"SHERLOCKING" AND INFORMATION DESIGN BY HYBRID PLATFORMS*

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Abstract. Platform-run marketplaces may exploit third-party sellers' data to develop competing products, but potential for future competition can deter sellers' enrty. We explore how this trade-off affects the platform's referral fee and its own entry decision. We first characterize the platform's optimal referral fee under full commitment on entry decision, and study its economic implications. We then analyze the extent to which the platform's own information sharing policy substitutes for its commitment to entry. We characterize the platform's optimal information policy and examine how it interacts with the platform's fee structure. Our findings highlight the importance of considering the platform's fee structure as a regulatory response in the policy debates on marketplace regulation.

1. Introduction

Platform-run marketplaces facilitate product discovery by enabling consumers to find obscure niche offerings that match their preferences.[1] However, it is common practice for such platforms to adopt a hybrid business model where they not only earn revenue from charging a referral fee from their third-party sellers, but also compete with the sellers by introducing their own private-label products. Often accused of being "both player and referee," the hybrid marketplace platform can be an uneven playing field. The platform can potentially use the third-party sellers' market data to design its own products and promote its private-label products over the competitors.[2]

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[1]The rapid increase in the availability of product variety and expansion of market share of niche products with the emergence of online retailers is often referred to as the "long tail" effect (Anderson, 2006), and has been explored by several scholars in both economics and management literature (Brynjolfsson et al., 2011; Yang, 2013; Goldfard and Tucker, 2019).

[2]To quote Margrethe Vestager, European Competition Commissioner and Vice-President of the European Commission, "the decisions that gatekeepers take, about how to rank different companies in search results,

For example, consider Apple's App Store, a gatekeeper marketplace for iPhone and iPad platforms. The App Store is the only channel through which app developers can distribute their apps to the end users. However, Apple is also a provider of apps that potentially compete against third-party apps, and has been accused of engaging in anti-competitive conduct often referred to as "Sherlocking": it allegedly uses market data to target and copy profitable third-party apps, rendering them obsolete and driving the third-party developers out of business.[3] Similar concerns were also raised for the leading e-commerce platform, Amazon Marketplace, as reportedly it has improperly shared the data about third-party sellers with the division in charge of private-level product developments (Mattiolli, 2020).

This type of predatory behavior by dominant platforms and its associated concerns have led to a variety of policy proposals to limit such exploitative conducts and ensure fair competition. For example, in the U.S., policy makers have proposed structural separation that would prohibit hybrid business models by dominant platforms (Warren, 2019). The proposal would designate large tech platforms as "Platform Utilities" whereby Amazon Marketplace and Basics, and Google's ad exchange and businesses on the exchange would be split apart into separate companies. The EU Digital Markets Act, in contrast, calls for behavioral restriction on the use of proprietary data generated through activities by the sellers and end users on gatekeeper platforms. It calls for a ban on using the data of business users when gatekeepers compete with them on their own platform and ranking the gatekeeper's own products or services in a more favorable manner compared to those of third parties.

However, even though "Sherlocking" may be ex-post optimal to the platform, it may deter sellers' entry, which is not only socially sub-optimal but also detrimental to the platform. Indeed, the platform may want to limit the extent of its own entry if it can commit to one, so as to balance its ex-post gains from imitating the sellers' product against ex-ante loss from reduced entry by the third-party sellers.[4] The choice of its referral fee also accounts

can make or break businesses in dozens of markets that depend on the platform. And if platforms also compete in those markets themselves, they can use their position as player and referee to help their own services succeed, at the expense of their rivals." Speech by Executive Vice-President Margrethe Vestager: Building trust in technology, 29 October, 2020.

(Available at: https://ec.europa.eu/commission/commissioners/2019-2024/vestager/announcements/speech-executive-vice-president-margrethe-vestager-building-trust-technology_en)

[3]The term was coined in early 2000 when Apple updated its own app "Sherlock", a search tool on its desktop operating system, to subsume all features that a third-party app named "Watson" was offering on its platform.

[4]For example, Gower and Henderson (2007) find that Intel tends to avoid competing directly with third-party developers of devices built on its microprocessors so as to stimulate entry of potential developers.

for this trade-off, and any policy regulation on the platform's behavior must account for its regulatory response in fee structure.

The goal of this paper is to analyze the interplay among the platform's fee structure, entry decision and its data usage policy in the face of this trade-off. We first analyze the optimal fee structure of the platform when it can commit to its entry policy. Next, we explore how and when the platform can use its data sharing/usage policy to achieve the outcome under entry commitment (when it cannot directly commit to its entry policy). Finally, we draw out the implications of our findings for some of the key policy proposals on platform regulation.

We develop a tractable model of a platform-run marketplace where the platform charges a referral fee to the sellers for access to the marketplace and may also subsequently launch its own private-label product by copying the seller's. The game unfolds as follows: first, a third-party seller privately observes his "type" and, given the referral fee, decides whether to enter the marketplace by incurring a fixed cost. The seller's type determines the profitability of his product. After the seller's entry, the platform may subsequently observe the type, and decide whether to enter the market by imitating the seller's product.

Our first set of results explore how the optimal referral fee varies with the platform's commitment power over its entry decision. If the platform operates only as a marketplace (instead of operating in hybrid mode) its optimal fee trades off extracting more from the entering sellers and encouraging the sellers' entry. How does the fee structure change when platform adopts a hybrid mode? It depends on the platform's commitment power over its own entry policy.

If the platform can enter but cannot commit to its entry policy, then it sets an exceedingly high referral fee, which can serve as a commitment device. In absence of any entry commitment, the platform would enter whenever it is profitable to do so. Anticipating that, a seller, particularly the lucrative one, may stay out of the marketplace as he expects the platform to subsequently imitate his product and put him out of business. But a high referral fee acts as a commitment device for the platform not to enter the market because it raises the platform's own opportunity cost of entry. By raising the referral fee, the platform loses the relatively less profitable sellers, but the resulting loss is more than compensated as the high fee would encourage the more lucrative seller types to enter (ones who were staying out due to imitation threat). We show that the platform may raise its fee to a sufficiently high level—one that is at least as large as the optimal fee when the platform operated only as a marketplace—so that product imitation is never optimal regardless of the seller's type.

However, if the platform can use, and commit to, a sophisticated (type-contingent) entry policy, then it would set a fee that is even lower than the one it would have set if it were to operate only as a marketplace. When the platform can commit to its entry decision, it can induce even a high-type seller—ones that are more vulnerable to imitation—to enter by limiting its own likelihood of entry, and extract (a part of) the rent the seller earns on the marketplace. In particular, the platform extracts all rents from the high-type sellers who are worth imitating, and have a stronger incentive to induce more seller types to enter. Consequently, it further lowers its referral fees.

