and the second by $X_P = \{G, \emptyset\}$. It is routine to check that the firm's ex-ante payoff given wage contracts $\{w_{iF}, \Delta_{iC}, \Delta_{iS}\}$, $i = 1,2$, effort levels e_1 and e_2, and protocol X_P, is:

$$\begin{aligned} \Pi^T \quad &:= \Pr\left(x^T \in X_P\right) \times \\ &\left[\Pr\left(\omega = G \mid x^T \in X_P\right)\left(y - \sum_i \Delta_{iS}\right) + \Pr\left(\omega = B \mid x^T \in X_P\right)\left(-\mu \sum_i \Delta_{iS}\right)\right] \sum_k e_k \\ &\qquad + \Pr\left(x^T \notin X_P\right)\left[\underline{\pi} - \sum_i \Delta_{iC}\right] - \sum_i w_{iF}. \end{aligned}$$

Agent i's participation constraint requires:

$$(IR_i^T) \qquad \begin{aligned} U_i^T := \; & \Pr\left(x^T \in X_P\right) \times \\ & \left[\Pr\left(\omega = G \mid x^T \in X_P\right) + \mu \Pr\left(\omega = B \mid x^T \in X_P\right)\right] \Delta_{iS} \sum_k e_k \\ & \qquad + \Pr\left(x^T \notin X_P\right) \Delta_{iC} + w_{iF} - \tfrac{1}{2} e_i^2 \quad \geq 0. \end{aligned}$$

In the optimal contracting problem, there are two key differences relative to the case of individual assignment. The first one relates to the incentive constraints that ensure it is optimal for the principal to proceed with the project if x^T is in X_P and to cancel it otherwise:

$$(IC_P^T\text{-}1) \qquad \begin{aligned} & \left[\Pr\left(\omega = G \mid x^T \in X_P\right) \left(y - \sum_i \Delta_{iS}\right)\right. \\ & \qquad \left. + \Pr\left(\omega = B \mid x^T \in X_P\right) \left(-\mu \sum_i \Delta_{iS}\right)\right] \sum_k e_k \geq \underline{\pi} - \sum_i \Delta_{iC}, \end{aligned}$$

and

$$(IC_P^T\text{-}2) \qquad \begin{aligned} & \left[\Pr\left(\omega = G \mid x^T \notin X_P\right) \left(y - \sum_i \Delta_{iS}\right)\right. \\ & \qquad \left. + \Pr\left(\omega = B \mid x^T \notin X_P\right) \left(-\mu \sum_i \Delta_{iS}\right)\right] \sum_k e_k \leq \underline{\pi} - \sum_i \Delta_{iC}. \end{aligned}$$

In contrast to the case of individual assignment, the project's success and cancellation both would require the principal to pay the corresponding wage premium to both agents. The need for such "double payment" generates diseconomies of scope in incentive provision under team assignment that, as discussed below in more detail, may affect her ability to provide incentives.

The second difference emerges in the agents' incentive compatibility constraints. As before, the constraint would require that neither of the two agents can gain by unilaterally deviating to a different effort choice and reporting policy. However, under team assignment, an agent chooses the effort in only one of the two tasks, and reports only one of the two signals on the project's underlying state. Thus, an agent cannot fully influence the project's output and the associated performance measure, nor can he fully control the information on the underlying state that may be communicated to the principal.

Let $U_i\left(e_i, \rho_i; e_{-i}, \rho_{-i}\right)$ be the agent $\mathcal{A}_i$'s payoff given the two agents' efforts and reporting policies (fixing the wage contracts and the principal's continuation policy). The agent's on-path payoff U_i^T must be the largest payoff attainable for any feasible choice of effort profile and reporting policy (given the other agent's equilibrium effort and reporting policy). So, the constraint requires:

$$U_i^T = \max_{e_i', \rho_i'} \; U_i\left(e_i', \rho_i'; e_{-i}, \rho_{-i}\right). \tag{2}$$

As before, it is sufficient to consider only two types of deviation: (i) the agent follows his equilibrium reporting policy ρ_i but deviates on his effort level, (ii) the agent reports $\emptyset$ regardless of his observation, and chooses his effort level accordingly. We again denote the latter reporting policy (given in (ii)) as $\rho_\emptyset$. Thus, the incentive compatibility constraint (2) for agent $\mathcal{A}_i$ $(i = 1, 2)$ is equivalent to the following two conditions:

$$e_i = \arg\max_{e_i'} U_i\left(e_i', \rho_i; e_{-i}, \rho_{-i}\right) \tag{2a}$$

and

$$U_i^T \geq \max_{e_i'} \; U_i\left(e_i', \rho_\emptyset; e_{-i}, \rho_{-i}\right). \tag{2b}$$

Now, $(2a)$ implies that e_i satisfies the following first-order condition: for $i = 1, 2$,

$$e_i = \Pr\left(x^T \in X_P\right)\left[\Pr\left(\omega = G \mid x^T \in X_P\right) + \mu \Pr\left(\omega = B \mid x^T \in X_P\right)\right] \Delta_{iS}. \tag{$IC_{A_i}^T$-1}$$

Also, $(2b)$ can be simplified in the same fashion in which we streamlined its counterpart under individual assignment. However, one needs to account for the fact that under team assignment, an agent's attempt to conceal information may be undermined by the report of the other agent. In parallel to our analysis of individual assignment, let $p_\emptyset^T$ be the probability that the project continues when agent i deviates to the reporting policy $\rho_\emptyset$, given the equilibrium communication protocol. Also, denote

$$p^T := \Pr\left(x^T \in X_P\right),$$

and

$$\begin{aligned} P^T &:= \Pr\left(\omega = G \mid x^T \in X_P\right) + \mu \Pr\left(\omega = B \mid x^T \in X_P\right), \\ P_\emptyset^T &:= \Pr\left(\omega = G \mid \rho_\emptyset, \rho_{-i}, \mathcal{C}\right) + \mu \Pr\left(\omega = B \mid \rho_\emptyset, \rho_{-i}, \mathcal{C}\right), \end{aligned}$$

where $\Pr\left(\omega \mid \rho_\emptyset, \rho_{-i}, \mathcal{C}\right)$ denotes the probability of the state ω conditional on the event that the project proceeds under the communication protocol $\left\{\rho_\emptyset, \rho_{-i}, \mathcal{C}\right\}$. Now, plugging in the agent's on- and off-path payoffs, condition $(2b)$ can be stated as:

$$(IC^T_{A_i}\text{-}2) \qquad \tfrac{1}{2}\left[\left(p^T P^T\right)^2 - \left(p^T_\emptyset P^T_\emptyset\right)^2\right]\Delta^2_{iS} + \left[\left(p^T P^T\right)^2 - \left(p^T_\emptyset P^T_\emptyset\right)\left(p^T P^T\right)\right]\Delta_{iS}\Delta_{-iS} \geq \left(p^T - p^T_\emptyset\right)\Delta_{iC}.$$

Thus, the optimal wage contract under team assignment that supports a communication protocol given by $X_P \in \{\{G\}, \{G, \emptyset\}\}$ solves the following program:

$$\mathcal{P}^T: \max_{\{w_{iF}, \Delta_{iC}, \Delta_{iS}\}, e_i:\ i=1,2} \Pi^T \ \ s.t. \ \left(IR^T_i\right), \ \left(IC^T_P\text{-}1\right), \ \left(IC^T_P\text{-}2\right), \ \left(IC^T_{A_i}\text{-}1\right), \text{ and } \left(IC^T_{A_i}\text{-}2\right).$$

Lemma 4. *The following holds:*

(i) If $\mathcal{P}^T$ admits a solution, it also admits a symmetric solution where both agents are offered the same contract, i.e., $w_{1F} = w_{2F} = w_F$, $\Delta_{1S} = \Delta_{2S} = \Delta_S$ and $\Delta_{1C} = \Delta_{2C} = \Delta_C$.

(ii) The program $\mathcal{P}^T$ always admits a solution for $X_P = \{G, \emptyset\}$ and admits a solution for $X_P = \{G\}$ if and only if both α and μ are sufficiently large.

When the program $\mathcal{P}^T$ admits a solution for both $X_P = \{G\}$ and $X_P = \{G, \emptyset\}$, the optimal contract induces the communication protocol under which the value of the program $\mathcal{P}^T$ is larger. As is the case under individual assignment, when $\mathcal{P}^T$ admits no solution for $X_P = \{G\}$, such communication protocol cannot be sustained in a PBE, and the optimal contract induces communication protocol $X_P = \{G, \emptyset\}$.[8]

5. Optimal job design

By comparing the principal's payoffs associated with the optimal contracts under team and individual assignment, we now characterize the optimal job design.

Proposition 2. ***(Optimal job design)*** *There exist two thresholds μ_0 and μ_1 (given α), $\mu_0 < \mu_1$, such that:*

[8]The reason why a solution need not always exist for $X_P = \{G\}$ is similar to that mentioned in the context of individual assignment (Lemma 2). However, under team assignment, nonexistence of solution may also arise due to the diseconomies of scope effect that tighten the principal's incentive compatibility constraints. When μ is small, this effect is more pronounced, and a contract that induces the agents to exert effort and follow the communication protocol violates the incentive constraint of the principal.

(i) If $\mu < \mu_0$, the optimal job design is individual assignment with the following communication protocol: the agent reports B if he observes the state to be B, and reports $\emptyset$ otherwise; the principal proceeds with the project unless the report is B. The associated optimal contract is efficient and the principal's payoff is S^* (as defined in Proposition 1).

(ii) If $\mu > \mu_1$, the optimal job design is team assignment with the following communication protocol: each agent reports B if he observes the state to be B, and reports $\emptyset$ otherwise; the principal proceeds with the project if no agent reports B. The associated optimal contract is efficient and the principal's payoff is S^*.

(iii) If $\mu_0 \le \mu \le \mu_1$, the principal is indifferent between team and individual assignments: both designs, along with their corresponding communication protocol as stated in parts (i) and (ii), yield the same payoff of $S^*$to the principal.

Moreover, the parameter thresholds μ_0 and μ_1 vary with α in the following manner.

Proposition 3. ***(Comparative statics)*** *(i) The threshold μ_0 is increasing in α. (ii) There exists a cutoff $\alpha^* \in \left(1 - \frac{1}{\sqrt{2}}, 1\right)$ such that $\mu_1 = 1$ for $\alpha \le \alpha^*$ and μ_1 is strictly decreasing in α for $\alpha \ge \alpha^*$.*

Propositions 2 and 3 (illustrated in Figure 1) show how the optimal job design is driven by the "availability" of the agents' signal (captured by α) and the "alignment" of the performance measure with the project's output (captured by μ). For low α (i.e., $\alpha \le \alpha^*$), individual assignment is always optimal: when μ is relatively small (i.e., $\mu < \mu_0$) individual assignment strictly dominates team assignment, otherwise (i.e., $\mu \ge \mu_0$) both designs yield the same (optimal) payoff. However, when α is large (i.e., $\alpha > \alpha^*$) team assignment can be strictly optimal: for μ sufficiently high (i.e., $\mu > \mu_1$), team assignment strictly dominates individual assignment, for low μ (i.e., $\mu < \mu_0$) individual assignment is strictly optimal, and for intermediate values of μ the two designs are payoff equivalent.

The intuition for this result is as follows. Recall that our setup highlights two key frictions. First, the principal lacks information on the project's viability and must elicit it from the agents. Second, even though the principal's continuation decision depends on the agents' information, she cannot commit to a continuation policy ex-ante, and her decision to continue the project must be sequentially rational. These two frictions give rise to a trade-off that drives the optimal job design: relative to individual assignment, team assignment facilitates information elicitation but suffers from diseconomies of scope in incentive provision which could weaken incentives due to the principal's lack of commitment power.

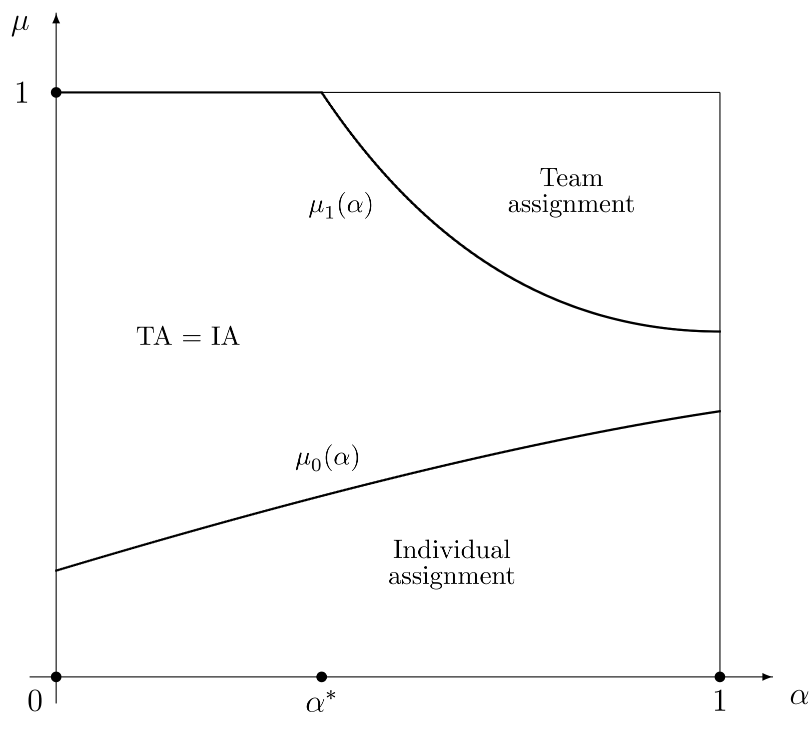

Figure 1: Optimal job design.

