SKILL ACQUISITION UNDER IMPLICIT CONTRACT

ARIJIT MUKHERJEE

Abstract. Firm specific skill acquisition involves dual moral hazard problem. Existing literature suggests that explicit incentive through promotion offer can solve this problem under some conditions. But these solutions necessarily yield inefficiency. In a repeated game framework we consider explicit incentives along with implicit incentive through relational contract. This paper models the interplay between the explicit and implicit incentive for firm specific skill aquisition. We find that here explicit and implicit incentive are substitutes. The more the firm relies on implicit incentive the less is the explicitly contracted wage differential across jobs. Implicit contract increases the total surplus when coupled with the explicit contract. It may induce skill acquisition even when the promotion rule by itself can't create enough incentives. Moreover, for a sufficiently patient firm the first best is attained. Compensation differential across jobs vanishes completely at the first best.

1. Introduction

The literature in firm specific human capital accumulation identifies the following incentive problem: unless the skill acquisition is enforceable by contract there is a dual moral hazard problem. The worker needs to be paid a premium for acquisition of skill. At the same time, firm has no incentives to pay the premium once the skill is acquired. Therefore the worker never acquires skill in a subgame perfect equilibrium.

Existing literature shows that if job assignment is verifiable, explicit contracts involving promotions *may* induce skill acquisition. Prendergast (1993) showed that up-or-stay promotion rule may create incentives both for the worker (to acquire skill) and the firm (to reward skill acquisition).

But there are two problems with this solution. First, when workers are heterogeneous in their productivity in each of the two jobs, then such mechanism yields misallocation of workers. Second, this solution requires that the return to human capital in the two jobs to be sufficiently different. Hence the two jobs have to do with different technologies rather than with two mere job titles. But often promotion actually means only a change in the job title[1].

Date: *First Draft:* June, 2003; *This Draft:* May 2004.

I am indebted to Alessandro Pavan and Michael Whinston for their invaluable guidance throughout this project. I have greatly benefited from the discussions with Scott Scheafer. I would also like to thank Heski Bar-Isaac, Maria Goltsman, Jonathan Levin, and Marcin Peski for their helpful comments and suggestions. All remaining errors are mine.

[1]For example, in Microsoft, software developers go trough "ladder levels". A fresh recruit joins in level 30, say, moves to level 31 after demonstrating sufficient programming skills. Then to level 32 and so on. Each

Kahn and Huberman (1988) suggested up-or-out promotion scheme as an incentive device to induce skill acquisition. Here jobs need not have to be different as is the case in Prendergast (1993). But this solution may yield inefficiency if the level of skill acquisition is a stochastic function of the investments made to acquire skills[2].

In such an environment, use of promotion to provide incentive is necessitated by the *nonverifiability* of skill acquisition and workers productivity. Though unobserved by a third party, both of these are often observed by the 'insiders'. This opens up a possibility to provide incentives through relational or implicit contract. Firm can credibly commit to a bonus payment geared to the acquisition of skill if it accumulates enough 'reputational capital' (a la Rayo (2002)), i.e., the surplus generated through the repeated interaction of between the firm and the workers. Here we model the interplay between the explicit incentive of up-or-stay promotion and the implicit incentive of bonus payment as instruments for inducing skill acquisition.

We find that these two sources of incentive are substitutes. There is a loss of efficiency associated with explicit incentive due to misallocation of workers between jobs. Implicit incentive has no such 'shadow cost'. The power of implicit incentive is limited by the size of 'reputational capital'. Hence, the more the 'reputational capital' the firm accumulates, the more it will use implicit incentive relative to the explicit incentive. With sufficiently high 'reputational capital' the firm will do away with explicit incentive altogether and incentive will be provided through implicit contract only. Therefore the aforementioned inefficiency a la Prendergast disappears.

The intuition for this result is the following: the inefficiency in Prendergast's (1993) model arises from the fact that the firm has to give a wage premium in the difficult job to provide incentive. First best requires that the promotion should be offered whenever the worker is more productive in job D relative to job E. But for such premium to be incentive compatible for the firm, promotion is offered only if the worker is *sufficiently more* productive in the difficult job relative to the easy one.

With implicit contract the firm, as before, promotes a worker only if the productivity differential outweighs the compensation differential in the two jobs. But here compensation differential in the two jobs consists of wage differential and differential in bonus payments. With no room for reputation, compensation differential must exists. Else, there is no way to provide incentive for skill acquisition. With reputational concern, bonus payments can be tied to skill acquisition. So if the bonus payments are sufficiently large even with little

level is more difficult to cross compared to its predecessor. These levels are simply promotion offers that come with a raise in the wage. But all the levels are basically different job titles, the job stays the same. If we consider such "ladder levels" as an incentive for the worker to acquire firm specific skill, Prendergast's (1993) model fails to provide any justification.