While the platform is better off when it can commit to its entry decision, in many scenarios it may lack such commitment power, especially when the platform can capture a third-party seller's market share by introducing a close substitute that may not be a direct imitation of the seller's product. For example, when Apple allegedly "sherlocked" Watson, it did not develop a new app by copying Watson's, but updated one of its existing tools (named "Sherlock") to offer the same functionality that Watson offered. Such possibility of "inventing around" may pose a challenge in enforcing the platform's commitment to its entry decision due to ex-post verifiability of imitation. Indeed, the "doctrine of equivalents" in the enforcement of patents extends the patentee's rights beyond the literal limits of the written claims to regulate imitations that deliberately design around an invention.[5] However, this doctrine creates significant legal uncertainty and makes patent disputes inevitable.

However, it might be possible for the platform to commit to its data usage policy—the degree to which its marketplace division shares third-party seller's data with its product division—as the nature of the shared data maybe discoverable and verifiable in court. Indeed, various regulatory proposals on data usage, e.g., the EU Digital Markets Act, that seek to regulate the level of aggregation and anonymity in third-party data shared between the platform divisions rely on the verifiability of the shared data.

If the platform cannot commit to entry, can it use its data sharing policy to implement the entry-commitment outcome? To explore this question, we model the platform's data sharing policy as an information design problem. Instead of directly observing the seller's type, the platform commits to observe only a signal on the seller's type that is generated from a pre-specified signal structure. We assume that the platform chooses its referral fee and the signal structure at the beginning of the game.

We derive a general condition under which the platform can secure its entry-commitment payoff through its information sharing policy. The condition holds if the cost of entry is

[5]According to the Supreme Court, the doctrine of equivalents is needed because "to permit imitation of a patented invention which does not copy every literal detail would be to convert the protection of the patent grant into a hollow and useless thing. Such a limitation would leave room for—indeed encourage—the unscrupulous copyist to make unimportant and insubstantial changes." (Graver Tank & Mfg Co v Linde Air Prods Co, 339 US 605, 607 (1950).)

relatively large and it takes relatively longer for the platform to imitate the seller's product. If this condition fails, then the platform must exclude some of the intermediate types of the seller. The optimal referral fee now balances a trade-off between the gains from enhanced entry by the seller (particularly of the intermediate types), and the loss due to distortions from the optimal fee under entry commitment.

Our findings have sharp implications for key policy recommendations on regulation of hybrid marketplaces, such as ban on hybrid mode and ban on the use of third-party sellers' information for the launch of private-level products. Due to the regulatory response of the platform in its choice of referral fee, the welfare implications of such policies are often ambiguous, and under some settings, they could be welfare reducing.

For example, as discussed earlier, under no regulation on the mode of operation and information usage, in equilibrium, the platform may be able to implement the entry-commitment outcome through its information policy. Moreover, the associated referral fee is lower than its counterpart when the platform operates only as a marketplace. Thus, banning hybrid mode may result in a much higher referral fee that stifles sellers' entry and reduces the welfare for both the sellers and the consumers.

A policy that only prohibits information usage (i.e., allows hybrid mode as long as the platform does not use its proprietary data to for its own entry decision) may also have a negative welfare implication. In this case, the platform would infer the sellers' type from their entry decision, and we show that depending on the parameter, one of two equilibria may be played. In one, the marketplace may cease to exist as no seller type enters, and in the other some types of the seller enter but the platform never finds it optimal to imitate the seller. If the former equilibrium is played, the policy is clearly welfare reducing. And even if the latter equilibrium is played, the welfare implication of such a policy remains ambiguous, as the optimal referral fee under marketplace mode and hybrid mode cannot be ranked a priori.

Related literature: Our paper is related to several recent papers that address various issues associated with platform marketplaces. Madsen and Vellodi (2022) analyze implications of platforms' use of proprietary marketplace sales data to target the introduction of private-level products. They explore how innovation incentives in digital markets can be shaped by various policies/regulations on platforms' data usage. They propose "data patents," which restrict data usage for a limited time, as a policy option that can improve upon a total ban on data usage by marketplaces. Even though there are some parallels between the two papers in terms of motivations and questions addressed, our paper differs from theirs in two major respects. First, they focus on the case where ad-valorem referral fee is fixed and do not characterize the optimal referral fee. In contrast, the interplay between the optimal referral fee choice and various entry configurations with data usage is one of the main foci of our paper. Second, they consider the seller's ex ante entry incentive (without knowing his type), while we focus on the seller's interim incentive. This drives several significant differences in the results and analysis.

Hagiu et al. (2022) build a model of platform that can choose to offer a marketplace to innovative third-party sellers and convenience benefits to consumers for transactions. They consider three types of platform business models: marketplace mode, seller mode, and dual mode where a platform sells on its own marketplace.[6] Given antitrust concerns about dominant platforms' adoption of the dual mode along with product imitation and self-preferencing, they analyze the effects of various policy options such as a ban on the dual mode. They show that the policy outcomes crucially depend on the platform's policy-induced endogenous choice of business models. In an extension of their baseline model, they also consider the possibility of product imitation and self-preferencing by marketplace platforms. However, the setup and focus of their model are very different from ours. For instance, the dual mode platform in their model can still sell its own existing product in the absence of entry by an innovative seller, and in their "exploitative" equilibrium a referral fee is set high to shield its own product from the innovative seller. In our model, entry by a third party is essential for the platform to sell its own product because we focus on the product discovery aspect of marketplaces; we envision a situation with so many potential products that platforms are not expected to possess information about each individual product's market demand and it is not in the interest of the platform to sell a product without observing its market demand first through a marketplace. Thus, the role of a referral fee is very different in our model in that it serves as a commitment device not to imitate the third-party seller's product to induce more entry.

In focusing on the product discovery aspect of marketplaces and addressing the issue of free-riding on the information provided by third-party sellers, our paper is related to Hervas-Drane and Shelegia (2022). They also assume the existence of "unobserved" products whose existence can be identified only through running a marketplace with dispersed information. In such a framework, they investigate incentives for a retailer to switch from a pure seller to a dual mode and the competitive effects from third parties associated with such a switch. One feature of their model is that the entry decision depends only on the referral fee, but is *independent of the platform's ex post entry decision* because they assume no fixed cost

[6]Platforms in dual mode are also called "hybrid platforms" (Anderson and Bedre-Defolie, 2021) or "retailer-led marketplaces" (Hervas-Drane and Shelegia, 2021). For papers that study the choice of business models, but without the possibility of dual mode, see Hagiu and Wright (2015, 2019) and Johnson (2017).

of entry. Thus, the issue of commitment does not arise in the model of Hervas-Drane and Shelegia (2022). In addition, their paper does not address the relevant issues from the information design perspective.

Finally, Zhu and Liu (2018) investigate Amazon's entry pattern into third-party sellers' product spaces and show that Amazon is more likely to target successful product spaces, which empirically validates our modeling approach.

This paper is structured as follows. Section 2 presents our model. We present two benchmark analyses in Section 3 that highlight how the potential of platform's entry (or lack thereof) affects the seller's entry decision. Section 4 explores the case where the platform has full commitment power over its entry decision. The optimal information policy is analyzed in Section 5. Section 6 discusses the welfare implications of some key policy recommendations in light of our findings. A final section, Section 7, presents a conclusion. All proofs are given in the Appendix.