Team assignment helps in information elicitation as a single agent cannot fully control the outcome of the project and its associated performance measure. Even if an agent attempts to suppress information and adjusts his effort (in his assigned task) accordingly, his gains from such deviations are muted by the fact that his teammate may still reveal the information to the principal. Also, the agent cannot control the level of effort on the task that is performed by his teammate. However, under individual assignment an agent fully controls what the principal gets to learn about the project's underlying state and how much effort is exerted on both tasks of the project. Thus, a "double deviation" where an agent concurrently manipulates his reporting and effort level becomes more profitable for him under individual assignment than under team assignment. In fact, he stands to profit the most from it when both α and μ are large.

When α is large, the agent is more likely to observe the state, and therefore it is more valuable for him to have the option to fully control the project's continuation by hiding any unfavorable information. In particular, the agent has a strong incentive to conceal the bad state (and let the project continue) if he expects to earn a large payoff even if the project fails. This is indeed the case when μ is large, i.e., the performance measure is significantly misaligned with the project's outcome. When μ is large, in a bad state the measure is more likely to indicate success (given the effort levels) even though the project is sure to fail. Moreover, should the agent deviate from his reporting policy and hide the bad state, he may also exert more effort (vis-a-vis the on-path effort levels) so as to further increase his gains from deviation. Thus, when α and μ are both large, deterring the agent from a double-deviation becomes harder under individual assignment, and the advantage of team assignment over individual assignment in information elicitation becomes stronger.

However, team assignment lacks economies of scope in incentive provision. Under individual assignment a single wage premium for success ($\Delta_S := w_S - w_F$) incentivizes the agent to exert efforts on all tasks of the project. In contrast, in a team assignment, because each agent is assigned to exactly one of the two tasks, the principal needs to incentivize the two agents separately. Hence, relative to individual assignment, if the principal were to induce the same level of effort in both tasks of the project, the required wage premium doubles.

Such *dis*economies of scope may be costly to the principal. As the principal lacks commitment power over the continuation policy, her (IC_P) constraints must hold. That is, for any given job design with communication protocol given by X_P, (i) the principal's expected payoff from proceeding when the agents' observation is in X_P must be larger than her payoff from cancelling the project, and (ii) the payoff from cancelling must be larger than her expected payoff from proceeding with the project if the agents' observation is not in X_P. Thus, any feasible contract must ensure that the principal earns more from proceeding when the signal is in X_P than when it is not.

Under individual assignment, this requirement implies that the gain in output from continuing the project when the "news is good" relative to when the "news is bad" (i.e., $x \in X_P$ rather than $x \notin X_P$)

$$\left[\left[\Pr\left(\omega = G \mid x \in X_P\right) - \Pr\left(\omega = G \mid x \notin X_P\right) \right] \sum_k e_k \right] y, \tag{3}$$

is greater than the loss due to the increased wage payout

$$\left[(1-\mu) \left[\Pr\left(\omega = G \mid x \in X_P\right) - \Pr\left(\omega = G \mid x \notin X_P\right) \right] \sum_k e_k \right] \Delta_S. \tag{4}$$

In the case of team assignment, the only substantive difference is that in the above expression (4), Δ_S is replaced by $(\Delta_{1S} + \Delta_{2S})$ as the expected wage payout accounts for the fact that the wage premium for success must be paid twice.[9] Hence, the above condition is harder to satisfy under team assignment than under individual assignment (as $\Pr(\omega = G \mid x \in X_P) > \Pr(\omega = G \mid x \notin X_P)$ for any $X_P \in \{\{G\}, \{G, \emptyset\}\}$).

Note that team assignment's relative disadvantage (due to diseconomies of scope) becomes more acute when μ is low (i.e., the measure is well-aligned with the project's output). First, for a given effort level and wage premium for success, the loss due to increased wage payout decreases in μ. When μ is small, the expected wage payout becomes more sensitive to the underlying state, and hence the difference in wage payout conditional on "good" and "bad" news (expression (4)) increases, potentially offsetting the difference in output gains (expression (3)). Second, when μ is small, the agent is less likely to earn a reward for success when the state is bad, and the wage premium for success needs to be sufficiently large so as to incentivize him to exert the required level of effort. And if the principal needs to pay such large premiums twice—as is the case under team assignment—her continuation policy is less likely to remain credible: inducing high effort would compromise the project's continuation decision.

To sum, team assignment facilitates information elicitation but suffers from diseconomies of scope in incentive provision. For large α and μ, information elicitation gets harder under individual assignment but diseconomies of scope are less acute and do not distort incentive provision under team assignment. This is why team assignment strictly dominates individual assignment when α and μ are high. But when μ is small, provision of incentives under team assignment gets compromised due to severe diseconomies of scope, whereas incentives under individual assignment remain sharp as information elicitation is relatively easy. This explains why individual assignment dominates team assignment when μ is low.

Notice that when the job design is chosen optimally, the associated contract yields the efficient level of surplus as obtained in the public information benchmark (in Section 3.1). However, this observation critically hinges on our modeling assumption that the agents' observation on the state does not contain any noise (conditional on observing it in the first place). As we discuss in the next section, when the agent's signal is noisy, the optimal job design may entail inefficiencies both in the principal's continuation policy and in the agents' effort levels.

[9]The comparison of expected output and wage payment follows from the fact that the $(IC_P^I\text{-}1)$ and $(IC_P^I\text{-}2)$ are satisfied only when the left-hand side of the first is larger than the left-hand side of the second (similarly for (IC^T)s).

6. Discussion and conclusion

Although our model adopts a stylized information setup for analytical tractability, the key trade-off that we highlight here (between information elicitation and diseconomies of scope in incentive provision) may continue to shape the firm's job design decision in some related and more general settings. We consider two such extensions of our model. First, we relax the assumption that an informed agent observes the state without any noise, and assume that an agent's signal may be imprecise. Next, we relax the assumption that the observability of the underlying state of the project in each of its two tasks is statistically independent, and explore the case where they are mutually exclusive. We also discuss the implications of our results for the elicitation of "early warning signs" (EWS) in complex projects.

6.1. Imprecise signals.

In our model, the agent, conditional on observing the state, always observes it without any noise. Although this assumption improves the analytical tractability of the model, it is conceivable that the agents may not be able to directly observe the state but only acquire an imprecise signal on the same. How would our characterization of the optimal job design change if the agents' information were noisy?

In order to explore this issue, we consider the following modification to our model: Suppose that the state $\omega \in \{G, B\}$ associated with the project is never directly observed, but the agents may observe a signal $z \in \{G, B\}$ that is informative of ω. Let

$$\Pr(\omega = G \mid z = G) = \Pr(\omega = B \mid z = B) = \theta,$$

where $\theta \in (1/2, 1)$ reflects the precision of the signal. In parallel with the information structure of our model, we assume that an agent assigned to a task privately observes z with probability α. And we denote the agent $\mathcal{A}_i$'s observation of the signal z as $x_i \in \{G, B, \emptyset\}$, where $x_i = \emptyset$ if $\mathcal{A}_i$ does not observe z in any of his assigned tasks. We keep all other aspects of our model unaltered. Notice that our main model corresponds to the case where $\theta = 1$.

Though a complete characterization of the optimal job design for this case appears analytically intractable, the following proposition suggests that our main result is robust to some noise in the agents' signal.

Proposition 4. *There exists a threshold $\theta^* < 1$ such that for $\theta > \theta^*$, the qualitative characterization of the optimal job design is the same as its counterpart in our main model (as given in Proposition 2), and the optimal contract is always efficient.*

However, if the agents' signal becomes sufficiently noisy (i.e., when θ is sufficiently low) our main result may no longer hold. Recall that under the optimal contract in our main model, the project proceeds even when the agents fail to reveal their signal, i.e., the project continues unless the agent(s) report(s) a bad state. But when the agents' signal is sufficiently noisy, information elicitation becomes harder. An agent now has a stronger incentive to hide a bad signal and let the project pass, because with some probability, a bad signal may still be associated with a good state.

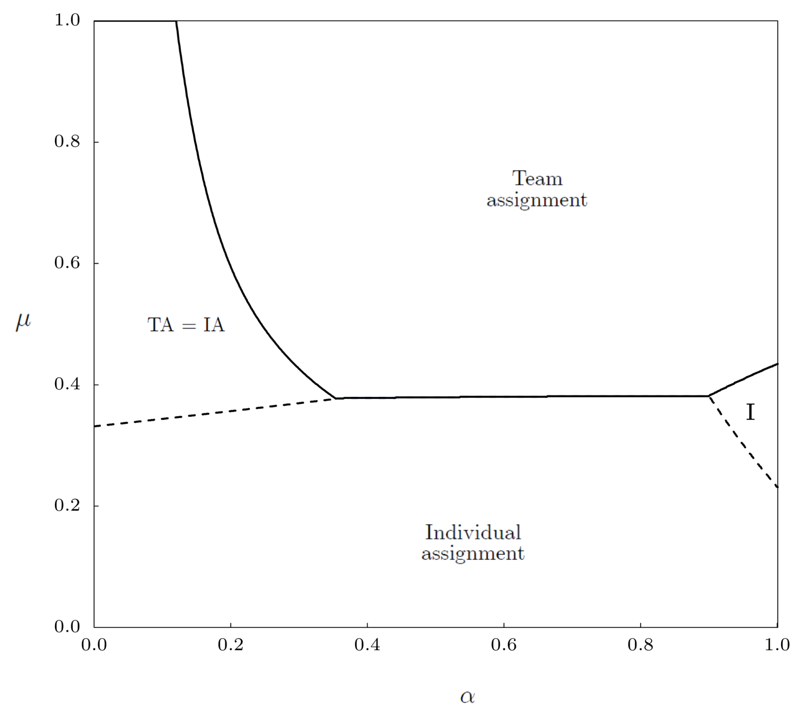

Figure 2: Optimal job design with imprecise signal ($\theta = 0.77$).
In region I individual assignment is optimal but the continuation decision is inefficient: the project continues only if the report is good.

This effect may introduce two sources of inefficiencies. First, the principal may reduce the effort incentives so as to mitigate the agent's incentive to hide a bad signal. Recall that as the agent's effort increases, the performance measure is more likely to indicate success. Thus when the efforts are high, the agent has stronger incentive to continue the project under a bad signal. Second, if such distortions to the effort level are too costly, the principal may also distort her continuation policy: the project may proceed only if the signal is good. And at the extreme, i.e., when θ is low enough, it is optimal for the principal to proceed with all projects without soliciting any information from the agents (or, equivalently, to settle for the outside option). These inefficiencies are illustrated in Figure 2, which presents a numerical solution for the optimal job design problem.

6.2. **Exclusive signals.** So far, we have assumed that the observability of the underlying state of the project in each of its two tasks is statistically independent. Such a setup may reflect a scenario where each task associated with the project gives access to a different (and independent) source of information, each of which may reveal the state ω with probability α. But it is conceivable that the informativeness of these sources may not be independent. In this subsection, we focus on one such scenario: sources being mutually exclusive in terms of their informativeness. An exploration of this case further illustrates how the agents' ability to control the outcome of the project through their efforts may affect the optimal job design.

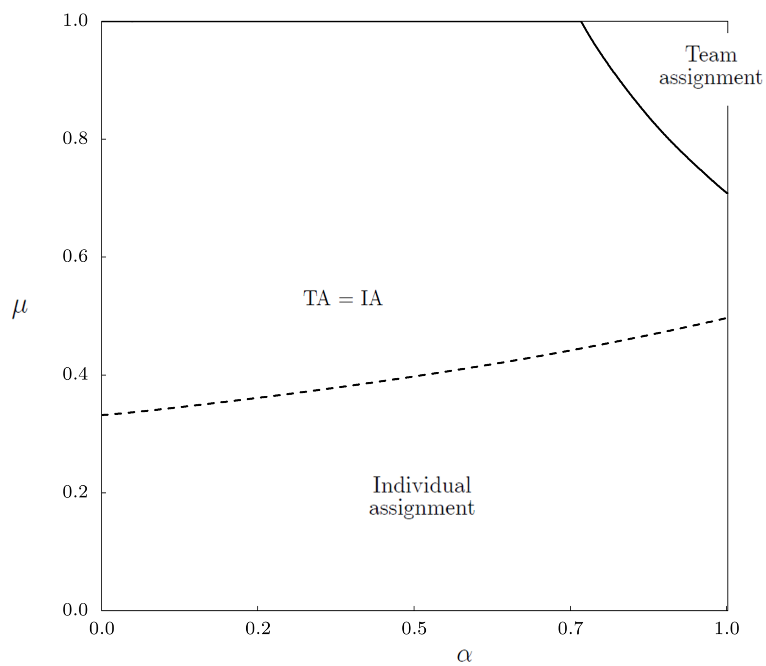

Figure 3: Optimal job design under mutually exclusive signals across tasks.

To formalize this idea, we make the following modification to our model. We assume that exactly one of the two tasks associated the project may yield information about its underlying state, and a priori the players do not know which task is informative. In particular, with probability 1/2, only task T_1 can yield information: the agent performing task T_1 observes the state with probability α, whereas the agent performing T_2 never observes it. And with probability 1/2, only T_2 is informative: the agent performing task T_1 never observes the state, whereas the agent performing T_2 observes it with probability α. We keep all other aspects of the model unchanged.

In this setup, under individual assignment, the probability that agent $\mathcal{A}_1$ observes the state of the project is α. And this is also the probability that under team assignment at least one of the two agents observes the state. However, in this setting, team assignment appears to lose its advantage in information elicitation: because the observability of the state is mutually exclusive between tasks, should an agent observe an unfavorable information he can completely suppress it, as his teammate would necessarily be uninformed.