[2]See Gibbons (1998) for a brief discussion on these issues.

dispersion across jobs, it provides enough incentive for the worker while allowing the firm to reduce explicit wage differential. Therefore total differential in compensation reduces. Consequently, required productivity differential across jobs that makes promotion viable, decreases.

With discount factor being zero the only sustainable equilibrium is the one discussed in Prendergast (1993). As the discount factor of the firm goes from zero to one, the compensation differential reduces. Hence misallocation of worker is less severe and total surplus increases monotonically. Moreover difference in wages between the two jobs shrinks. This is to say that implicit incentive substitutes its explicit counterpart to the maximum feasible level. We will show that in this environment the first best can be supported if the discount factor is sufficiently high.

We further show that implicit contract coupled with explicit contract increases total surplus. It may induce skill acquisition even when explicit contract by itself is not sufficient. There are two observations that lead to this result. First, implicit contract substitutes explicit contract. So by using them together the firm gains on efficiency ground. Second, the power of these two incentives is limited by two independent factors. How much explicit incentive the firm can provide depends the underlying technology. In contrast, implicit incentive is limited by the reputational capital of the firm. Hence explicit incentive can be inadequate even when implicit incentive is strong enough to induce skill acquisition.

Related Literature

There are several surveys of the literature on incentives in organizations. Gibbons (1998), Gibbons and Waldman (1998) Malcomson (1998) and Prendergast (1993) are to name a few. A related survey in executive compensation is done by Murphy (1998). As we have already mentioned, Prendergast (1993) shows that up-or-stay promotion scheme can induce skill acquisition in the presence of the dual moral hazard problem. The basic idea is as follows: let there be two types of jobs - difficult (D) and easy (E). A worker is relatively more productive in D if she acquires skill and in E if she doesn't. The firm announces wages for the two jobs and is committed to pay so. D carries a wage premium. Given that an appointment of a skilled worker in job D is more profitable than appointing in job E, the firm has an incentive to promote the skilled worker. This foresight along with the wage premium drives the worker to acquire skill. Kahn and Huberman (1988) shows that up-or-out promotion may create incentives even when the up-or-stay promotion is ineffective. After a stipulated time period the firm either promote the worker with a higher wage or fires him. One of the early works in implicit contract is by Bull (1987). Bull provides the condition under which a implicit contract can be sustained if players adopt trigger strategies. MacLeod and Malcomson (1989) characterize the optimal implicit contract when employees' performance is observable but not verifiable. They show that there may exist a variety of self enforcing implicit contracts as perfect equilibria of a repeated game. Baker et al. (1994) provide an analysis of the optimal contract when the firm can reward the worker on the basic of both verifiable and non-verifiable measures. A more general approach to the issue of implicit contract is by Levin (2003). He characterizes the optimal contract in an asymmetric information setting. This is to say when the effort (or type of the worker) is not observed by the principal leading to a moral hazard (adverse selection) problem.

This paper is organized as follows. In section 2 the basic model is set up and some benchmark cases are discussed. Section 3 deals with implicit contract in this environment and contains a few results that are useful in solving the model. The optimal contract is discussed in section 4. A final section concludes.

2. The model

2.1. **Organizational Form.** Our model is an extension of Prendergast (1993) where we put the initial model in a repeated game framework and allow for bonus payments. Consider a repeated game where one long run player, namely the firm, faces a sequence of short run players, namely the workers. Both set of players are risk neutral. In every even period a new worker is hired who stays with the firm for two periods. Let k be the index of time $(k = 0, 1, 2, ...)$. Generation t worker is hired in $k = 2t$ period, $(t = 0, 1, 2, ...)$. Workers have an outside option that gives her 0 utility. The firm can't fire workers. To keep things simple we assume that the workers do not discount future. The firm discounts the payoff from each generation by the factor of δ.[3]

Workers are drawn from a pool with known distribution of productivity. Workers in their first period are ex ante identical with no firm-specific skills. However, in the first period of their career, workers can acquire firm specific human capital. Skill acquisition requires effort $e \in \{0, 1\}$. Skill is acquired only if $e = 1$. Disutility from effort is c. To focus on the issue of skill acquisition we assume that no effort is needed to produce output.

There are two types of job: difficult (D) and easy (E). A new worker can only join in job E. At the beginning of the second period the firm may promote her to job D if it finds that profitable. Type of a worker (θ), governs her productivity in the two jobs. At the first period of her career, neither the worker nor the firm knows her θ. Type is revealed only at the beginning of the second period and is observed by both parties. We assume θ to be uniformly distributed over the interval $[0, 1]$.

[3] i.e., payoff for $k = 2$ and 3 is discounted by δ, for $k = 4$ and 5 is discounted by δ^2 and so on. Discounting every 'generation' rather than every 'period' makes the subsequent analysis simpler.