2. Model

We describe the model below by elaborating on its three key components: *players*, *market interactions*, and *payoffs*:

Players: A platform owner P runs a marketplace where a (representative) seller S may bring his product to reach potential customers.

Market interactions: The marketplace operates for two periods, where the time lengths of the first and second periods are $1-\delta$ and δ, respectively.[7] At the beginning of the game, the platform posts an ad velorem referral fee $r \in [0,1]$ on the seller's profit. The seller incurs a fixed cost $K > 0$ to enter the marketplace, and once he enters, he earns a profit of $\theta \in [0,1]$ per unit time if he stays as a monopolist on the marketplace. Crucially, θ is privately known to the seller before he enters the marketplace, and therefore, influences his entry decision. For simplicity, we assume that the seller does not incur variable costs and θ is drawn according to the uniform distribution over $[0,1]$.

If the seller enters, in the first period, he earns $(1-r)(1-\delta)\theta$, while the platform owner P obtains $r(1-\delta)\theta$ as referral fees. At the beginning of the second period, P observes θ and decides whether to launch its own product by imitating the seller's item. Should P decide to imitate, it faces the same cost as the seller (i.e., only pays a fixed cost K) and earns

[7]We do not consider any time discounting.

$\delta\theta$ by forcing the seller to exit. We assume that S does not have access to any alternative marketplace, and the platform cannot develop the product on its own.

PAYOFFS: If the seller S does not enter, all players earn 0. If S enters, his (ex-post) aggregate payoff is[8]

$$\pi_S(r,\theta) := (1-\delta)(1-r)\theta - K + (1-\mathbf{1}_E)\,\delta(1-r)\theta,$$

and P's (ex-post) payoff is

$$\pi_P(r,\theta) := (1-\delta)\,r\theta + (1-\mathbf{1}_E)\,\delta r\theta + \mathbf{1}_E(\delta\theta - K),$$

where $\mathbf{1}_E$ is 1 when the platform enters the market in period 2 and is 0 otherwise.

TIME LINE: The timeline of the game is summarized below:

- *Period* 1. The principal posts a referral fee $r \geq 0$. The seller observes his "type" θ and the referral fee r, and decides whether or not to enter. Period-one payoffs are realized.
- *Period* 2. P observes the realization of θ and decides whether to launch its own product. If P enters, S's product is taken off the platform. Period-two payoffs are realized.

To streamline our analysis, we impose the following parametric restriction.

Assumption 1. $1-\delta < K < \delta$.

The first inequality implies that even the highest type seller ($\theta = 1$) does not enter the marketplace if he expects the platform to steal his business for sure. Notice that if the platform enters for sure then the entering seller's payoff is equal to $(1-\delta)(1-r)\theta - K \leq (1-\delta) - K$. Now, if the platform enters in the second period then its payoff is equal to $\delta\theta - K$. Thus, the second inequality ensures that in absence of any referral fee (i.e., when $r = 0$), the platform finds it profitable to enter if θ is sufficiently large. Note that this assumption implies $\delta > 1/2$, i.e., for both parties, the future (period 2) payoff is at least as important as their current (period 1) payoff.

[8]The indicator function $\mathbf{1}_X$ takes the value 1 when X holds and 0 otherwise.

We use (weak) Perfect Bayesian Equilibrium as the solution concept.

We conclude this section with the following remarks on our modeling specifications. First, one may interpret the seller's type in terms of the demand state of his product. For example, suppose that the demand function for the seller's product is invariant over time and is given by $D(p,\theta) := \theta d(p)$ where p is the unit price and the parameter θ captures the market thickness. The seller, if selling on the marketplace, would charge the monopoly price $p^m := \arg\max_p \; pd(p)$ that is independent of the market size and the referral fee, and the same holds for the platform if it launches its own product in the second period. One obtains our modeling setup by normalizing the monopoly profit $p^m d(p^m)$ to 1. Second, the weights assigned to the payoffs from the two periods capture relative "lengths" of the two periods. Specifically, $1-\delta$ can be interpreted as the relative length of time it takes for the platform to acquire relevant information on the profitability of the product and δ reflects the time span over which the platform gets to exploit this information. Finally, in our setting, ad valorem fee levied on the seller's profit is quantitatively equivalent to levying it on price as we have assumed the seller's marginal cost of production to be zero. A strictly positive marginal cost compromises the algebraic tractability of our model as it leads to a double marginalization problem.

3. Benchmarks on entry by the platform

We begin by analyzing two benchmark cases, one in which the platform never imitates the seller's product and the other in which the platform imitates the seller's profit whenever it is profitable to do so. Note that the first case can be interpreted as the case in which the platform commits to not imitate the seller's product, while the second case reflects the scenario where the platform has no commitment power over its own entry decision.

3.1. **No entry by platform.** If S does not face any threat of entry from P, P's action affects S's entry only through the referral fee. Given r, S enters if and only if

$$(1-r)\,\theta \geq K \Leftrightarrow \theta \geq \theta_S(r) := \min\left\{\frac{K}{1-r}, 1\right\}. \tag{1}$$

Therefore, P's problem is

$$\mathcal{P}_{NE}: \; \max_r \quad \Pi_P^{NE}(r) := \int_{\theta_S(r)}^1 \theta r d\theta,$$

and its interior solution, denoted by r^{NE}, satisfies

$$\frac{1}{2}\left(1-\theta_S\left(r^{NE}\right)^2\right)=r^{NE}\theta_S\left(r^{NE}\right)\theta_S'\left(r^{NE}\right).$$

Intuitively, an increase of r allows the seller to extract more from the seller conditional on entry (i.e., $\theta \in (\theta_S, 1]$). However, it reduces the seller's entry incentives and the platform loses the referral fee from the marginal seller type θ_S. In the above equation, the left-hand side captures the former marginal benefit, while the right-hand side represents the latter marginal cost. As the optimal fee is strictly positive, there is too little entry compared to the socially efficient level (note that it is efficient for all $\theta \geq K$ to enter).

3.2. **Entry by platform whenever profitable.** Next, consider the case where P can imitate the seller's product and enter the market whenever profitable. Given θ, P prefers to enter if and only if

$$\delta\theta - K \geq \delta r\theta \Leftrightarrow \theta \geq \theta_P\left(r\right) := \min\left\{\frac{K}{\delta\left(1-r\right)}, 1\right\}. \tag{2}$$

So, given r, the type-θ seller's payoff from entry is:

$$\pi_S\left(r;\theta\right)=\begin{cases}\left(1-r\right)\theta & if\ \theta\leq\theta_P\left(r\right)\\ \left(1-\delta\right)\left(1-r\right)\theta & otherwise\end{cases}.$$

By Assumption 1, no seller type above θ_P wishes to enter. This implies that the seller enters if and only of $\theta \in [\theta_S(r), \theta_P(r))$ where the cutoff $\theta_S(r)$ is as defined in (1). Since $\theta_S(r) \leq \theta_P(r)$ for any r, P's problem is

$$\mathcal{P}_{NC}:\ \max_r\ \ \Pi_P^{NC}\left(r\right):=\int_{\theta_S(r)}^{\theta_P(r)} r\theta d\theta.$$

Let r^{NC} be the solution to $\mathcal{P}_{NC}$. Then we have the following result.