One may anticipate that such complete control over the information on the state may make team assignment suboptimal to individual assignment, as team assignment still continues to suffer from diseconomies of scope in incentive provision. However, this intuition is incomplete. Recall that an agent controls the outcome of the project in two ways: through his reporting on the state that affects the project's continuation probability, and also through his effort(s) that affect(s) the project's output and the performance metric (should the project proceed). When the signals are mutually exclusive, the advantage of team assignment in muting the former channel is indeed diminished. However, team assignment may still help information elicitation as the agent cannot control the effort in all tasks that are associated with the project. Numerical results suggest (see Figure 3) that team assignment's advantage in information elicitation remains sufficiently strong even under mutually exclusive signals and, as in our main model, it may still dominate individual assignment when both α and μ are sufficiently large.

6.3. **Implications for eliciting EWS.** As mentioned in the Introduction, our analysis may be particularly relevant for complex project management where elicitation of "early warning signs" (EWS) is a matter of paramount importance. The extant literature focuses on certain barriers to detection of EWS that are exogenous to the production environment, such as behavioral biases, excessive reliance on formal assessments despite their limited scope, and sheer complexity of the project that makes EWS hard to foresee (Haji-Kazemi et al. 2013).

Our article complements this literature by highlighting that such barriers can emerge endogenously as incentives for effort provision and truthful reporting become intertwined when the project managers cannot commit to how they might act on the observed EWS. Thus, the provision of effort incentives in this context must strike a delicate balance between effort inducement and information elicitation without compromising the firm's project continuation policy.

Our findings illustrate how the problem of effective detection and use of EWS can be mitigated through strategic job design. The optimal design is shaped by a trade-off between information elicitation and diseconomies of scope in incentive provision. To ascertain the optimal design for a given project, the firm ought to consider two important aspects of its production environment: how likely it is that the workers would observe the EWS and how well-aligned the workers' performance measures are with the firm's objective. For example, when the workers are quite likely to observe the EWS but their performance measure is not well-aligned with the firm's value, information elicitation from the workers becomes harder and team assignment becomes the preferred design. However, when the firm has access to performance measures that can fairly reflect its own objectives, individual assignment is the optimal job design.

6.4. **Conclusion.** When effective managerial decision-making requires local information, the incentive structure in an organization must meet two goals at once: induce the workers to exert costly effort, and induce the workers to truthfully report their information, even if the information may be detrimental to their own interest. This article explores how such intertwined incentives may be sharpened through job design, i.e., the allocation of tasks among the workers. We argue that the optimal job design is shaped by a novel trade-off between the ease of information elicitation and diseconomies of scope in incentive provision. And this trade-off, in turn, is driven by the interplay between the "availability" of the workers' information and the "alignment" of their performance measure with the firm's objective. Our analysis sheds light on real-world settings where the employees working on a project are likely to learn information about its future viability, and such information is critical for managerial decision-making. Our results also suggest a novel explanation of why team assignment can offer better incentives even when measures of individual performance remain available.

7. Appendix

This appendix contains the proofs omitted in the text.

Proof of Proposition 1. Consider first the case of individual assignment. The optimal contracting program given continuation policy X_P:

$$\max_{\mathbf{e},w_1^I(\cdot)} \Pi^I := \mathbb{E}\left[Y - w_1^I(M) \mid \mathbf{e},\ X_P\right]$$

$$s.t.$$

$$\mathbf{e} = \arg\max_{\mathbf{e}'} U_1^I(\mathbf{e}', X_P) \qquad (IC^I)$$

$$U_1^I(\mathbf{e}, X_P) \geq 0 \qquad (IR^I)$$

By standard argument, (IR^I) must bind in any solution to the problem. Furthermore, as long as X_P is nonempty, which means that the project proceeds with a strictly positive probability, any effort profile can be implemented (i.e., made to satisfy the $\left(IC^I\right)$) by choosing the wage schedule $w_1^I(M)$ appropriately. Thus, the program boils down to:

$$\max_{\mathbf{e}} \quad \mathbb{E}\left[Y \mid \mathbf{e}, X_P\right] - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2.$$

Denote $\pi(X_P) := \max_{\mathbf{e}} \ \mathbb{E}\left[Y \mid \mathbf{e}, X_P\right] - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$, and it is routine to check that

$$\pi(X_P) := \begin{cases} (1+(2\alpha-\alpha^2)(2\alpha-\alpha^2-\frac{1}{2}))\underline{\pi} & if \ X_P = \{G\} \\ (1+\alpha-\frac{1}{2}\alpha^2)\underline{\pi} & if \ X_P = \{G,\emptyset\} \\ \underline{\pi} & if \ X_P = \{G,\emptyset,B\} \end{cases}.$$

It is also routine to check that any other continuation policy is dominated by $X_P = \{G\}$, $X_P = \{G,\emptyset\}$, or $X_P = \{G,\emptyset,B\}$. Comparing the values, we obtain that $X_P = \{G,\emptyset\}$ is optimal. That is, under individual assignment, in the optimal contract the principal proceeds with the project if and only if the bad state is not observed, and obtains payoff $S^* = \left(1+\alpha-\frac{1}{2}\alpha^2\right)\underline{\pi}$ from the project.

Similarly, under team assignment, the optimal contracting program given continuation policy X_P is:

$$\max_{\mathbf{e},\{w_1^T(\cdot),w_2^T(\cdot)\}} \quad \Pi^T := \mathbb{E}\left[Y - \left(w_1^T(M) + w_2^T(M)\right) \mid \mathbf{e}, X_P\right]$$

$$s.t.$$

$$e_i = \arg\max_{e_i'} \ U_i^T\left(e_i', e_{-i}, X_P\right), \quad i = 1,2 \qquad \left(IC^T\right)$$

$$U_i^T\left(e_i, e_{-i}, X_P\right) \geq 0, \quad i = 1,2 \qquad \left(IR^T\right)$$

As in the case of individual assignment, $\left(IR^T\right)$ must bind for both agents, any effort profile can be induced by an appropriate choice of wage contracts, and the program boils down to:

$$\max_{e_1,e_2} \ \mathbb{E}\left[Y - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 \mid e_1, e_2, X_P\right].$$

Thus, given X_P, the principal's payoff is exactly the same as that in the case of individual assignment, and so the claim follows. □

Proof of Lemma 1. Recall that a reporting policy for the agent is a mapping $\rho : \{G,\emptyset,B\} \to \{G,\emptyset,B\}$ that associates a report r to an observation x. As the agent's observation is hard information, the reporting policy must satisfy $\rho(\emptyset) = \emptyset$ and $\rho(x) \in \{x,\emptyset\}$ for all $x \in \{G,B\}$. Similarly, a continuation policy for the principal is a mapping $\mathcal{C} : \{G,\emptyset,B\} \to \{cancel, proceed\}$ that associates a report r from the agent to a decision on the continuation of the project.

Trivially, the continuation policy $\mathcal{C}(r) = \textit{cancel}$ for all $r \in \{G, \emptyset, B\}$ yields a payoff of $\underline{\pi}$ to the principal (she gets her outside option); and the continuation policy $\mathcal{C}(r) = \textit{proceed}$ for all $r \in \{G, \emptyset, B\}$ also yields $\underline{\pi}$ (by Assumption 1). Thus, in any PBE where the principal's payoff exceeds $\underline{\pi}$ (if it exists), the ex-ante probability that the project continues must be strictly between 0 and 1. Fix one such PBE, chosen arbitrarily. We present a proof of our claim in the following two steps.

Step 1. *We can restrict attention to only two continuation policies.*

Let Π_ω^P denote the expected payoff to the principal from proceeding with the project given that the state is ω (for all $\omega = G, B$); and let Π^C denote the payoff to the principal from cancelling the project. Hence, given a report r from the agent, the principal's expected payoff if she proceeds with the project is given by

$$\Pi^P(r) := \Pr(\omega = G \mid r)\, \Pi_G^P + \Pr(\omega = B \mid r)\, \Pi_B^P.$$

Now, observe that we cannot have $\Pi_G^P > \Pi^C$ and $\Pi_B^P > \Pi^C$, otherwise $\Pi^P(r) > \Pi^C$ for all $r \in \{G, \emptyset, B\}$ and the principal would always proceed with the project. Similarly, we cannot have $\Pi_G^P \leq \Pi^C$ and $\Pi_B^P \leq \Pi^C$, otherwise $\Pi^P(r) \leq \Pi^C$ for all $r \in \{G, \emptyset, B\}$ and the principal would always cancel the project. (As a tie-breaking rule, assume that the principal cancels the project when she is indifferent between proceeding with the project and canceling it.) Hence, we must have:

$$\max\{\Pi_G^P, \Pi_B^P\} > \Pi^C \geq \min\{\Pi_G^P, \Pi_B^P\}. \tag{5}$$

This condition implies that $\Pi_G^P \neq \Pi_B^P$, and we can assume that $\Pi_G^P > \Pi_B^P$. (In equilibria where $\Pi_G^P \leq \Pi_B^P$, the probability that the state is G when the project proceeds is weakly lower than the prior, 1/2, and by Assumption 1 such equilibria cannot generate a payoff to the principal higher than $\underline{\pi}$ while satisfying the agent's participation constraint.)

As the agent cannot misreport his observation, we must have (in any equilibrium)

$$\Pr(\omega = G \mid r = G) > \Pr(\omega = G \mid r = \emptyset) > \Pr(\omega = G \mid r = B).$$

These inequalities, together with the facts that (i) $\Pi_G^P > \Pi_B^P$ and (ii) in equilibrium the ex-ante probability that the project continues is strictly between 0 and 1, imply that

$$\Pi^P(r = G) > \Pi^C \geq \Pi^P(r = B).$$

But this implies that the continuation policy of the principal must satisfy $\mathcal{C}(G) = \textit{proceed}$ and $\mathcal{C}(B) = \textit{cancel}$. And there are only two continuation polices that satisfy these conditions: (i) $\mathcal{C}(r) = \textit{proceed}$ if and only if $r = G$, and (ii) $\mathcal{C}(r) = \textit{proceed}$ if and only if $r \in \{G, \emptyset\}$.

Thus, without loss of generality, we can focus on equilibria that support only one of these two continuation policies.

Step 2. *For each of the two continuation policies stated in Step 1, we can restrict attention to only one reporting policy.*

Step 2a. Suppose that the principal's continuation policy is (i), i.e., $\mathcal{C}(r) = \textit{proceed}$ if and only if $r = G$. As in equilibrium the ex-ante probability that the project proceeds is strictly positive, the reporting policy must satisfy $\rho(G) = G$. Thus, two reporting strategies are possible depending on what the agent reports when $x = B$: $\rho(G) = G$, $\rho(\emptyset) = \emptyset$, $\rho(B) = B$ and $\rho(G) = G$, $\rho(\emptyset) = \emptyset$, $\rho(B) = \emptyset$. Both yield the same payoff and induce the same probability that the project proceeds. However, the latter policy relaxes the principal's incentive constraints relative to the former one as

$$\Pr(\omega = G \mid x = \emptyset) \geq \Pr(\omega = G \mid x \in \{\emptyset, B\}) \geq \Pr(\omega = G \mid x = B).$$

Thus, if there exists a PBE where the continuation policy (i) is played, then there also exists a payoff equivalent PBE where the associated reporting policy is $\rho(G) = G$ and $\rho(x) = \emptyset$ if $x \in \{\emptyset, B\}$.

Step 2b. Now suppose that the principal's continuation policy is (ii), i.e., $\mathcal{C}(r) = \textit{proceed}$ if and only if $r \in \{G, \emptyset\}$, is played. As in equilibrium the ex-ante probability that the project proceeds is strictly smaller than one, the reporting policy must satisfy $\rho(B) = B$. Thus, two reporting strategies are possible depending on what the agent reports when $x = G$: $\rho(G) = G$, $\rho(\emptyset) = \emptyset$, $\rho(B) = B$ and $\rho(G) = \emptyset$, $\rho(\emptyset) = \emptyset$, $\rho(B) = B$. As in the previous step, the two reporting strategies yield the same payoff and induce the same probability that the project proceeds. But the policy $\rho(B) = B$ and $\rho(x) = \emptyset$ if $x \in \{G, \emptyset\}$ relaxes the incentive constraints relative to the policy $\rho(x) = x$ for all $x \in \{G, B\}$. Thus, if there exists a PBE where the continuation policy (ii) is played, then there also exists a payoff equivalent PBE where the associated reporting strategy is $\rho(B) = B$ and $\rho(x) = \emptyset$ if $x \in \{\emptyset, G\}$.

Steps 1 and 2, taken together, imply that if there exists a PBE where the principal's payoff exceeds her outside option, then there also exists a payoff equivalent PBE where the associated communication protocol is one of the following: (i) If the state is observed to be G, report G, otherwise report $\emptyset$; proceed with the project if and only if $r = G$. (ii) If the state is observed to be B, report B, otherwise report $\emptyset$; proceed with the project if and only if $r \neq B$. □

Proof of Lemma 2. For brevity, we rewrite the objective function and all the constraints of program $\mathcal{P}^I$ using the notations p^I, $p^I_\emptyset$, P^I and $P^I_\emptyset$ (as defined in Section 4.1) and $P^I_C := \Pr(\omega = G \mid x \notin X_P) + \mu \Pr(\omega = B \mid x \notin X_P)$. The program $\mathcal{P}^I$ boils down to:

$$\begin{aligned}
\max_{\substack{w_F,\Delta_C,\Delta_S,\\ e_1,e_2}} \quad & \Pi^I := p^I \left[\Pr(\omega = G \mid x \in X_P)y - P^I\Delta_S\right] \sum_k e_k \\
& +(1-p^I)[\underline{\pi} - \Delta_C] - w_F \\
& s.t. \\
& p^I P^I \Delta_S \sum_k e_k + (1-p^I)\Delta_C + w_F - \tfrac{1}{2}\sum_k e_k^2 \geq 0 && (IR^I) \\
& [\Pr(\omega = G \mid x \in X_P)y - P^I\Delta_S] \sum_k e_k \geq \underline{\pi} - \Delta_C && (IC^I_P\text{-}1) \\
& [\Pr(\omega = G \mid x \notin X_P)y - P^I_C\Delta_S] \sum_k e_k \leq \underline{\pi} - \Delta_C && (IC^I_P\text{-}2) \\
& e_k = p^I P^I \Delta_S, \quad k = 1,2 && (IC^I_A\text{-}1) \\
& \left[\left(p^I P^I\right)^2 - \left(p^I_\emptyset P^I_\emptyset\right)^2\right] \Delta_S^2 \geq \left(p^I - p^I_\emptyset\right)\Delta_C. && (IC^I_A\text{-}2)
\end{aligned}$$

Recall that the program $\mathcal{P}^I$ ignores the bounds on e_k but we will show below that at the optimal contract $e_k \in (0, 1/2)$ and hence, the use of first-order condition in $\left(IC^I_A\text{-}1\right)$ and $\left(IC^I_A\text{-}2\right)$ does not entail any loss of generality.