Effort (e) and type (θ) are *observable to both parties but not verifiable.* But job assignment is both *observable and verifiable.* We further assume that future worker can *observe* past sequence of e but *not* θ.

Given the type θ and the effort e, let $y_E(\theta, 0)$ be the output of the worker in the first period and $\langle y_i(\theta, e)\rangle_{i=D,E}$ be the output in the second periods in job D and E. We assume the following on the y values.

Assumption 1: $i)$ $y_E(\theta, 1) - \max_{j\in\{D,E\}} \{y_j(\theta, 0)\} \geq c\ \forall\theta$

$ii)$ $y_D(\theta, 1) - y_E(\theta, 1) \geq y_D(\theta, 0) - y_E(\theta, 0)\ \forall\theta$

$iii)$ $y_{D_1}(\theta, e) \geq y_{E_1}(\theta, e) > 0\ \forall e$

These assumptions have the following interpretations: $i)$ implies that skill acquisition is efficient even if the skilled worker is never promoted, $ii)$ implies increasing difference between productivity in the two jobs *w.r.t.* e for a given type θ and $iii)$ implies increasing difference between productivity in the two jobs *w.r.t.* θ. Observe that $iii)$ refers to the single crossing property of the output functions[4]. Define θ^{FB} and $\bar{\theta}$ by the following equations: $y_D(\theta^{FB}, 1) - y_E(\theta^{FB}, 1) = y_D(\bar{\theta}, 0) - y_E(\bar{\theta}, 0) = 0$. We assume that θ^{FB} & $\bar{\theta} \in (0, 1)$. From assumption $ii)$ and $iii)$ we conclude that $\theta^{FB} < \bar{\theta}$. Figure 1 depicts the outputs as a function of θ for each of the effort levels.

For every generation of worker, at the beginning of the her career the firm announces the wages for both the jobs in an explicit contract. Let the wage in job E for generation-t be $\omega_t \in \mathbb{R}$ while wage in job D be $\omega_t + w_{Dt}$ where $w_{Dt} \geq 0$. w_{Dt} can be interpreted as a wage premium in the difficult job[5],[6].

[4]Define $y_i(\theta, e) = y(j, e \mid \theta)$ where $j = 1$ if $i = D$ and $j = 0$ otherwise. These assumptions implies that $y(j, e \mid \theta)$ is supermodular in (j, e).

[5]Observe that the base wage can be both negative or positive. Negative wage can be interpreted as a payment from the worker to the firm. All that the worker cares about is the expected wage in the two periods of her career. As long as that is positive a negative base wage is possible as an equilibrium outcome.

[6]We keep the wages in the two jobs remain the same in both the periods in a workers career. Given our information structure this restriction, this is done without any loss of generality.

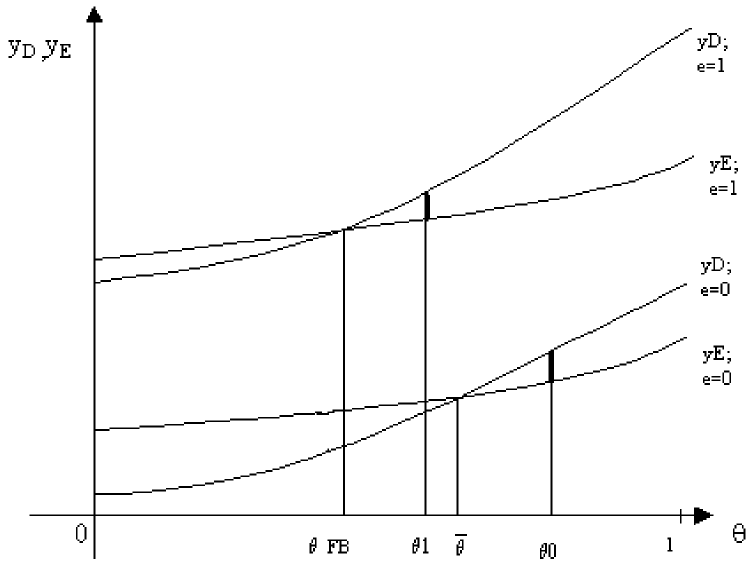

*Figure*1 :Output as a function of θ

In the second period of the worker's career there are two channels of providing incentives for skill acquisition. The firm can either promote the worker in job D or can pay a bonus to a worker. Let the bonus for generation t in job E be b_{Et} and that in job D be b_{Dt}. The bonus payment is sustained by reputation only and hence is a part of implicit contract. The only contract that can be written with generation t, for any t, specifies only the wages in each job for the two periods of the workers career. Figure 2 depicts the timing in a stage game.