Proposition 1. $r^{NC} \geq r^{NE}$.

In other words, the optimal referral fee set by the platform when it has no commitment power over its own entry decision is at least as large as the optimal fee it would set when it commits to not enter by imitating the seller's product.

To see the intuition for this result, recall that a lack of commitment by the platform causes the most lucrative seller types ($\theta \geq \theta_P$) not to enter. Raising r helps reduce associated losses, because it lowers the platform's own entry incentive, thereby encouraging third-party sellers' entry; that is, a higher r helps the platform mitigate its own no-commitment problem. Specifically, as r rises, θ_P increases faster than θ_S, so the absolute length of the interval $[\theta_S(r), \theta_P(r)]$ expands, provided that $\theta_P(r) < 1$. This expansion is profitable: even if the average entering type is constant, it would increase the platform's profit. But, the expansion even raises the average entering type, as it replaces lower types (around $\theta_S(r)$) with higher types (around $\theta_P(r)$).[9] Consequently, the platform would raise r up to the point where $\theta_P(r) = 1$.

An important implication of this result is that the "no-entry" case explored earlier Pareto dominates the "no-commitment" benchmark. The comparison of the seller's payoff is immediate from Proposition 1. When the platform cannot commit to its entry policy, in equilibrium, it does not enter, and the higher referral fee further stifles the seller's entry. The platform's payoffs in the two cases also have the same ranking as with no commitment. As no seller types above θ_P enter, in equilibrium the platform does not benefit from its ability to enter. Finally, consumes are better off because the seller is more likely to enter (i.e., $\theta_S\left(r^{NE}\right) \leq \theta_S\left(r^{NC}\right)$).

4. Full commitment to entry

Section 3 highlights the importance of the platform's commitment to its entry policy. This section studies the platform's optimal strategy when it has full commitment power to its entry policy. In particular, we assume that the platform can choose, and commit to, a function $\alpha : [0,1] \times \mathbb{R} \to [0,1]$ where $\alpha(\theta, r)$ represents the probability that it enters conditional on the seller's type being θ.

Given r and $\alpha(\cdot)$, the seller enters if

$$(E) \qquad [(1-\delta) + \delta(1-\alpha(\theta, r))](1-r)\theta \geq K.$$

This implies that the platform's problem can be written as:

[9] This result is not guaranteed if the distribution of seller types is not uniform. However, it would hold as long as the density function does not decrease too fast near 1.

$$\max_{r,\alpha(\cdot)} \int_{\Theta_E} \left[(1-\alpha(\theta,r))\, r\theta + \alpha(\theta,r)\left((1-\delta)\, r\theta + \delta\theta - K\right)\right] d\theta$$

where

$$\Theta_E = \left\{\theta \in [0,1] \mid (E) \text{ is satisfied}\right\}.$$

As before, given r, it is profitable for the platform to enter if and only if $\theta \geq \theta_P(r)$. So, for $\theta < \theta_P(r)$, it is optimal for the platform to set $\alpha(\theta,r) = 0$ (i.e., not to enter), and the seller enters as long as $\theta \in [\theta_S(r), \theta_P(\theta))$. For $\theta \geq \theta_P(r)$ it is optimal for the platform to raise $\alpha(\theta,r)$ as much as possible subject to the seller's entry constraint (E). This implies that the optimal α is given by

$$\alpha(\theta;r) = \frac{1}{\delta}\left(1 - \frac{K}{(1-r)\,\theta}\right). \tag{3}$$

It is worth noting that $\theta_P(r)$ and $\theta_S(r)$ are increasing in r, whereas $\alpha(\theta;r)$ is decreasing in r and increasing in θ. Also, it is routine to check that under Assumption 1, $0 \leq \alpha(\theta;r) < 1$. This implies that the platform's optimal commitment problem reduces to:

$$\mathcal{P}_C : \max_{r\in[0,1]} \; \Pi_P^C(r) := \int_{\theta_S(r)}^{\theta_P(r)} r\theta d\theta + \int_{\theta_P(r)}^{1} \begin{bmatrix} (1-\delta)\, r\theta + \\ \left[\alpha(\theta;r)(\delta\theta - K) + (1-\alpha(\theta;r))\,\delta r\theta\right] \end{bmatrix} d\theta.$$

Let r^C be the solution to $\mathcal{P}_C$. Notice that

$$\Pi_P^C(r) - \Pi_P^{NE}(r) = \int_{\theta_P(r)}^{1} \alpha(\theta;r)\left[\delta(1-r)\,\theta - K\right] d\theta \quad \begin{cases} > 0 & if\ r < \overline{r} \\ = 0 & otherwise \end{cases},$$

where $\theta_P(\overline{r}) = 1$. This observation leads to the following result.

Proposition 2. *$r^C \leq r^{NE}$.*

The result above states that the optimal referral fee set by the platform when it has full commitment power over its own entry decision is no larger than the optimal fee it would set when it commits to not enter the marketplace.

In the no-entry benchmark case, the seller obtains positive rents whenever $\theta \geq \theta_S(r)$, whereas in the commitment case, the seller receives no rents whenever $\theta \geq \theta_P(r)$. This implies that the marginal gain of increasing r is higher under the no-entry case (extracting more from seller types in $[\theta_S(r), 1])$ than under the case where the platform can commit to its entry decision (extracting more from seller types in $[\theta_S(r), \theta_P(r))$). The result that $r^C \leq r^{NE}$ follows, because the corresponding marginal cost of increasing r—$\theta_S(r)$ type not entering—is identical between the two cases. Notice that under full commitment, the seller's entry incentives are stronger as the referral fee is lower. Also the threat of business stealing does not thwart the seller's entry as, in equilibrium, the platform's likelihood of entry is set at the level where the seller breaks even if he decides to enter.

Compared to the two benchmark cases (in Section 3), with commitment to entry the platform and the consumers are both better off. It is trivial that commitment to entry would benefit the platform as it can always implement the market equilibrium under the "no entry" or "no commitment" case. The consumers are better off as there is more entry due to lowered referral fee (and they pay the same (monopoly) price regardless of platform's subsequent entry decision). However, the seller's payoff vis-a-vis the benchmark cases cannot be ranked as it is affected by two countervailing effects: a lower referral fee induces more entry by the seller (θ_S is lower) but it also makes the platform's entry more likely. Interestingly, this observation opens up the possibility that, relative to the case when the platform cannot enter, some types of the seller may be better off when the platform can commit to its entry decision.

5. Optimal information policy

We now turn to the situation in which the platform does not have full commitment power, but can optimally choose its information policy. To evaluate the full potential of the platform's information policy, as in the recent literature on information design, we endow the platform with full flexibility in its choice of information. The following result, however, shows that in our environment, it suffices to restrict attention to binary signals.