By a standard argument, we obtain that $\left(IR^I\right)$ must bind in any solution to the problem. Using $\left(IR^I\right)$ and $(IC^I_A\text{-}1)$, we can eliminate w_F and the e_is, and the program further simplifies to:

$$\begin{aligned}
\max_{\Delta_C,\Delta_S} \quad & 2\left(p^I\right)^2 P^I \Pr(\omega = G \mid x \in X_P)y\Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I \Delta_S\right)^2 \\
& s.t. \\
& [\Pr(\omega = G \mid x \in X_P)y - P^I\Delta_S]\left(2p^I P^I \Delta_S\right) \geq \underline{\pi} - \Delta_C && (IC^I_P\text{-}1) \\
& [\Pr(\omega = G \mid x \notin X_P)y - P^I_C\Delta_S]\left(2p^I P^I \Delta_S\right) \leq \underline{\pi} - \Delta_C && (IC^I_P\text{-}2) \\
& \left[\left(p^I P^I\right)^2 - \left(p^I_\emptyset P^I_\emptyset\right)^2\right] \Delta_S^2 \geq \left(p^I - p^I_\emptyset\right)\Delta_C && (IC^I_A\text{-}2)
\end{aligned}$$

Case 1: Consider first the case where $X_P = \{G, \emptyset\}$. In this case, $p^I_\emptyset = 1$. Using this fact and rearranging the constraints, the program becomes:

$$\mathcal{P}^I_{\{G,\emptyset\}} : \begin{cases}
\max_{\Delta_C,\Delta_S} \quad 2\left(p^I\right)^2 P^I \Pr(\omega = G|x \in X_P)y\Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I \Delta_S\right)^2 \\
\qquad s.t. \\
\Delta_C \geq l_P := \underline{\pi} - [\Pr(\omega = G|x \in X_P)y - P^I\Delta_S]\left(2p^I P^I \Delta_S\right) & (IC^I_P\text{-}1) \\
\Delta_C \leq u_P := \underline{\pi} - [\Pr(\omega = G|x \notin X_P)y - P^I_C\Delta_S]\left(2p^I P^I \Delta_S\right) & (IC^I_P\text{-}2) \\
\Delta_C \geq l_A := \left[\left(P^I_\emptyset\right)^2 - \left(p^I P^I\right)^2\right] \frac{\Delta_S^2}{1-p^I} & (IC^I_A\text{-}2)
\end{cases}$$

As Δ_C does not appear in the objective function, we can rewrite the program as:

$$\max_{\Delta_S} \quad 2\left(p^I\right)^2 P^I \Pr(\omega = G \mid x \in X_P) y \Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I \Delta_S\right)^2$$

$$s.t.$$

$$u_P \geq l_A \Leftrightarrow \underline{\pi} \geq \left[2p^I \Pr(\omega = G \mid x \notin X_P) P^I y\right] \Delta_S - \left[2p^I P^I P_C^I - \frac{\left(P_\emptyset^I\right)^2 - \left(p^I P^I\right)^2}{1-p^I}\right] \Delta_S^2 \ ,$$

$$u_P \geq l_P \Leftrightarrow \Delta_S \leq \tfrac{y}{1-\mu}$$

where we use the fact that $P^I - P_C^I = (1-\mu)[\Pr(\omega = G \mid x \in X_P) - \Pr(\omega = G \mid x \notin X_P)]$ and $\Pr(\omega = G \mid x \in X_P) > \Pr(\omega = G \mid x \notin X_P)$ when $X_P = \{G, \emptyset\}$ to simplify the constraint $u_P \geq l_P$.

By routine calculation one obtains that $\Pr(\omega = G \mid x \in X_P) = 1/\left(2-\alpha'\right)$, $\Pr(\omega = G \mid x \notin X_P) = 0$, and

$$p^I = 1 - \frac{1}{2}\alpha', \ P^I = \frac{1}{2-\alpha'} + \mu\left(1 - \frac{1}{2-\alpha'}\right), \ P_C^I = \mu,$$

where $\alpha' := 1 - (1-\alpha)^2$. Also, we can plug in $\underline{\pi} = \frac{1}{4}y^2$. So, the program $\mathcal{P}^I$ boils down to:

$$\max_{\Delta_S} \quad -\left[\tfrac{1}{2}(1+\mu(1-\alpha'))\Delta_S - \tfrac{1}{2}y\right]^2 + \left(\tfrac{1}{4} + \tfrac{1}{8}\alpha'\right)y^2$$

$$s.t. \qquad .$$

$$\tfrac{1}{4}y^2 \geq \tfrac{1}{2}\alpha'\left(\mu\Delta_S\right)^2 \text{ and } \tfrac{y}{1-\mu} \geq \Delta_S$$

Notice that the objective function is strictly concave with peak at $\Delta_S = \frac{y}{1+\mu(1-\alpha')}$ and the feasible set is always non-empty. Thus the solution always exists and is given by

$$\Delta_S^* = \begin{cases} \frac{y}{1+\mu(1-\alpha')} & \text{if } \frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \frac{1}{2} \\ \frac{y}{\mu\sqrt{2\alpha'}} & \text{otherwise} \end{cases}.$$

The associated value is:

$$(6) \qquad V_{\{G,\emptyset\}}^I = \begin{cases} \left(\frac{1}{4} + \frac{1}{8}\alpha'\right)y^2 & \text{if } \frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \frac{1}{2} \\ \left(\frac{1}{4} + \frac{1}{8}\alpha'\right)y^2 - \frac{1}{4}\left[\frac{y}{\mu\sqrt{2\alpha'}}(1+\mu(1-\alpha')) - y\right]^2 & \text{otherwise} \end{cases}.$$

Finally, we check whether the effort choice is an interior point for $\Delta_S = \Delta_S^*$. On path, for $k = 1, 2$, $e_k = p^I P^I \Delta_S^* = \frac{1}{2}y \leq \frac{1}{4}$. And off-path, the effort level is

$$e_k' = \frac{p_\emptyset^I P_\emptyset^I}{p^I P^I}\left(\frac{1}{2}y\right) = \frac{1+\mu}{1+\mu(1-\alpha')}\left(\frac{1}{2}y\right) \leq y \leq \frac{1}{2}$$

as $\frac{1+\mu}{1+\mu(1-\alpha')} \leq 2$. Hence the effort choice is always in the interior of $[0, 1/2]$.

Case 2: Consider now the case where $X_P = \{G\}$. In this case $p^I_\emptyset = 0$, and the program becomes:

$$\mathcal{P}^I_{\{G\}}: \begin{cases} \max_{\Delta_C,\Delta_S} \quad 2\left(p^I\right)^2 P^I \Pr(\omega = G \mid x \in X_P) y \Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I \Delta_S\right)^2 \\ \qquad s.t. \\ \Delta_C \geq l_P := \underline{\pi} - \left[\Pr(\omega = G \mid x \in X_P) y - P^I \Delta_S\right]\left(2p^I P^I \Delta_S\right) \qquad (IC^I_P\text{-}1) \\ \Delta_C \leq u_P := \underline{\pi} - \left[\Pr(\omega = G \mid x \notin X_P) y - P^I_C \Delta_S\right]\left(2p^I P^I \Delta_S\right) \qquad (IC^I_P\text{-}2) \\ \Delta_C \leq u_A := p^I \left(P^I\right)^2 \Delta_S^2 \qquad (IC^I_A\text{-}2) \end{cases} .$$

As in Case 1, Δ_C does not enter into the objective function, and we can further simplify the program as:

$$\begin{gathered} \max_{\Delta_S} \quad 2\left(p^I\right)^2 P^I \Pr(\omega = G \mid x \in X_P) y \Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I \Delta_S\right)^2 \\ s.t. \\ l_P \leq u_A \Leftrightarrow \underline{\pi} \leq \left[2p^I \Pr(\omega = G \mid x \in X_P) P^I y\right]\Delta_S - \left[p^I \left(P^I\right)^2\right]\Delta_S^2 \\ l_P \leq u_P \Leftrightarrow \Delta_S \leq \tfrac{y}{1-\mu} \end{gathered} ,$$

and plugging the values for the probabilities and for $\underline{\pi}$, we obtain:

$$\begin{gathered} \max_{\Delta_S} \quad \tfrac{1}{2}\alpha'^2 \Delta_S \left(y - \tfrac{1}{2}\Delta_S\right) + \tfrac{1}{4}(1 - \tfrac{1}{2}\alpha')y^2 \\ s.t. \\ \alpha' \Delta_S \left(y - \tfrac{1}{2}\Delta_S\right) \geq \tfrac{1}{4}y^2 \text{ and } \Delta_S \leq \tfrac{y}{1-\mu} \end{gathered} .$$

The feasible set is non-empty if and only if $\alpha' \geq 1/2$ (equivalently, $\alpha \geq 1 - 1/\sqrt{2}$), and the objective function is concave with peak at y. Thus, the solution of the program and the value would be:

$$\Delta_S^* = y \text{ and } V^I_{\{G\}} = \frac{1}{4}y^2 + \frac{1}{4}\alpha'\left(\alpha' - \frac{1}{2}\right)y^2 \text{ if } \alpha' \geq \frac{1}{2} \tag{7}$$

and no solution otherwise.

It is routine to check that the agent's effort choice is always in the interior of $[0, 1/2]$. Trivially, on path, for $k = 1, 2$, $e_k = p^I P^I \Delta_S^* = \frac{1}{2}\alpha' y \leq \frac{1}{4}$; and off-path, the effort level is $e'_k = \frac{p^I_\emptyset P^I_\emptyset}{p^I P^I}\left(\frac{1}{2}\alpha' y\right) = 0$. □

Proof of Lemma 4. The proof is similar to that of Lemma 2. We begin by writing the objective function and all the constraints of program $\mathcal{P}^T$ using the notations p^T, $p^T_\emptyset$, P^T and $P^T_\emptyset$ (as defined in Section 4.2) and $P^T_C := \Pr\left(\omega = G \mid x^T \notin X_P\right) + \mu \Pr\left(\omega = B \mid x^T \notin X_P\right)$.

The program boils down to:

$$\max_{\substack{\Delta_{iC},\Delta_{iS}\\ w_{iF},e_i}} \Pi^T = p^T\left[\Pr(\omega=G\mid x^T\in P)y - P^T\sum_i \Delta_{iS}\right]\sum_k e_k$$
$$+(1-p^T)\left[\underline{\pi}-\sum_i\Delta_{iC}\right]-\sum_i w_{iF}$$
$$s.t.\ \ \forall i\in\{1,2\}$$

$$p^TP^T\Delta_{iS}\sum_k e_k+(1-p^T)\Delta_{iC}+w_{iF}-\tfrac{1}{2}e_i^2\geq 0 \qquad (IR_i^T)$$
$$\left[\Pr(\omega=G\mid x^T\in X_P)y-P^T\sum_i\Delta_{iS}\right]\sum_k e_k\geq\underline{\pi}-\sum_i\Delta_{iC} \qquad (IC_P^T\text{-}1)$$
$$\left[\Pr(\omega=G\mid x^T\notin X_P)y-P_C^T\sum_i\Delta_{iS}\right]\sum_k e_k\leq\underline{\pi}-\sum_i\Delta_{iC} \qquad (IC_P^T\text{-}2)$$
$$e_i=p^TP^T\Delta_{iS} \qquad (IC_{A_i}^T\text{-}1)$$
$$\tfrac{1}{2}\left[\left(p^TP^T\right)^2-\left(p_\emptyset^TP_\emptyset^T\right)^2\right]\Delta_{iS}^2+\left[\left(p^TP^T\right)^2-\left(p_\emptyset^TP_\emptyset^T\right)\left(p^TP^T\right)\right]\Delta_{iS}\Delta_{-iS}\geq(p^T-p_\emptyset^T)\Delta_{iC} \qquad (IC_{A_i}^T\text{-}2)$$

We can eliminate w_{iF} and the e_is using (IR_i^T) (that must bind) and $(IC_{A_i}^T\text{-}1)$, and the program further simplifies to:

$$\max_{\Delta_{iC},\Delta_{iS}} \ \left(p^T\right)^2P^T\Pr(\omega=G\mid x^T\in X_P)y\sum_i\Delta_{iS}+(1-p^T)\underline{\pi}-\tfrac{1}{2}\left(p^TP^T\right)^2\sum_i\Delta_{iS}^2$$
$$s.t.$$
$$\left[\Pr(\omega=G\mid x^T\in X_P)y-P^T\sum_i\Delta_{iS}\right]p^TP^T\sum_i\Delta_{iS}\geq\underline{\pi}-\sum_i\Delta_{iC} \qquad (IC_P^T\text{-}1)$$
$$\left[\Pr(\omega=G\mid x^T\notin X_P)y-P_C^T\sum_i\Delta_{iS}\right]p^TP^T\sum_i\Delta_{iS}\leq\underline{\pi}-\sum_i\Delta_{iC} \qquad (IC_P^T\text{-}2)$$
$$\tfrac{1}{2}\left[\left(p^TP^T\right)^2-\left(p_\emptyset^TP_\emptyset^T\right)^2\right](\Delta_{1S})^2+\left[\left(p^TP^T\right)^2-p_\emptyset^TP_\emptyset^Tp^TP^T\right]\Delta_{1S}\Delta_{2S}\geq(p^T-p_\emptyset^T)\Delta_{1C} \qquad (IC_{A_1}^T\text{-}2)$$
$$\tfrac{1}{2}\left[\left(p^TP^T\right)^2-\left(p_\emptyset^TP_\emptyset^T\right)^2\right](\Delta_{2S})^2+\left[\left(p^TP^T\right)^2-p_\emptyset^TP_\emptyset^Tp^TP^T\right]\Delta_{1S}\Delta_{2S}\geq(p^T-p_\emptyset^T)\Delta_{2C} \qquad (IC_{A_2}^T\text{-}2)$$

Part (i). We now prove that if $\mathcal{P}^T$ admits a solution, it also admits a symmetric solution where $\Delta_{1S}=\Delta_{2S}=\Delta_S$ and $\Delta_{1C}=\Delta_{2C}=\Delta_C$. The proof is given in the following five steps.