Lemma 1. *Consider any set X of signal realizations and a (measurable) signal $\sigma : [\theta_S, 1] \to \Delta(X)$, where $\Delta(X)$ denotes the set of all probability distributions over X. There exists a* *binary signal* $\hat{\sigma} : [\theta_S, 1] \to \Delta(\{0,1\})$ *that implements the same equilibrium outcome as the signal* σ*.*

To understand this result, notice that the platform's payoff is linear in θ, so its entry decision depends only on the conditional expectation of θ—in particular, whether it exceeds θ_P or not. All signal realizations leading to the posterior mean below (above) θ_P can be pooled. Therefore, the outcome by any signal can be replicated by a binary signal, whose realization can be interpreted as an action (entry or not) recommended to the platform.

5.1. **Implementing full commitment outcome.** We first explore if and when it may be possible for the platform to implement the full commitment outcome characterized in Section 4.

To implement the full commitment outcome, it is necessary and sufficient that the platform enters with probability $\alpha(\theta, r)$ when the seller's type is θ. With a binary signal, this outcome can be induced if and only if the platform uses the following signal structure:

$$\Pr(x = 1 \mid \theta) = \begin{cases} \alpha(\theta, r) & if\ \theta \geq \theta_P(r) \\ 0 & otherwise \end{cases} . \tag{4}$$

In other words, the platform should observe $x = 1$, and so enters, with probability $\alpha(\theta, r)$ whenever $\theta \geq \theta_P(r)$; otherwise, the platform should observe $x = 0$ and not enter. By construction, this signal induces the full commitment outcome in Section 4 if and only if the following two obedience constraints hold:

$$\mathbb{E}[\theta \mid x = 0;\ \theta \geq \theta_S(r)] \leq \theta_P(r), \tag{OC_0}$$

and

$$\mathbb{E}[\theta \mid x = 1;\ \theta \geq \theta_S(r)] > \theta_P(r). \tag{OC_1}$$

That is, conditional on $x = 0$, the expectation of θ should be less than $\theta_P(r)$ so that the platform has no incentive to enter. On the contrary, conditional on $x = 1$, the expectation of θ should exceed $\theta_P(r)$ so that the platform wishes to enter.

Note that for any r, the constraint (OC_1) is always satisfied under the signal structure (4). This implies the following result (we omit the proof as it is immediate from the discussion above).

Proposition 3. *The full commitment outcome is implementable if and only if the constraint* (OC_0) *holds for* $r = r^C$*, which is equivalent to:*

$$\int_{\theta_P(r^C)}^{1} (\theta - \theta_P\left(r^C\right))(1 - \alpha(\theta, r^C))d\theta \leq \int_{\theta_S(r^C)}^{\theta_P\left(r^C\right)} (\theta_P\left(r^C\right) - \theta)d\theta. \tag{5}$$

Figure 1 shows the parameter region under which (OC_0) holds—so, the commitment outcome can be implemented by an information policy. If δ is sufficiently small, full commitment outcome is always implementable for any K (satisfying Assumption 1), whereas for large δ, we require K to be above a threshold (that itself varies with δ).

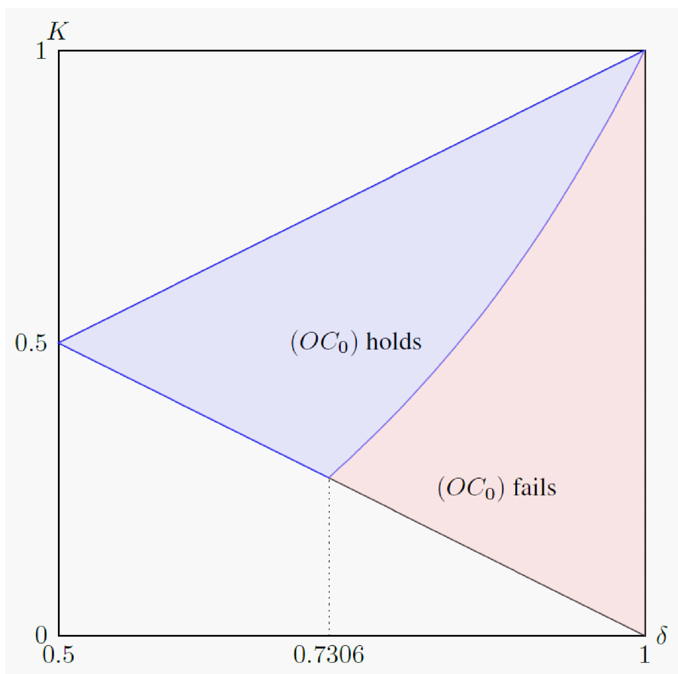

Figure 1: Set of K and δ where full commitment outcome can be implemented via information design (i.e., (OC_0) holds under $r = r^C$).

In order to understand the pattern, note that one may interpret the right-hand side of (5) as the platform's "budget for (no entry) obedience" while the left-hand side as the corresponding "spending". Recall that for a given r, θ_P is increasing in K and decreasing in δ, whereas α is decreasing in both K and δ. If K is large and δ is relatively small then $\theta_P(r)$ is close to 1 and α remains moderate. In this case, the left-hand side ("spending") is small, so

(OC_0) holds. Similarly, if δ is small and close to $1/2$ then $\theta_P(r) - \theta_S(r)$ is large and so is α. Consequently the right-hand side ("budget") of (OC_0) becomes larger whereas the left-hand side ("spending") remains moderate, and (OC_0) continues to hold. But (OC_0) fails when K is relatively small and δ is relatively large. The left-hand side of (OC_0) becomes large (as θ_P and α are both relatively small) and the right-hand side becomes small (as $\theta_P(r) - \theta_S(r)$ is small). As a result, (OC_0) fails.

5.2. **Optimal signal when full commitment outcome is infeasible.** What is the optimal information policy for the platform when the the full commitment outcome cannot be implemented, i.e., what if (5) fails? As before, we explore this case in two steps. First, we study the optimal information policy for a given referral fee, and then explore the profit maximizing fee for the platform.

Fix a value of r (recall that it also determines the cutoffs θ_S and θ_P). For any seller type θ who enters, let $\gamma(r, \theta)$ denote the probability that the platform receives $x = 1$, and so also enters, conditional on θ. For the optimal signal, we must have $\gamma = \alpha$ for all $\theta > \theta_P(r)$; if $\gamma(r, \theta) > \alpha(r, \theta)$, type θ would not enter, and if $\gamma(r, \theta) < \alpha(r, \theta)$, raising γ would relax (OC_0) and yield a higher payoff for the platform. This implies that whenever (OC_0) is violated under the signal structure (4), the optimal signal should dissuade entry for some seller types above θ_P. The following result shows that it is optimal for the platform to deter entry by intermediate seller types.

Proposition 4. *Given a referral fee r such that (OC_0) fails, the optimal binary signal is given as follows: there exist θ_* $(> \theta_P)$ and $\theta^* \in [\theta_*, 1]$ such that*

$$\Pr(x = 1 \mid \theta) = \begin{cases} \alpha(\theta, r) & if\ \theta \in [\theta_P(r), \theta_*) \cup [\theta^*, 1] \\ 1 & if\ \theta \in [\theta_*, \theta^*) \\ 0 & otherwise \end{cases} . \tag{6}$$

Facing this signal, the seller enters if and only if $\theta \in [\theta_S, \theta_*] \cup [\theta^*, 1]$.

To understand this result, let $\phi : [\theta_P, 1] \to \{0, 1\}$ denote the entry decision of the seller above θ_P induced by the platform's information policy (and resulting entry decision). Given this, (OC_0) can be written as

$$\int_{\theta_P}^{1} \phi(\theta)(\theta - \theta_P)(1 - \alpha(\theta, r))d\theta \leq \int_{\theta_S}^{\theta_P} (\theta_P - \theta)d\theta.$$

Notice that this inequality necessarily holds if $\phi(\theta) = 0$ for all $\theta \geq \theta_P$ and fails if $\phi(\theta) = 1$ for all $\theta \geq \theta_P$. The platform should find the set of seller types for which $\phi(\theta) = 0$, whose decision depends on how much a seller type would contribute to the platform's profit as well as the type's implicit cost via (OC_0). Proposition 4 shows that the platform should exclude intermediate types $[\theta_*, \theta^*)$, that is, induce entry for seller types below θ_* or above θ^* (see the left panel of Figure 2). Seller types just above θ_P cost little in terms of (OC_0)—$(\theta - \theta_P)(1 - \alpha(\theta, r))$ is close to 0—but yield non-negligible profits to the platform. The highest seller types (above θ^*) are costly, but they are particularly lucrative to the platform.

Next, consider the optimal referral fee r^I (say) when the platform uses the associated optimal signal design. The platform's optimal referral fee solves (recall that θ_S, θ_P, and α also depend on r):

$$\mathcal{P}_I : \max_{r \in [0,1]} \quad \Pi_P^I(r) := \int_{\theta_S}^{\theta_P} r\theta d\theta + \int_{[\theta_P, \theta_*] \cup [\theta^*, 1]} \left[(1 - \alpha(\theta))r\theta + \alpha(\theta)\left((1 - \delta)r\theta + \delta\theta - K\right)\right] d\theta.$$

Let r^I be the solution to $\mathcal{P}_I$. Notice that

$$\Pi_P^I(r) = \Pi_P^C(r) - \int_{\theta_*(r)}^{\theta^*(r)} \left[r\theta + \alpha(\theta; r)\left[(1 - r)\,\delta\theta - K\right]\right] d\theta. \tag{7}$$

As noted earlier, if (OC_0) does not bind at r^C, the optimal fee is the same as that under full commitment case, i.e., $r^I = r^C$. In this case, $\theta_*(r^C) = \theta^*(r^C) = 1$ and the platform implements the full commitment outcome. Otherwise, if the platform sets $r = r^C$, in order to meet the (OC_0) constraint it must exclude entry for some of the intermediate types of the seller (i.e., for all types $\theta \in [\theta_*(r), \theta^*(r)]$). Thus, the optimal fee, r^I, may differ from r^C. As (4) indicates, r^I balances the trade-off between the loss relative to the full-commitment payoff and gains from relaxing (OC_0) that may allow for more entry from the intermediate types.

It is reasonable to conjecture that $r^I > r^C$. Raising r from r^C would have a second-order effect on Π_P^C (the first term in the expression of Π_P^I in (4)) but it increases θ_P and may relax (OC_0), consequently, having a first-order effect on the loss due to fewer entry of the intermediate types of the sellers (captured by the second term in (4)). Unfortunately, the comparative statics for the cutoffs $\theta_*(r)$ and $\theta^*(r)$ appear to be analytically intractable, and hence, a formal proof of this conjecture remains elusive. But numerical solution to the platform's program $(\mathcal{P}_I)$ conforms to our conjecture.

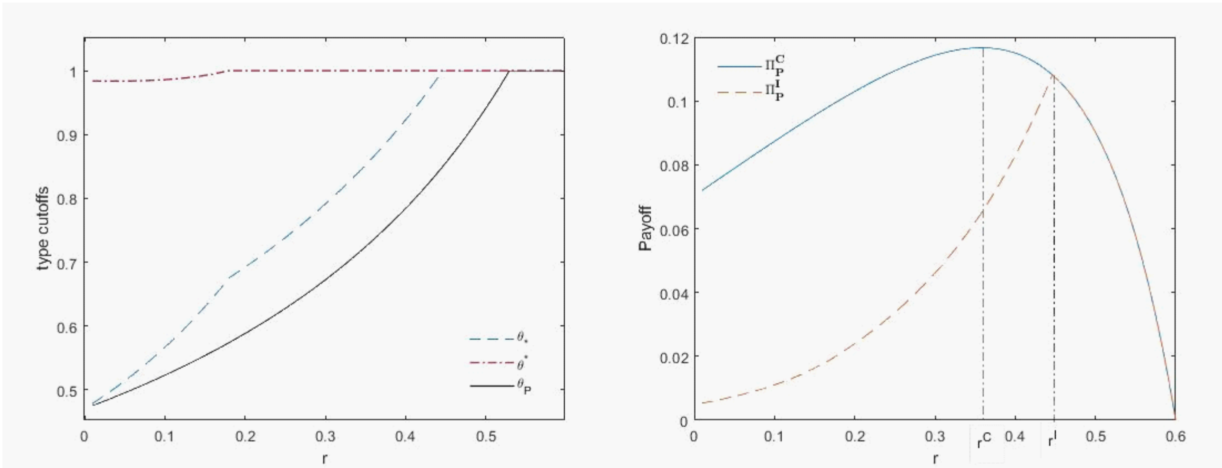

Figure 2: Left panel – θ_* and θ^* as function of r. Right panel – platform's payoff as a function of referral fee ($K = 0.4$ and $\delta = 0.85$).

As shown in Figure 2, increasing r tend to foster more entry as the set of excluded types $[\theta_*(r), \theta^*(r)]$ gets smaller with r. Moreover, the optimal fee r^I is the smallest value such that the obedience constraint (OC_0) does not bind. That is, at the optimum, the seller's entry decision continues to follow the cutoff rule where all types $\theta \geq \theta_S$ enter. While there is no exclusion of the intermediate types, the higher fee does restrict entry of the low types by raising the entry cutoff θ_S.

6. Implications for regulatory measures

In light of our above discussion we can now explore the welfare implications of two salient policy proposals for regulation of platform-run marketplaces: prohibition on imitating the third-party sellers' products, and prohibition on use of proprietary data on sellers' products. The implications for the former policy—ban on imitation—can be readily obtained from our analysis of the case where the platform operates only as a marketplace. The welfare implication of such a policy depends on the relative ranking of—r^I and r^{NE}—the equilibrium referral fees when there are no regulations and when the platform commits not to enter. Clearly, when the full commitment case can be implemented through information policy, i.e., we have $r^I = r^C < r^{NE}$, prohibition on platform's entry is welfare reducing for the consumers. But otherwise, r^I and r^{NE} cannot be ordered a priori, and when $r^I > r^{NE}$, a ban on imitation improves welfare.