Step 1: Suppose $\Delta^*:=(\Delta_{1S}^*,\Delta_{2S}^*,\Delta_{1C}^*,\Delta_{2C}^*)$ is a solution to $\mathcal{P}_T$. If $\Delta_{1S}^*=\Delta_{2S}^*=\Delta_G^*$ (say), we argue that there also exists a symmetric solution $(\Delta_S^*,\Delta_S^*,\Delta_C^*,\Delta_C^*)$ where

$$\Delta_C^*=\frac{1}{2}\sum_i\Delta_{iC}^*.$$

To see this, notice that under Δ^*, ($IC^T_{A_i}$-2)s imply:

$$\begin{aligned}\left[\tfrac{3}{2}\left(p^T P^T\right)^2 - \tfrac{1}{2}\left(p^T_\emptyset P^T_\emptyset\right)^2 - p^T_\emptyset P^T_\emptyset p^T P^T\right](\Delta^*_S)^2 &\geq \max\{(p^T - P^T_\emptyset)\Delta^*_{1C}, (p^T - P^T_\emptyset)\Delta^*_{2C}\}\\ &\geq \tfrac{1}{2}(p^T - P^T_\emptyset)(\Delta^*_{1C} + \Delta^*_{2C})\\ &= (p^T - P^T_\emptyset)\Delta^*_C.\end{aligned}$$

Thus, $(\Delta^*_S, \Delta^*_S, \Delta^*_C, \Delta^*_C)$ is also a solution as it satisfies ($IC^T_{A_i}$-2) and does not affect (IC^T_P-1) and (IC^T_P-2).

Step 2: Denote

$$\begin{aligned}\Pi^T(\Delta_{1S}, \Delta_{2S}) \;&:= \left(p^T\right)^2 P^T \Pr(\omega = G|x^T \in X_P)y\sum_i \Delta_{iS} + (1-p^T)\underline{\pi}\\ &\quad -\tfrac{1}{2}\left(p^T P^T\right)^2 \sum_i \Delta^2_{iS}.\end{aligned}$$

Suppose $\Delta^* := (\Delta^*_{1S}, \Delta^*_{2S}, \Delta^*_{1C}, \Delta^*_{2C})$ is a solution to $\mathcal{P}_T$ but $\Delta^*_{1S} \neq \Delta^*_{2S}$. Without loss of generality, assume $\Delta^*_{1S} > \Delta^*_{2S}$. We argue that then Δ^* cannot be a solution. In particular, there exists $\varepsilon > 0$ and cancellation premiums Δ'_{iC}s such that $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta'_{1C}, \Delta'_{2C})$ is feasible and

$$\Pi^T(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon) > \Pi^T(\Delta^*_{1S}, \Delta^*_{2S}).$$

Observe that $\Pi_T(\Delta_{1S}, \Delta_{2S})$ is symmetric and concave in $(\Delta_{1S}, \Delta_{2S})$ with peak at

$$\Delta_{1S} = \Delta_{2S} = \frac{y}{P^T}\Pr\left(\omega = G|x^T \in X_P\right).$$

Also, the following holds: take any $(\Delta_{1S}, \Delta_{2S})$ such that $\Delta_{1S} \neq \Delta_{2S}$, $\Delta_{1S} > \Delta_{2S}$, say. Then, there exists $\varepsilon > 0$ such that

$$\Pi^T\left(\Delta_{1S} - \varepsilon, \Delta'_{2S} + \varepsilon\right) > \Pi^T\left(\Delta_{1G}, \Delta_{2G}\right).$$

So, we only need to show that there exists an $\varepsilon > 0$, and $\Delta'_{1C}, \Delta'_{2C}$ values such that $(\Delta^*_{1S} - \varepsilon, \Delta^*_{2S} + \varepsilon, \Delta'_{1C}, \Delta'_{2C})$ is feasible. In order to prove this claim, it is worthwhile to first establish a few properties of the ($IC^T_{A_i}$-2) constraints, as given in the next step.

Step 3: Denote

$$L_i(\Delta_{1S}, \Delta_{2S}) := A\left(\Delta_{iS}\right)^2 + B\Delta_{1S}\Delta_{2S},$$

where

$$A := \frac{\left(p^T P^T\right)^2 - \left(p^T_\emptyset P^T_\emptyset\right)^2}{2\left(p^T - p^T_\emptyset\right)} \text{ and } B := \frac{\left(p^T P^T - p^T_\emptyset P^T_\emptyset\right)p^T P^T}{p^T - p^T_\emptyset}.$$

Note that the ($IC^T_{A_i}$-2) constraints can be written as:

$$L_i\left(\Delta_{1S}, \Delta_{2S}\right) \geq \Delta_{iC} \text{ if } p^T - p^T_\emptyset > 0, \text{ and } L_i\left(\Delta_{1S}, \Delta_{2S}\right) \leq \Delta_{iC} \text{ otherwise.}$$

Also,

$$(B-A)=\frac{\left(p^{T}P^{T}-p_{\emptyset}^{T}P_{\emptyset}^{T}\right)^{2}}{2\left(p^{T}-p_{\emptyset}^{T}\right)},$$

and hence,

$$\operatorname{sign}(B-A)=\operatorname{sign}\left(p^{T}-p_{\emptyset}^{T}\right)$$

It is routine to check that for $X_P=\{G\}$, $p^T-p_{\emptyset}^T>0$ and $p^TP^T-p_{\emptyset}^TP_{\emptyset}^T>0$, whereas for $X_P=\{G,\emptyset\}$, $p^T-p_{\emptyset}^T<0$ and $p^TP^T-p_{\emptyset}^TP_{\emptyset}^T<0$. Thus,

$$A>0, B>0.$$

In the next two steps, we consider the two cases $p^T-p_{\emptyset}^T>0$ and <0, and show that the claim in Step 2 above holds in both cases.

Step 4: Suppose $p^T-p_{\emptyset}^T>0$. So, ($IC_{A_i}^T$-2)s are given as:

$$L_i(\Delta_{1S},\Delta_{2S})\geq\Delta_{iC}.$$

There are three possibilities:

Case 1: Both ($IC_{A_i}^T$-2)s are slack at $(\Delta_{1S}^*,\Delta_{2S}^*,\Delta_{1C}^*,\Delta_{2C}^*)$. Consider the solution

$$(\Delta_{1S}^*-\varepsilon,\Delta_{2S}^*+\varepsilon,\Delta_{1C}^*,\Delta_{2C}^*)$$

where $\varepsilon>0$. This solution leaves $\left(IC_P^T\right)$s unaffected, for sufficiently small ε, both ($IC_{A_i}^T$-2)s remain slack, and yields a higher value of Π^T(from Step 2).

Case 2: Exactly one of the two ($IC_{A_i}^T$-2)s is slack at $(\Delta_{1S}^*,\Delta_{2S}^*,\Delta_{1C}^*,\Delta_{2C}^*)$. Suppose only ($IC_{A_1}^T$-2) is slack, say, (hence, (IC_{A_2}-2) is binding). Set

$$\Delta_{1C}'=\Delta_{1C}^*+\delta,\ \Delta_{2C}'=\Delta_{2C}^*-\delta$$

where $\delta>0$. For δ sufficiently small, at $(\Delta_{1S}^*,\Delta_{2S}^*,\Delta_{1C}',\Delta_{2C}')$, both ($IC_{A_i}^T$-2) become slack and $\left(IC_P^T\right)$s are unaffected, and hence, it is feasible. But then, as argued in Case 1, the solution $(\Delta_{1S}^*-\varepsilon,\Delta_{2S}^*+\varepsilon,\Delta_{1C}',\Delta_{2C}')$ is also feasible for $\varepsilon>0$ sufficiently small, and attains a higher value of Π^T.

Case 3: Both ($IC_{A_i}^T$-2)s are binding at $(\Delta_{1S}^*,\Delta_{2S}^*,\Delta_{1C}^*,\Delta_{2C}^*)$. Consider changing $(\Delta_{1S}^*,\Delta_{2S}^*)$ to $(\Delta_{1S}^*-\varepsilon,\Delta_{2S}^*+\varepsilon)$. The left-hand side of $\left(IC_{A_i}^T\text{-}2\right)$ changes by

$$\delta_i:=L_i\left(\Delta_{1S}^*-\varepsilon,\Delta_{2S}^*+\varepsilon\right)-L_i\left(\Delta_{1S}^*,\Delta_{2S}^*\right)$$

where

$$\begin{aligned}\delta_1 &= -\varepsilon\left(2A\left(\Delta_{1S}^* - \frac{1}{2}\varepsilon\right) - B\left(\Delta_{1S}^* - \left(\Delta_{2S}^* + \varepsilon\right)\right)\right), \\ \delta_2 &= \varepsilon\left(2A\left(\Delta_{2S}^* + \frac{1}{2}\varepsilon\right) + B\left(\Delta_{1S}^* - \left(\Delta_{2S}^* + \varepsilon\right)\right)\right).\end{aligned}$$

Note that by $A > 0$, $B > 0$ and ε small enough, $\delta_2 > 0$.

So, if $\delta_1 > 0$, the perturbation relaxes both $\left(IC_{A_i}^T\text{-}2\right)$s and by the argument given in Case 1, $(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon, \Delta_{1C}^*, \Delta_{2C}^*)$ is an improvement.

If $\delta_1 < 0$, $\left(IC_{A_1}^T\text{-}2\right)$ is now violated, but $(IC_{A_2}\text{-}2)$ has become slack. Also note that by $B - A > 0$,

$$\delta_2 + \delta_1 = 2\varepsilon\left(B - A\right)\left(\Delta_{1S}^* - \left(\Delta_{2S}^* + \varepsilon\right)\right) > 0.$$

Now, set

$$\Delta_{1C}' = \Delta_{1C}^* + \delta_1, \;\; \Delta_{2C}' = \Delta_{2C}^* - \delta_1.$$

Note that

$$L_1\left(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon\right) = L_1\left(\Delta_{1S}^*, \Delta_{2S}^*\right) + \delta_1 = \Delta_{1C}^* + \delta_1 = \Delta_{1C}',$$

and

$$\begin{aligned}L_2\left(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon\right) &= L_2\left(\Delta_{1S}^*, \Delta_{2S}^*\right) + \delta_2 \\ &= L_2\left(\Delta_{1S}^*, \Delta_{2S}^*\right) - \delta_1 + \left(\delta_2 + \delta_1\right) \\ &> L_2\left(\Delta_{1S}^*, \Delta_{2S}^*\right) - \delta_1 \\ &= \Delta_{2C}^* - \delta_1 = \Delta_{2C}'.\end{aligned}$$

Hence, $(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon, \Delta_{1C}', \Delta_{2C}')$ is feasible (note that the $\left(IC_P^T\right)$s are unaltered by construction), and for $\varepsilon > 0$ sufficiently small, attains a higher value of Π^T.

Step 5: Suppose $p^T - p_\emptyset^T < 0$. Thus, the $(IC_{A_i}^T\text{-}2)$s are

$$L_i\left(\Delta_{1S}, \Delta_{2S}\right) \leq \Delta_{iC}.$$

As before, there are three possibilities:

Case 1: Both $(IC_{A_i}^T\text{-}2)$s are slack at $(\Delta_{1S}^*, \Delta_{2S}^*, \Delta_{1C}^*, \Delta_{2C}^*)$. By argument in case 1 in Step 4, this solution can be improved upon.

Case 2: Exactly one of the two $(IC_{A_i}^T\text{-}2)$s is slack at $(\Delta_{1S}^*, \Delta_{2S}^*, \Delta_{1C}^*, \Delta_{2C}^*)$. Suppose only $(IC_{A_1}^T\text{-}2)$ is slack, say, (hence, $(IC_{A_2}^T\text{-}2)$ is binding). Set

$$\Delta_{1C}' = \Delta_{1C}^* - \delta, \;\; \Delta_{2C}' = \Delta_{2C}^* + \delta$$

where $\delta > 0$. As in case 2 in Step 4, the solution $(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon, \Delta_{1C}', \Delta_{2C}')$ is also feasible for $\varepsilon > 0$ sufficiently small, and attains a higher value of Π^T.

Case 3: Both $(IC_{A_i}^T\text{-}2)$*s are binding at* $(\Delta_{1S}^*, \Delta_{2S}^*, \Delta_{1C}^*, \Delta_{2C}^*)$. Consider changing $(\Delta_{1S}^*, \Delta_{2S}^*)$ to $(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon)$. As in case 3 in Step 4, the left-hand side of $\left(IC_{A_i}^T\text{-}2\right)$ changes by

$$\delta_i := L_i\left(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon\right) - L_i\left(\Delta_{1S}^*, \Delta_{2S}^*\right)$$

where $\delta_2 > 0$ and

$$\delta_2 + \delta_1 = 2\varepsilon\left(B - A\right)\left(\Delta_{1S}^* - \left(\Delta_{2S}^* + \varepsilon\right)\right) < 0.$$

for ε small enough.

So, if $\delta_1 > 0$, the perturbation relaxes both $\left(IC_{A_i}^T\text{-}2\right)$s and by argument given in Case 1, $(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon, \Delta_{1C}^*, \Delta_{2C}^*)$ is an improvement.

If $\delta_1 < 0$, $\left(IC_{A_2}^T\text{-}2\right)$ is now violated, but $\left(IC_{A_1}^T\text{-}2\right)$ has become slack. Now, set

$$\Delta_{1C}' = \Delta_{1C}^* - \delta_2,\ \ \Delta_{2C}' = \Delta_{2C}^* + \delta_2.$$

Note that

$$\begin{aligned} L_1\left(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon\right) &= L_1\left(\Delta_{1S}^*, \Delta_{2S}^*\right) + \delta_1 \\ &= L_1\left(\Delta_{1S}^*, \Delta_{2S}^*\right) - \delta_2 + \left(\delta_2 + \delta_1\right) \\ &< L_1\left(\Delta_{1S}^*, \Delta_{2S}^*\right) - \delta_2 \\ &= \Delta_{1C}^* - \delta_2 = \Delta_{1C}'. \end{aligned}$$

and

$$L_2\left(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon\right) = L_2\left(\Delta_{1S}^*, \Delta_{2S}^*\right) + \delta_2 = \Delta_{2C}^* + \delta_2 = \Delta_{2C}',$$

Hence, $(\Delta_{1S}^* - \varepsilon, \Delta_{2S}^* + \varepsilon, \Delta_{1C}', \Delta_{2C}')$ is feasible (note that $\left(IC_P^T\right)$s are unaltered by construction), and for $\varepsilon > 0$ sufficiently small, attains a higher value of Π^T.

Combining all cases stated above, we obtain that without loss of generality, we can focus on the solution where $\Delta_{1S} = \Delta_{2S} = \Delta_S$, $\Delta_{1C} = \Delta_{2C} = \Delta_C$. And from (IR_i^T), we obtain that under such a solution, we must have $w_{1F} = w_{2F} = w_F$. This observation completes the proof of part (i) of this lemma.

Part (ii). As we focus on $\Delta_{1S} = \Delta_{2S} = \Delta_S$ and $\Delta_{1C} = \Delta_{2C} = \Delta_C$, the program can be simplified as:

$$\mathcal{P}^T \begin{cases} \max\limits_{\Delta_C, \Delta_S} & 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1 - p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2 \\ \\ s.t. & \left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p_\emptyset^T P_\emptyset^T\right)^2 - p_\emptyset^T P_\emptyset^T p^T P^T\right] (\Delta_S)^2 \geq (p^T - p_\emptyset^T)\Delta_C \quad (IC_A^T\text{-}2) \\ & 2\left[\Pr(\omega = G \mid x^T \in X_P) y - 2P^T \Delta_S\right] p^T P^T \Delta_S \geq \underline{\pi} - 2\Delta_C \quad (IC_P^T\text{-}1) \\ & 2\left[\Pr(\omega = G \mid x^T \notin X_P) y - 2P_C^T \Delta_S\right] p^T P^T \Delta_S \leq \underline{\pi} - 2\Delta_C \quad (IC_P^T\text{-}2) \end{cases}$$

As in the case of individual assignment, we have two cases: $X_P = \{G, \emptyset\}$ and $X_P = \{G\}$.

Case 1: $X_P = \{G, \emptyset\}$. Here, $p^T - p^T_\emptyset < 0$; so we have:

$$\mathcal{P}^T_{\{G,\emptyset\}}\begin{cases} \max\limits_{\Delta_C,\Delta_S} \quad 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1-p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2 \\ \\ s.t. \qquad \Delta_C \geq l_A := \left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p^T_\emptyset P^T_\emptyset\right)^2 - p^T_\emptyset P^T_\emptyset p^T P^T\right]\frac{\Delta_S^2}{p^T - p^T_\emptyset} \qquad (IC^T_A\text{-}2) \\ \qquad \Delta_C \geq l_P := \frac{1}{2}\underline{\pi} - \left[\Pr(\omega = G \mid x^T \in X_P)y - 2P^T\Delta_S\right] p^T P^T \Delta_S \qquad (IC^T_P\text{-}1) \\ \qquad \Delta_C \leq u_P := \frac{1}{2}\underline{\pi} - \left[\Pr(\omega = G \mid x^T \notin X_P)y - 2P^T_C\Delta_S\right] p^T P^T \Delta_S \qquad (IC^T_P\text{-}2) \end{cases}.$$

Notice that Δ_C is not in the objective function, and we can further simplify the program as:

$$\begin{gathered} \max_{\Delta_S} \quad 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1-p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2 \\ s.t. \\ u_P \geq l_A \Leftrightarrow \frac{1}{2}\underline{\pi} \geq \left[\Pr(\omega = G \mid x^T \notin X_P)y - 2P^T_C\Delta_S\right] p^T P^T \Delta_S \\ +\frac{\Delta_S^2}{p^T - p^T_\emptyset}\left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p^T_\emptyset P^T_\emptyset\right)^2 - p^T_\emptyset P^T_\emptyset p^T P^T\right] \\ \\ u_P \geq l_P \Leftrightarrow \Delta_S \leq \frac{y}{2(1-\mu)}. \end{gathered}.$$

By routine calculation, one obtains $\Pr(\omega = G|x^T \in X_P) = \frac{1}{2-\alpha'}$, $\Pr(\omega = G|x^T \notin X_P) = 0$, and

$$\begin{gathered} p^T = 1 - \tfrac{1}{2}\alpha'; \quad P^T = \mu + (1-\mu)\tfrac{1}{2-\alpha'}; \quad p^T_C = \mu; \\ p^T_\emptyset = 1 - \tfrac{1}{2}\alpha; \quad P^T_\emptyset = \mu + (1-\mu)\tfrac{1}{2-\alpha}, \end{gathered}$$

where $\alpha' := 1 - (1-\alpha)^2$. Also, as in the proof of Lemma 2, plugging in $\underline{\pi} = \frac{1}{4}y^2$ and the probability values, the program becomes:

$$\begin{aligned} &\max_{\Delta_S} \quad -\tfrac{1}{4}\left(1 + \mu\left(1-\alpha'\right)\right)^2\left(\Delta_S - \tfrac{y}{1+\mu(1-\alpha')}\right)^2 + \tfrac{1}{4}(1 + \tfrac{1}{2}\alpha')y^2 \\ \\ &\text{s.t.} \qquad \Delta_S^2 \leq \tfrac{y^2}{2\mu^2\alpha(1-\alpha)} \text{ and } \Delta_S \leq \tfrac{y}{2(1-\mu)} \end{aligned}.$$

The solution is given as:

$$\Delta_S^* = \begin{cases} \frac{y}{1+\mu(1-\alpha)^2} & \text{if } \frac{1}{1+\mu(1-\alpha)^2} \leq \frac{1}{2(1-\mu)} \\ \frac{y}{2(1-\mu)} & \text{otherwise} \end{cases},$$

and the associated value is:

$$V^T_{\{G,\emptyset\}} = \begin{cases} \frac{1}{4}y^2 + \frac{1}{8}\alpha(2-\alpha)y^2 & \text{if } \frac{1}{1+\mu(1-\alpha)^2} \leq \frac{1}{2(1-\mu)} \\ \frac{1+\mu(1-\alpha)^2}{4(1-\mu)}\left[1 - \frac{1+\mu(1-\alpha)^2}{4(1-\mu)}\right]y^2 + \frac{1}{8}\alpha(2-\alpha)y^2 & \text{otherwise} \end{cases}. \tag{8}$$

We now check whether the agent's effort choice is always at the interior of $[0, 1/2]$. On path, for $k = 1, 2$, $e_k = p^T P^T \Delta_S^* = \frac{1}{2}y \leq \frac{1}{4}$. And off-path, the effort level is $e'_k = \frac{p^T_\emptyset P^T_\emptyset}{p^T P^T}\left(\frac{1}{2}y\right) \leq \frac{p^I_\emptyset P^I_\emptyset}{p^I P^I}\left(\frac{1}{2}y\right) \leq y \leq \frac{1}{2}$Note that $p^I P^I = p^T P^T = \frac{1}{2}\left(1 + \mu\left(1-\alpha'\right)\right)$ and $p^T_\emptyset P^T_\emptyset =$

$\frac{1}{2}(1+\mu(1-\alpha)) \leq p^I_\emptyset P^I_\emptyset = \frac{1}{2}(1+\mu)$. Hence the effort choice is always in the interior of $[0, 1/2]$.

Case 2: $X_P = \{G\}$. Here, $p^T - p^T_\emptyset > 0$, so we have:

$$\mathcal{P}^T_{\{G\}} \begin{cases} \max_{\Delta_C, \Delta_S} & 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1-p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2 \\ \\ s.t. & \Delta_C \leq u_A := \left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p^T_\emptyset P^T_\emptyset\right)^2 - p^T_\emptyset P^T_\emptyset p^T P^T\right] \frac{\Delta_S^2}{p^T - p^T_\emptyset} \quad (IC^T_A\text{-}2) \\ & \Delta_C \geq l_P := \frac{1}{2}\underline{\pi} - \left[\Pr(\omega = G \mid x^T \in X_P) y - 2P^T \Delta_S\right] p^T P^T \Delta_S \quad (IC^T_P\text{-}1) \\ & \Delta_C \leq u_P := \frac{1}{2}\underline{\pi} - \left[\Pr(\omega = G \mid x^T \notin X_P) y - 2P^T_C \Delta_S\right] p^T P^T \Delta_S \quad (IC^T_P\text{-}2) \end{cases}.$$

As Δ_C does not appear in the objective function, we can replace the constraints by requiring $l_P \leq u_A$ and $l_P \leq u_P$, and the program simplifies to:

$$\begin{gathered} \max_{\Delta_S} \quad 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y \Delta_S + (1-p^T)\underline{\pi} - \left(p^T P^T\right)^2 (\Delta_S)^2 \\ s.t. \\ \frac{1}{2}\underline{\pi} \leq \left[\Pr(\omega = G \mid x^T \in X_P) y - 2P^T \Delta_S\right] p^T P^T \Delta_S \\ + \left[\frac{3}{2}\left(p^T P^T\right)^2 - \frac{1}{2}\left(p^T_\emptyset P^T_\emptyset\right)^2 - p^T_\emptyset P^T_\emptyset p^T P^T\right] \frac{\Delta_S^2}{p^T - p^T_\emptyset} \\ \Delta_S \leq \frac{y}{2(1-\mu)} \end{gathered}.$$

Plugging $\underline{\pi} = y^2/4$ and the values for the probabilities, we obtain:

$$\begin{gathered} \max_{\Delta_S} \quad \Pi^T_{\{G\}}(\Delta_S) := -\frac{1}{4}\left(\alpha'(\Delta_S - y)\right)^2 + \frac{1}{4}\left(1 - \alpha'\left(\frac{1}{2} - \alpha'\right)\right) y^2 \\ s.t. \\ \alpha(2-\alpha) y \Delta_S - \frac{1}{2}\alpha(1-\alpha)\Delta_S^2 \geq \frac{1}{4} y^2 \\ \Delta_S \leq \frac{y}{2(1-\mu)} \end{gathered}.$$

Let $\hat{\alpha} := 0.12445$ and $K(\alpha) := \frac{1}{1-\alpha}\left(2 - \alpha - \sqrt{(2-\alpha)^2 - \frac{1-\alpha}{2\alpha}}\right)$. It is routine to check that the program does not admit a solution if $\alpha < \hat{\alpha}$ or $K(\alpha) > 1/2(1-\mu)$. Otherwise, the solution is as follows:

$$\Delta_S^* = \begin{cases} y & \text{if } \alpha \geq \hat{\alpha} \textit{ and } K(\alpha) \leq 1 \leq \frac{1}{2(1-\mu)} \\ \frac{y}{2(1-\mu)} & \text{if } \alpha \geq \hat{\alpha} \textit{ and } K(\alpha) \leq \frac{1}{2(1-\mu)} < 1 \\ \tilde{\alpha} y & \text{if } \alpha \geq \hat{\alpha} \textit{ and } 1 < K(\alpha) \leq \frac{1}{2(1-\mu)} \end{cases},$$

and the associated value function is

$$V^T_{\{G\}} = \begin{cases} \Pi^T_{\{G\}}(y) & \text{if } \alpha \geq \hat{\alpha} \textit{ and } K(\alpha) \leq 1 \leq \frac{1}{2(1-\mu)} \\ \Pi^T_{\{G\}}\left(\frac{y}{2(1-\mu)}\right) & \text{if } \alpha \geq \hat{\alpha} \textit{ and } K(\alpha) \leq \frac{1}{2(1-\mu)} < 1 \\ \Pi^T_{\{G\}}(\tilde{\alpha} y) & \text{if } \alpha \geq \hat{\alpha} \textit{ and } 1 < K(\alpha) \leq \frac{1}{2(1-\mu)} \end{cases}. \tag{9}$$

Finally, we again check whether the agent's effort choice is at the interior of $[0,1/2]$. On path, for $k=1,2$, $e_k = p^T P^T \Delta_S^* \leq \frac{1}{2}y \leq \frac{1}{4}$. And off-path, the effort level is $e'_k = \frac{p_\emptyset^T P_\emptyset^T}{p^T P^T} e_k = \frac{\alpha}{\alpha'} e_k \leq e_k$ (note that $p^T = \frac{1}{2}\alpha'$, $P^T = 1$, $p_\emptyset^T = \frac{1}{2}\alpha$, and $P_\emptyset^T = 1$). Hence the effort choice is always in the interior of $[0,1/2]$.