The implications for the latter policy—ban on the use of (entry) decision-relevant information from the third-party sellers—is more subtle. Our model implies that welfare implications of such a policy is again ambiguous, and may in fact lead to a welfare loss.

If information usage is ruled out by regulators, but there is no restriction on imitation, the platform's entry decision would be only based on the seller's entry threshold. In what follows, we first characterize the platform's entry decision for a given referral fee. Let $\alpha \in [0,1]$ denote the probability that the principal launches its own product by imitating the seller. Given (r, α), a type-θ seller enters if and only if

$$(1-\alpha\delta)(1-r)\theta \geq K \Leftrightarrow \theta \geq \theta_S(r,\alpha) := \frac{K}{(1-\alpha\delta)(1-r)}.$$

As discussed earlier, for $r \geq \overline{r} := 1 - \frac{K}{\delta}$, $\theta_P(r) = 1$, i.e., the platform never enters. Therefore, for any such $r \geq \overline{r}$, the seller's entry threshold is

$$\hat{\theta}_S(r) = \min\{\theta_S(r,0), 1\},$$

and the principal's profit is given by

$$\Pi_P^{NI}(r) := \int_{\hat{\theta}_S(r)}^{1} r\theta d\theta = r\mathbb{E}\left[\theta \mid \theta \geq \hat{\theta}_S(r)\right].$$

For $r < \overline{r}$ (equivalently, $\theta_P(r) < 1$), there are multiple equilibria. One can construct a "futile" equilibrium where the seller never enters. In this equilibrium, given r $(< \overline{r})$ the seller believes that the platform will enter with probability 1 (i.e., $\alpha = 1$). If so, the seller would not enter regardless of the his type. And this outcome can be supported as a perfect Bayesian Equilibrium with the platform's belief that assigns probability 1 to $\theta = 1$ as the entering seller's type.

Now we consider the case in which (in equilibrium) the platform does not enter (i.e., $\alpha = 0$). This equilibrium exists if and only if

$$\mathbb{E}\left[\theta \mid \theta \geq \hat{\theta}_S(r)\right] \leq \theta_P(r) \Leftrightarrow \int_{\hat{\theta}_S(r)}^{1} (\theta - \theta_P(r))\, d\theta \leq 0. \tag{8}$$

This inequality reduces to

$$\frac{1}{2}(1-\theta_P)^2 - \frac{1}{2}\left(\hat{\theta}_S - \theta_P\right)^2 \leq 0 \Leftrightarrow r \geq \underline{r} := 1 - \frac{2-\delta}{\delta}K. \tag{9}$$

It is easy to see that $\underline{r} < \overline{r}$. Also note that

$$\underline{r} \geq 0 \Leftrightarrow K \geq \frac{\delta}{2-\delta}.$$

As $K \in (1-\delta, \delta)$, we therefore have the following two cases: if $K \in \left(\frac{\delta}{2-\delta}, \delta\right)$ then $\underline{r} \leq 0$, so this equilibrium exists for all $r < \overline{r}$. Otherwise (i.e., $K \in \left(1-\delta, \frac{\delta}{2-\delta}\right)$), this equilibrium does not exist for $r < \underline{r}$.

One may explore if there could be a mixed strategy equilibrium where the platform enters with probability α, particularly when the "no entry" equilibrium for the platform does not exist. For such a mixed strategy equilibrium to exist, the platform should be indifferent between entering and not entering. Therefore, it must be that

$$E\left[\theta \mid \theta \geq \theta_S(r, \alpha)\right] = \theta_P(r) \Leftrightarrow \int_{\theta_S(r,\alpha)}^{1} \left(\theta - \theta_P(r)\right) d\theta = 0.$$

But this equality cannot hold whenever $r < \underline{r}$. From (8) and (9) it follows that for any such r we would have

$$\mathbb{E}\left[\theta \mid \theta \geq \theta_S(r, 0)\right] > \theta_P(r),$$

and since $\theta_S(r, \alpha)$ is increasing in α, for all $\alpha > 0$, we have

$$\mathbb{E}\left[\theta \mid \theta \geq \theta_S(r, \alpha)\right] > \mathbb{E}\left[\theta \mid \theta \geq \theta_S(r, 0)\right] > \theta_P(r).$$

This observation implies that if $0 \leq r < \underline{r}$, then the above futile equilibrium is the only equilibrium. Therefore, with a prohibition on information sharing, either the market ceases to exist or the platform never enters. Consequently, the welfare implication of such a policy cannot be ascertained a priori. Even in the equilibrium where the seller enters, the optimal referral fee would be r^{NE} (as the platform does not enter), and as we have already argued, in general, r^{NE} and r^I cannot be ordered. And if $r^I < r^{NE}$, prohibition on information sharing would reduce welfare.

7. Conclusion

The use and misuse of proprietary data on third-party sellers by hybrid platforms have drawn considerable attention from the regulatory agencies. Leading platform-run marketplaces, such as Apple App Store and Amazon Marketplace, are alleged to use third-party sellers' product data to target and copy successful products forcing the third-party sellers to exit. Such a practice by the platforms, often termed as "Sherlocking", may stifle innovation and entry by third-parties. The anticompetitive implications of this practice have led several regulatory agencies in the U.S. and the European Union to propose limits on the platform's behavior, including ban on imitation and ban on date sharing between the platform's marketplace and product division.

However, the policy on third-party sellers' data usage is not the only strategic lever that a platform has at its disposal. The platform also chooses the referral fee that the sellers must pay to sell their product on the marketplace and decides on its own entry policy. Thus, in order to assess the platform's response to a regulatory limit on its data policy, one must analyze the interplay between its fee structure, data policy, and entry decision. In this article, we present a stylized model of hybrid platform to explore this issue. Our findings indicate that in a broad range of parameters, the platform's commitment to data policy can be a substitute for commitment to entry. And an outright ban either on entry by imitation or data sharing, in general, may be welfare reducing as the platform adjusts its referral fee in response to such policies.

8. Appendix

This Appendix contains the proofs omitted in the text.

Proof of Proposition 1. Notice that

$$\Pi_P^{NE}(r) - \Pi_P^{NC}(r) = \int_{\theta_P(r)}^{1} r\theta d\theta \quad \begin{cases} > 0 & if\ r < \overline{r} \\ = 0 & otherwise \end{cases},$$

where $\overline{r}$ denote the value such that $\theta_P(\overline{r}) = 1$, that is, $\overline{r} = 1 - K/\delta$. It is immediate that $r^{NC} = r^{NE}$ if $r^{NE} \geq \overline{r}$.