Thus, we conclude that the program $\mathcal{P}^T$ always admits a solution for $X_P = \{G,\emptyset\}$ and admits a solution for $X_P = \{G\}$ if and only if α and μ are sufficiently large. □

Proof of Proposition 2. The proof is given in three steps.

Step 1. First, observe that an upper bound for the principal's payoff in programs $\mathcal{P}^I_{\{G,\emptyset\}}$ and $\mathcal{P}^I_{\{G\}}$ (stated in the proof of Lemma 2) and in programs $\mathcal{P}^T_{\{G,\emptyset\}}$ and $\mathcal{P}^T_{\{G\}}$ (stated in the proof of Lemma 4) is

$$S^* = \frac{1}{4}\left(1+\alpha-\frac{1}{2}\alpha^2\right)y^2,$$

i.e., the principal's payoff in the benchmark case (recall that $\underline{\pi} = y^2/4$). Moreover, because the optimal continuation policy in the benchmark case is given by $X_P = \{G,\emptyset\}$, the principal's optimal payoff in programs $\mathcal{P}^I_{\{G\}}$ and $\mathcal{P}^T_{\{G\}}$, $V^I_{\{G\}}$ and $V^T_{\{G\}}$, respectively, must satisfy

$$V^I_{\{G\}} < S^* \text{ and } V^T_{\{G\}} < S^*.$$

Step 2. Recall that the solution for program $\mathcal{P}^I_{\{G,\emptyset\}}$ (see (6) in the proof of Lemma 2) stipulates that the principal's (optimal) payoff

$$V^I_{\{G,\emptyset\}} = \frac{1}{4}\left(1+\frac{1}{2}\alpha'\right)y^2 = \frac{1}{4}\left(1+\alpha-\frac{1}{2}\alpha^2\right)y^2 = S^*$$

if and only if $\frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \frac{1}{2}$. Now, observe that $\frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} \leq \frac{1}{2}$ if and only if $\mu \in [0,\mu_1]$, where μ_1 is defined as follows:

$$\mu_1 = \begin{cases} 1 & \text{if } \frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} < \frac{1}{2}\ \forall \mu \in [0,1] \\ \mu^*(\alpha') & \textit{otherwise} \end{cases}, \tag{10}$$

where

$$\mu^*(\alpha') = \frac{1-\alpha'+\sqrt{2\alpha'}}{2\alpha'-(1-\alpha')^2}$$

is the unique solution in $[0,1]$ to

$$\frac{\alpha'\mu^2}{(1+\mu(1-\alpha'))^2} = \frac{1}{2}.$$

Thus, $V^I_{\{G,\emptyset\}} = S^*$ for all $\mu \in [0,\mu_1]$ and $V^I_{\{G,\emptyset\}} < S^*$ for all $\mu \in (\mu_1, 1]$.

Similarly, the solution for program $\mathcal{P}^T_{\{G,\emptyset\}}$ (see (8) in the proof of Lemma 4) stipulates that the principal's (optimal) payoff

$$V^T_{\{G,\emptyset\}} = S^*$$

if and only if $\frac{1}{1+\mu(1-\alpha)^2} \leq \frac{1}{2(1-\mu)}$. This condition is satisfied if and only if $\mu \in [\mu_0, 1)$, where μ_0 is the solution to the equation

$$\frac{1}{1+\mu(1-\alpha)^2} = \frac{1}{2(1-\mu)};$$

that is,

$$\mu_0 = \frac{1}{2+(1-\alpha)^2} = \frac{1}{3-\alpha'}. \tag{11}$$

Thus, $V^T_{\{G,\emptyset\}} < S^*$ for $\mu \in [0, \mu_0)$ and $V^T_{\{G,\emptyset\}} = S^*$ for $\mu \in [\mu_0, 1)$.

Step 3. Observe that $\mu_0 < \mu_1$ $\forall \alpha \in [0,1]$ as using (11), one obtains

$$\frac{\alpha' \mu_0^2}{\left(1+\mu_0\left(1-\alpha'\right)\right)^2} = \frac{\alpha'}{4\left(2-\alpha'\right)^2} < \frac{1}{2}.$$

Combining the above observations we obtain: (i) if $\mu < \mu_0$, $V^I_{\{G,\emptyset\}} = S^* > \max\{V^T_{\{G,\emptyset\}}, V^I_{\{G\}}, V^T_{\{G\}}\}$; that is, individual assignment with $X_P = \{G, \emptyset\}$ is optimal; (ii) if $\mu > \mu_1$, $V^T_{\{G,\emptyset\}} = S^* > \max\{V^I_{\{G,\emptyset\}}, V^I_{\{G\}}, V^T_{\{G\}}\}$; that is, team assignment with $X_P = \{G, \emptyset\}$ is optimal; (iii) if $\mu_0 \leq \mu \leq \mu_1$, $V^I_{\{G,\emptyset\}} = V^T_{\{G,\emptyset\}} = S^* > \max\{V^I_{\{G\}}, V^T_{\{G\}}\}$; that is, both team and individual assignment with $X_P = \{G, \emptyset\}$ are optimal. □

Proof of Proposition 3. It follows directly from (11) that μ_0 is increasing in α. Regarding the threshold μ_1, consider its definition as given in (10). When $\alpha' < \frac{1}{2}$, we have

$$\frac{\alpha' \mu^2}{\left(1+\mu\left(1-\alpha'\right)\right)^2} \leq \alpha' \mu^2 \leq \alpha' < \frac{1}{2};$$

so, $\mu_1 = 1$. For $\alpha' \geq \frac{1}{2}$, we have

$$\mu_1 = \min\left\{1, \mu^*\left(\alpha'\right)\right\}.$$

Differentiating $\mu^*(.)$, we obtain

$$\frac{d}{d\alpha'}\mu^*\left(\alpha'\right) = -\frac{\left(1-\frac{1}{\sqrt{2\alpha'}}\right)\left(2\alpha' - \left(1-\alpha'\right)^2\right) + 2\left(2-\alpha'\right)\left(1-\alpha'+\sqrt{2\alpha'}\right)}{\left(2\alpha' - \left(1-\alpha'\right)^2\right)^2}.$$

For $\alpha' \in [\frac{1}{2}, 1)$, it is routine to check that $1 - \frac{1}{\sqrt{2\alpha'}} \geq 0$ and all the other three terms in the numerator are strictly positive. So, $\frac{d}{d\alpha'}\mu^*\left(\alpha'\right) < 0$. Because α' increases with α (recall that $\alpha' := 1 - (1-\alpha)^2$), then $\mu^*\left(\alpha'\right)$ is strictly decreasing in α when $\alpha \in \left[1 - 1/\sqrt{2}, 1\right]$.

Finally, note that when $\alpha = 1 - \frac{1}{\sqrt{2}}$, $\mu^*(\alpha') = 2$; and when $\alpha = 1$, $\mu^*(\alpha') = \frac{1}{\sqrt{2}}$. As $\mu^*(\alpha')$ is decreasing in α, by the Intermediate Value Theorem, there exists an α^* such that when $\alpha = \alpha^*$, $\mu^*(\alpha') = 1$. Also, when $\alpha < \alpha^*$, $\mu^*(\alpha') > 1$; and when $\alpha > \alpha^*$, $\mu^*(\alpha') < 1$. Thus, for $1 - \frac{1}{\sqrt{2}} \leq \alpha \leq \alpha^*$, $\mu_1 = \min\{1, \mu^*(\alpha')\} = 1$ and for $\alpha \geq \alpha^*$, μ_1 is decreasing in α. □

Proof of Proposition 4. **Step 1:** As Lemma 1 and 3 hold for any $\theta \in (\frac{1}{2}, 1)$ (note that the proofs of these lemmas presented above do not rely on any specific value of θ), we may continue to limit attention to the set of four programs $\mathcal{P}^I_{\{G,\emptyset\}}$, $\mathcal{P}^I_{\{G\}}$, $\mathcal{P}^T_{\{G,\emptyset\}}$, and $\mathcal{P}^T_{\{G\}}$ as defined in the proofs of Lemma 2 and 4. In this step, we compute the unconstrained maximum of these four programs. That is, for $\mathcal{P}^I_{X_P}$, $X_P \in \{\{G,\emptyset\},\{G\}\}$, we solve for

$$\overline{V}^I_{X_P} := \max_{\Delta_S} \;\; 2\left(p^I\right)^2 P^I \Pr(\omega = G \mid x \in X_P) y\Delta_S + (1-p^I)\underline{\pi} - \left(p^I P^I\right)^2 \Delta_S^2,$$

and for $\mathcal{P}^T_{X_P}$, $X_P \in \{\{G,\emptyset\},\{G\}\}$, we solve for

$$\overline{V}^T_{X_P} := \max_{\Delta_S} \;\; 2\left(p^T\right)^2 P^T \Pr(\omega = G \mid x^T \in X_P) y\Delta_S + (1-p^T)\underline{\pi} - \left(p^T P^T\right)^2 \Delta_S^2.$$

As all the objective functions are quadratic in Δ_S, a solution exists and is unique. Plugging in the values for all the probabilities and solving the optimization problem we obtain (recall that $\alpha' = 1 - (1-\alpha)^2$):

$$\overline{V}^I_{\{G,\emptyset\}} = \overline{V}^T_{\{G,\emptyset\}} = \frac{1}{4}\left[(1 - \alpha'(1-\theta))^2 + \frac{1}{2}\alpha'\right] y^2 =: \overline{V},$$

and

$$\overline{V}^I_{\{G\}} = \overline{V}^T_{\{G\}} = \frac{1}{4}\left[(\alpha'\theta)^2 + (1 - \frac{1}{2}\alpha')\right] y^2.$$

Note that

$$\overline{V} > \overline{V}^I_{\{G\}} = \overline{V}^T_{\{G\}}.$$

In what follows, we focus our attention on programs $\mathcal{P}^I_{\{G,\emptyset\}}$ and $\mathcal{P}^T_{\{G,\emptyset\}}$, as we show that for θ sufficiently large, at least one of them achieves the value $\overline{V}$.

Step 2: We show that for θ sufficiently large, there exists a cutoff $\mu_0(\alpha;\theta)$ such that $V^T_{\{G,\emptyset\}} < \overline{V}$ if $\mu < \mu_0(\alpha;\theta)$; and $V^T_{\{G,\emptyset\}} = \overline{V}$ otherwise. Plugging the values of the probabilities, the program $\mathcal{P}^{T}_{\{G,\emptyset\}}$ can be written as:

$$\mathcal{P}^{T}_{\{G,\emptyset\}}:\begin{cases}\max\limits_{\Delta_S}\ \overline{V}-\frac{1}{4}\left[(1-\alpha'(1-\theta)+\mu(1-\alpha'\theta))\,\Delta_S-(1-\alpha'(1-\theta))y\right]^2\\ \qquad s.t.\\ [1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)](1-\theta)y\Delta_S \\ \qquad +\frac{1}{2}\alpha(1-\alpha)(1-\theta\,(1-\mu))^2\Delta_S^2\leq\frac{1}{4}y^2 & \left(C_1^T\right)\\ \Delta_S\leq\frac{y}{2(1-\mu)} & \left(C_2^T\right)\end{cases}.$$

The objective function achieves its peak at $\Delta_S^*=\frac{1-\alpha'(1-\theta)}{1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)}y$. If Δ_S^* is feasible under constraints $\left(C_1^T\right)$ and $\left(C_2^T\right)$, $V^{T}_{\{G,\emptyset\}}=\overline{V}$; and $V^{T}_{\{G,\emptyset\}}<\overline{V}$ otherwise. Next, we analyze conditions under which this solution may be feasible.

Plugging Δ_S^* into $\left(C_2^T\right)$ and simplifying, we obtain:

$$\mu\geq\frac{1-\alpha'(1-\theta)}{3-\alpha'\theta-2\alpha'(1-\theta)}=:\mu_0(\alpha;\theta).$$

Thus, Δ_S^* satisfies constraint $\left(C_2^T\right)$ if and only if $\mu\geq\mu_0(\alpha;\theta)$.

We also claim that Δ_S^* satisfies $\left(C_1^T\right)$ if $\theta>0.85$. To see this, plug Δ_S^* into the left-hand side of $\left(C_1^T\right)$, and we obtain:

$$\begin{aligned}&(1-\theta)(1-\alpha'(1-\theta))+\tfrac{1}{2}\alpha(1-\alpha)(1-\alpha'(1-\theta))^2\left[\tfrac{1-\theta(1-\mu)}{1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)}\right]^2\\ &\leq(1-\theta)(1-\alpha'(1-\theta))+\tfrac{1}{2}\alpha(1-\alpha)(1-\alpha'(1-\theta))^2\left(\tfrac{1}{2-\alpha'}\right)^2\\ &\leq(1-\theta)+\tfrac{\alpha(1-\alpha)}{2(2-\alpha')^2}\\ &\leq\tfrac{1}{4}.\end{aligned}$$

The first inequality follows as the expression is increasing in μ, the second one follows as $1-\alpha'(1-\theta)\in[0,1]$, and the final one holds as $\frac{\alpha(1-\alpha)}{2(2-\alpha')^2}<0.1$ (for $\alpha\in[0,1]$).

Hence, for $\theta>0.85$, $V^{T}_{\{G,\emptyset\}}<\overline{V}$ when $\mu<\mu_0(\alpha;\theta)$, and $V^{T}_{\{G,\emptyset\}}=\overline{V}$ otherwise.

Step 3: We show that for θ sufficiently large, there exists a cutoff $\mu_1(\alpha;\theta)$ such that $V^{I}_{\{G,\emptyset\}}=\overline{V}$ when $\mu\leq\mu_1(\alpha;\theta)$ and $V^{I}_{\{G,\emptyset\}}<\overline{V}$ otherwise. The proof is analogous to the one given in Step 2 above.

Plugging the values of the probabilities, the program $\mathcal{P}^{I}_{\{G,\emptyset\}}$ can be written as:

$$\mathcal{P}^{I}_{\{G,\emptyset\}}:\begin{cases}\max\limits_{\Delta_S}\ \overline{V}-\frac{1}{4}\left[(1-\alpha'(1-\theta)+\mu(1-\alpha'\theta))\,\Delta_S-(1-\alpha'(1-\theta))y\right]^2\\ \qquad s.t.\\ (1-\theta)[1+\mu-\alpha'(1-\theta+\mu\theta)]y\Delta_S+\frac{1}{2}\alpha'\left(1-\theta\,(1-\mu)\right)^2\Delta_S^2\leq\frac{1}{4}y^2 & \left(C_1^I\right)\\ \Delta_S\leq\frac{y}{1-\mu} & \left(C_2^I\right)\end{cases}.$$

The objective function achieves its peak at $\Delta_S^* = \frac{1-\alpha'(1-\theta)}{1-\alpha'(1-\theta)+\mu(1-\alpha'\mu)} y$. If Δ_S^* is feasible under constraints $\left(C_1^I\right)$ and $\left(C_2^I\right)$, $V_{\{G,\emptyset\}}^I = \overline{V}$; and $V_{\{G,\emptyset\}}^I < \overline{V}$ otherwise.

It is routinely to check that Δ_S^* is always feasible under $\left(C_2^I\right)$:

$$\Delta_S^* = \frac{1-\alpha'(1-\theta)}{1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)} y \leq y \leq \frac{y}{1-\mu}.$$

Now, plugging Δ_S^* in the left-hand side of $\left(C_1^I\right)$ and removing term y^2, we get:

$$L(\mu;\alpha,\theta) := (1-\theta)(1-\alpha'(1-\theta)) + \frac{1}{2}\alpha'\left(1-\alpha'(1-\theta)\right)^2 \left(\frac{1-\theta+\mu\theta}{1-\alpha'(1-\theta)+\mu(1-\alpha'\theta)}\right)^2.$$

Note that $L(\mu;\alpha,\theta)$ is increasing in $\mu \in [0,1]$, so it achieves its maximum at $\mu = 1$, where:

$$L(1;\alpha,\theta) = (1-\theta)(1-\alpha'(1-\theta)) + \frac{1}{2}\alpha'\left(\frac{1-\alpha'(1-\theta)}{2-\alpha'}\right)^2.$$

Now, if $\theta > 0.85$, we have $L(1;0,\theta) = 1-\theta < \frac{1}{4}$ and $L(1;1,\theta) = \theta - \frac{1}{2}\theta^2 > \frac{1}{4}$; also

$$\frac{d}{d\alpha} L(1;\alpha,\theta) = \frac{2-2\alpha}{(2-\alpha')^3}\left[R_1+R_2+R_3+R_4\right],$$

where

$$R_1 = \tfrac{1}{2}(1-\theta)^2\left(2\alpha'+\alpha'^3\right), \quad R_2 = 3(1-\theta)^2(\alpha'-\alpha'^2),$$
$$R_3 = 8\left(\tfrac{3}{4}-\theta\right)^2\alpha', \qquad R_4 = 1-8(1-\theta)^2.$$

As $R_i \geq 0$ for $i = 1, ..., 4$, we have $\frac{d}{d\alpha}L(1;\alpha,\theta) \geq 0$. So by the Intermediate Value Theorem, there exists a unique $\alpha^*(\theta) \in (0,1)$ such that $L(1;\alpha^*(\theta),\theta) = \frac{1}{4}$.

Next, define $\mu_1(\alpha;\theta)$ as follows: for $\alpha \leq \alpha^*(\theta)$, let $\mu_1(\alpha;\theta) = 1$; and for $\alpha > \alpha^*(\theta)$, let $\mu_1(\alpha;\theta)$ be the solution to $L(\mu;\alpha,\theta) = \frac{1}{4}$. That is:

$$\mu_1(\alpha;\theta) := \begin{cases} 1 & if\ \alpha \leq \alpha^*(\theta) \\ \frac{(1-\alpha'(1-\theta))\left(\sqrt{K}-(1-\theta)\sqrt{\alpha'}\right)}{(1-\alpha'(1-\theta))\theta\sqrt{\alpha'}-(1-\alpha'\theta)\sqrt{K}} & otherwise \end{cases},$$

where $K := \frac{1}{2} - 2(1-\theta)(1-\alpha'(1-\theta))$.

Notice that when $\alpha \leq \alpha^*(\theta)$, for all $\mu \leq 1 = \mu_1(\alpha;\theta)$, $L(\mu;\alpha,\theta) \leq \frac{1}{4}$, i.e., Δ_S^* satisfies $\left(C_1^I\right)$; when $\alpha > \alpha^*(\theta)$, for all $\mu \leq \mu_1(\alpha;\theta)$, $L(\mu;\alpha,\theta) \leq \frac{1}{4}$, i.e., Δ_S^* satisfies $\left(C_1^I\right)$, and for all $\mu > \mu_1(\alpha;\theta)$, $L(\mu;\alpha,\theta) > \frac{1}{4}$, i.e., Δ_S^* always violate $\left(C_1^I\right)$. As Δ_S^* always satisfies $\left(C_2^I\right)$ we conclude: for $\theta > 0.85$, $V_{\{G,\emptyset\}}^T = \overline{V}$ when $\mu \leq \mu_1(\alpha;\theta)$ and $V_{\{G,\emptyset\}}^T < \overline{V}$ otherwise.

Step 4: Define θ^* as the largest solution in $[0,1]$ to the equation $\mu_0(1;\theta) = \mu_1(1;\theta)$; i.e.,

$$\theta^* := \frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right).$$

As $\theta^* > 0.85$, the definition of μ_0 and μ_1 are valid for $\theta > \theta^*$.

Step 5: Note that $\mu_0(\alpha;\theta)$ is increasing in both α and θ for $\theta \in (\theta^*, 1]$:

$$\frac{d}{d\alpha}\mu_0(\alpha;\theta) = \frac{(2\theta-1)(2-2\alpha)}{\left[3-\alpha'\theta-2\alpha'(1-\theta)\right]^2} \geq 0,$$

and

$$\frac{d}{d\theta}\mu_0(\alpha;\theta) = \frac{\alpha'(2-\alpha')}{\left[3-\alpha'\theta-2\alpha'(1-\theta)\right]^2} \geq 0.$$

Step 6: Next, we claim that $\mu_1(\alpha;\theta)$ is decreasing in α and increasing in θ for $\theta \in (\theta^*, 1]$.

Recall that for $\alpha \leq \alpha^*(\theta)$, $\mu_1(\alpha;\theta) = 1$; for $\alpha > \alpha^*(\theta)$, taking the derivative of $\mu_1(\alpha;\theta)$ with respect to α we obtain:

$$\frac{d}{d\alpha}\mu_1(\alpha;\theta) = -\left(S_1S_2 + S_3S_4\right)S_5,$$

where

$$\begin{aligned}
S_1 &:= (1-\theta)\left[(1-\alpha'(1-\theta))\theta\sqrt{\alpha'} - (1-\alpha'\theta)\sqrt{K}\right],\\
S_2 &:= \tfrac{1}{2\sqrt{K}}\left(1-6(1-\theta)(1-\alpha'(1-\theta))\right) + \tfrac{1}{2\sqrt{\alpha'}}\left(1-3\alpha'(1-\theta)\right),\\
S_3 &:= (1-\alpha'(1-\theta))\left[\sqrt{K}-(1-\theta)\sqrt{\alpha'}\right],\\
S_4 &:= \tfrac{1}{2\sqrt{\alpha'}}\theta(1-3\alpha'(1-\theta)) + \tfrac{1}{2\sqrt{K}}\left(\theta+2\theta^2+6\alpha'^2-2\right),\\
S_5 &:= (2-2\alpha)\Big/\left[(1-\alpha'(1-\theta))\theta\sqrt{\alpha'}-(1-\alpha'\theta)\sqrt{K}\right]^2.
\end{aligned}$$

It is routine to check that $S_i \geq 0$ for all $i = 1, ..., 5$. Hence, $\frac{d}{d\alpha}\mu_1(\alpha;\theta) \leq 0$.

Next, consider the derivative of μ_1 with respect to θ:

$$\frac{d}{d\theta}\mu_1(\alpha;\theta) = \frac{\frac{\sqrt{\alpha'}}{\sqrt{K}}T_1 + T_2}{\left[1-\alpha'(1-\theta)\theta\sqrt{\alpha'}-(1-\alpha'\theta)\sqrt{K}\right]^2},$$

where

$$\begin{aligned}
T_1 &:= \tfrac{5}{2}(1-\theta) + \tfrac{1}{2}(-19+33\theta-13\theta^2)\alpha' + (1-\theta)\left(11-17\theta+4\theta^2\right)\alpha'^2 - 4(1-\theta^3)\alpha'^3,\\
T_2 &:= -\tfrac{1}{2}\alpha'(2-\alpha') + (1-\alpha'(1-\theta))\left[2-(3-2\theta)\alpha'+(1-\theta)(8\theta-3)\alpha'^2\right].
\end{aligned}$$

Below, we show that $T_1 > 0$ and $T_2 \geq 0$, which implies that $\mu_1(\alpha;\theta)$ is increasing in θ.

Step 6a: To show $T_1 > 0$, we consider two cases: $A > 0$ and $A \leq 0$, where $A := -19 + 33\theta - 13\theta^2$.

Case 1: When $A > 0$, we have $11 - 17\theta + 4\theta^2 < 0$ and $13 - 17\theta + 4\theta^2 \geq 0$. Now,

$$\begin{aligned}
T_1 \geq\ & \tfrac{5}{2}(1-\theta) + \tfrac{1}{2}(-19+33\theta-13\theta^2)\alpha' + (1-\theta)\left(11-17\theta+4\theta^2\right) - 4(1-\theta^3)\\
=\ & \tfrac{1}{2}(-19+33\theta-13\theta^2)\alpha' + (1-\theta)\left(13-17\theta+4\theta^2\right) + (1-\theta)\left[\tfrac{1}{2}-4(1-\theta)^2\right]\\
>\ & 0.
\end{aligned}$$

Case 2: When $A \leq 0$ and $\theta > \theta^* > 0.85$, we have $11 - 17\theta + 4\theta^2 < 0$. Now,

$$\begin{aligned} T_1 \geq & \tfrac{5}{2}(1-\theta) + \tfrac{1}{2}(-19 + 33\theta - 13\theta^2) + (1-\theta)(11 - 17\theta + 4\theta^2) - 4\left(1 - \theta^3\right) \\ = & \tfrac{1}{2}\theta(5\theta - 4) > 0. \end{aligned}$$

Step 6b: To show $T_2 \geq 0$, we first define

$$\begin{aligned} T_3 &: = 2 - (3 - 2\theta)\alpha' + (1-\theta)(8\theta - 3)\alpha'^2, \\ T_4 &: = 2(1-\theta)(2 - \alpha') + \alpha'(1 - \alpha'(1-\theta)). \end{aligned}$$

Now,

$$\begin{aligned} T_2 = & -\tfrac{1}{2}\alpha'(2-\alpha') + (1 - \alpha'(1-\theta))T_3 \\ \geq & -\tfrac{1}{2}\alpha'(2-\alpha') + (1 - \alpha'(1-\theta))T_4 \\ \geq & -\tfrac{1}{2}\alpha'(2-\alpha') + \tfrac{1}{2}\left(2-\alpha'\right)^2 \\ = & (2-\alpha')(1-\alpha') \geq 0. \end{aligned}$$

The first inequality follows as $T_3 \geq T_4$ (routine to check). The second inequality follows from the fact that as we have $\alpha > \alpha^*(\theta)$, we have $L\left(1; \alpha, \theta\right) > \frac{1}{4}$. And,

$$L\left(1; \alpha, \theta\right) > \frac{1}{4} \Leftrightarrow (1 - \alpha'(1-\theta))T_4 > \frac{1}{2}\left(2 - \alpha'\right)^2.$$

Step 7: It is routine to check $\mu_1(1;\theta) > \mu_0(1;\theta)$. So, for any $\theta \in (\theta^*, 1]$, $\mu_1(\alpha;\theta) > \mu_0(\alpha;\theta)$ for all $\alpha \in [0,1]$ (as μ_0 is strictly increasing, and μ_1 is decreasing in α). Thus, from Step 2 and 3, we find for $\mu < \mu_0(\alpha;\theta)$, $V^I_{\{G,\emptyset\}} = \overline{V} > \max\left\{V^I_{\{G\}}, V^T_{\{G,\emptyset\}}, V^T_{\{G\}}\right\}$; for $\mu > \mu_1(\alpha;\theta)$, $V^T_{\{G,\emptyset\}} = \overline{V} > \max\left\{V^I_{\{G\}}, V^I_{\{G,\emptyset\}}, V^T_{\{G\}}\right\}$; otherwise, $V^I_{\{G,\emptyset\}} = V^T_{\{G,\emptyset\}} = \overline{V}$. Thus, the characterization of optimal job design is qualitatively identical to that in Proposition 2. □

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