Thus, it suffices to show that $r^{NC} \geq \overline{r}$: if $r^{NE} \geq \overline{r}$ then $r^{NE} = r^{NC}$, while if $r^{NE} < \overline{r}$ then $r^{NE} \leq r^{NC}$. For any $r < \overline{r}$, we have $\theta_S(r) < \theta_P(r)$ and

$$\theta_S'(r) = \frac{K}{(1-r)^2} < \theta_P'(r) = \frac{K}{\delta(1-r)^2} = \frac{1}{\delta}\theta_S'(r).$$

Therefore,

$$\frac{d}{dr}\Pi_P^{NC}(r) = \int_{\theta_S(r)}^{\theta_P(r)} \theta d\theta + r\left[\theta'_P(r)\theta_P(r) - \theta'_S(r)\theta_S(r)\right] > 0.$$

This means that $\Pi_P^{NC}(r)$ is strictly increasing whenever $r < \overline{r}$, so $r^{NC} \geq \overline{r}$. □

Proof of Proposition 2. Since the result is immediate if $r^C < \overline{r} \leq r^{NE}$, it suffices to consider the case where r^C, $r^{NE} < \overline{r}$. At any $r < \overline{r}$, we have

$$\frac{d}{dr}\left(\Pi_P^C(r) - \Pi_P^{NE}(r)\right) = \frac{d}{dr}\int_{\theta_P(r)}^1 \alpha(\theta; r)\left[\delta(1-r)\theta - K\right] d\theta < 0,$$

where the inequality holds because both $\alpha(\theta; r)$ and $\delta(1-r)\theta - K$ are positive and decreasing in r. In other words, $\Pi_P^C(r)$ is strictly decreasing whenever $\Pi_P^{NE}(r)$ is decreasing. Assuming that $\Pi_P^C(r)$ is quasi-concave, this implies that $r^C < r^{NE}$. □

Proof of Lemma 1. For each x, let $\beta(x)$ denote the probability that the principal enters. Clearly, (since the principal cannot commit to its launching policy)

$$\beta(x)\begin{cases} = 0 & \text{if } \mathbb{E}[\theta|x] < \theta_P \\ \in [0,1] & \text{if } \mathbb{E}[\theta|x] = \theta_P \\ = 1 & \text{if } \mathbb{E}[\theta|x] > \theta_P \end{cases}.$$

Notice that the probability that the principal enters conditional on θ is given by

$$\widehat{\beta}(\theta) = \int \beta(x) d\sigma(\theta).$$

Consider the following binary signal: $\widehat{X} = \{0,1\}$ and for each θ, $\sigma(\theta)$ assigns probability $1 - \widehat{\beta}(\theta)$ to 0 and probability $\widehat{\beta}(\theta)$. By construction, this signal induces the same outcome, provided that the principal's obedience constraint is satisfied (i.e., $\mathbb{E}[\theta|0] \leq \theta_P$ and $\mathbb{E}[\theta|1] \geq \theta_P$). To see that this last requirement automatically holds from our construction, divide the set X as follows:

$$\begin{aligned} X_< &:= \{x \in X : \mathbb{E}[\theta|x] < \theta_P\}, \\ X_= &:= \{x \in X : \mathbb{E}[\theta|x] = \theta_P\}, \\ \text{and } X_> &:= \{x \in X : \mathbb{E}[\theta|x] > \theta_P\}. \end{aligned}$$

By our construction, each $x \in X_<$ is mapped to 0 in our binary signal, each $x \in X_=$ is split between 0 and 1, and each $x \in X_>$ is mapped to 1. This implies that

$$\mathbb{E}[\theta|0] \leq \mathbb{E}[\theta|x \in X_< \cup X_=] \leq \theta_P \text{ and } \mathbb{E}[\theta|0] \geq \mathbb{E}[\theta|x \in X_= \cup X_>] \geq \theta_P.$$

□

Proof of Proposition 4. The proof is given by the following steps.

Step 1. Let $\phi : [0,1] \to \{0,1\}$ be the entry decision of the seller ($\phi = 1$ if the seller enters) induced by the information structure (and r). As aruged in Section 5.2, we have $\Pr(x = 1 \mid \theta) = \alpha(r, \theta)$ for all θ such that $\phi(\theta) = 1$. In addition, the platform prefers the seller to enter whenever $\theta \in [\theta_S(r), \theta_P(r)]$, as it not only yields direct benefits to the platform, but also relaxes (OC_0) by raising the right-hand side in (5). This implies that the platform's problem can be written as[10]:

$$\max_{\phi(\cdot)\in\{0,1\}} \int_{\theta_P}^{1} \phi(\theta)\left[(1-\alpha(\theta,r))r\delta\theta + \alpha(\theta)(\delta\theta - K)\right] d\theta$$

$$\text{s.t.} \quad \int_{\theta_P}^{1} \phi(\theta)(\theta-\theta_P)(1-\alpha(\theta,r))d\theta \leq \int_{\theta_S}^{\theta_P} (\theta_P - \theta)d\theta. \tag{10}$$

Step 2. Consider the associated Lagrangian:

$$\mathcal{L} = \int_{\theta_P}^{1} \phi(\theta)\left[(1-\alpha(\theta))r\delta\theta + \alpha(\theta)(\delta\theta - K) - \lambda(\theta-\theta_P)(1-\alpha(\theta))\right] d\theta + \lambda\int_{\theta_S}^{\theta_P} (\theta_P - \theta)d\theta.$$

Since this is linear in $\phi(\theta)$, it is immediate that $\phi(\theta) = 1$ if

$$H(\theta,\lambda) := (1-\alpha(\theta))\left(r\delta\theta - \lambda\left(\theta - \theta_P\right)\right) + \alpha(\theta)(\delta\theta - K) > 0,$$

and $\phi(\theta) = 0$ if $H(\theta, \lambda) \leq 0$.

Step 3. Routine calculation yields,

$$\frac{\partial^2}{\partial\theta^2}H(\theta,\lambda) = \frac{2}{\theta^3}\left(\left(1-r\right)\delta + \lambda\right)\left(\frac{K}{\left(1-r\right)\delta}\right)^2 > 0,$$

hence, H is convex in θ. Now $H(\theta_P, \lambda) = (1-\alpha(\theta_P))\left(r\delta\theta_P\right) + \alpha(\theta_P)(\delta\theta_P - K)$. Plugging $\alpha\left(\theta_P\right) = \frac{1}{\delta} - 1$, we obtain

$$H(\theta_P, \lambda) = \frac{rK}{1-r} > 0.$$

Thus, if $H \leq 0$ for some θ, we have an interval $[\theta_*, \theta^*]$ where $\theta_* > \theta_P$ and $\theta^* \in [\theta^*, 1]$ such that $H \leq 0$ if and only if $\theta \in [\theta_*, \theta^*]$. And if $H > 0$ for all θ, we set $\theta_* = \theta^* = 1$. Thus,

[10] To simplify notation, we do not explicitly note the dependency of θ_P, θ_S, and $\alpha(\theta)$ on r.

at the optimum, $\phi(\theta) = 1$ for all $\theta \in [\theta_P, \theta_*] \cup [\theta^*, 1]$ and $\phi(\theta) = 0$ for all $\theta \in [\theta_*, \theta^*]$. The signal structure (6) follows as having $x = 1$ with certainty for all $\theta \in [\theta_*, \theta^*]$ relaxes (10) and thwarts entry of the types in $[\theta_*, \theta^*]$. □

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