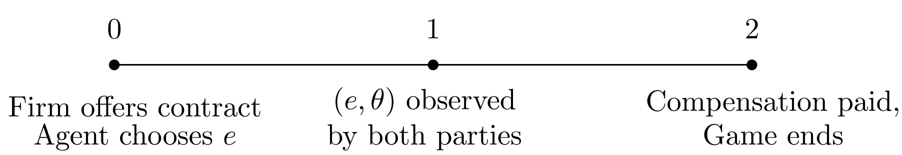

FIGURE 2

To formalize the compensation plan, consider the mapping $B_t : \Phi \to \mathbb{R}_+ \times \{D, E\}$ where Φ is a set of all possible (θ_t, e_t) tuples. $\mathcal{B}$ be the set of all such mapping B_t. $B_t \in \mathcal{B}$ assigns an incremental payment over and above ω_t and a job to each realization of (θ_t, e_t) . Let $B_t^1(\theta_t, e_t)$

denote the first component of $B_t(\theta_t, e_t)$ and is defined as follows:

$$B_t^1(\theta_t, e_t) = \begin{cases} b_{Et} & if \quad e_t = 1 \text{ and } \theta_t < \theta_{1t} \\ w_{D_t} + b_{Dt} & if \quad e_t = 1 \text{ and } \theta_t \geq \theta_{1t} \\ 0 & if \quad e_t = 0 \text{ and } \theta_t < \theta_{0t} \\ w_{D_t} & if \quad e_t = 0 \text{ and } \theta_t \geq \theta_{0t} \end{cases} \tag{1}$$

where θ_{1t} and θ_{0t} are to be solved endogenously. If skill is acquired (not acquired) the worker is promoted $iff\ \theta_t \geq \theta_{1t}\ (\theta_t \geq \theta_{0t})$.

Given the information structure in our model, the only class of compensation plan that can be sustained in equilibrium is the one in (1). This claim follows from two things. First, future workers can only observe past e and not the θ. Therefore the bonus amounts in any specific job can't vary with θ. Second, the form of the $y_D(\cdot)$ and $y_E(\cdot)$ functions - the single crossing and increasing difference properties as formalized in assumption 1 - ensures that the optimum promotion rule must involve a cut off strategy as in (1).

For the generation t worker, the compensation in the second period of her career is $W_t = \omega_t + B_t^1(\theta_t, e_t)$. The payoff for generation t worker over the two periods is $u_t = E_\theta(\omega_\tau + W_\tau) - ce_t$. In generation t, firm's payoff is $\pi^t = E_\theta \sum_{\tau=t}^{\infty} \delta^{\tau-t}\pi_t$ where $\pi_\tau = y_\tau - (\omega_\tau + W_\tau)$ is the profit from generation τ, y_τ being the output of generation τ.

We define expected social surplus from a generation-t worker as $S_t(e_t, \theta_t) = E_\theta y_t - ce_t = u_t + \pi_t$. That is, for every generation the surplus is the sum of the payoff of that generation's worker and the profit of the firm in that generation. An optimal contract is the one that maximizes the expected social surplus. As skill acquisition is efficient the first best effort level $e^{FB} = 1$.

2.2. **Benchmark cases.** Before we go into the discussion of the repeated game let us consider the following benchmark cases. First we will consider the case where only explicit incentive is feasible with no room for reputation. This static game is just the stage game of our dynamic model when the firm has no future concerns. Characterization of the equilibrium of this game will be useful in studying our dynamic game.

Second, we will explore case where only implicit incentive is feasible with no room for explicit contract. We will further put the restriction that future workers observe only the job placement, both e and θ are not observed .We will characterize the optimal payoff as a function of δ. This observation serves as a benchmark for our main comparative static result where we will study the same in the more general dynamic model with richer information structure.

2.2.1. *Equilibrium in Static Game.* Here the strategy of the firm is to offers a tuple $\sigma = (\theta_0, \theta_1, \omega, b_E, w_D, b_D)$. The worker observes σ and chooses e. Recall that the firm can't renege on the wages ω and w_D but can renege on the payment of the bonus b_E & b_D and the 'cut points' θ_0 & θ_1. In a subgame perfect equilibrium the firm doesn't reneges on the initial offer σ. We will investigate the equilibrium that maximizes the expected social surplus.

In absence of reputation concern the equilibrium resembles the one in Prendergast (1993). Observe that in equilibrium $b_E = b_D = 0$ as the firm has no incentive to pay the bonus in a 'one shot' game. A SPNE must satisfy the incentive compatibility and individual rationality constraints of the two players. First consider the worker's incentives. The worker's outside option is zero. So in an equilibrium where the worker acquires the skill, the participation constraints for the worker is:

$$2\omega + (1-\theta_1)\, w_D \geq c \tag{2}$$

Worker's incentive compatibility constraint requires that the expected gain from acquiring the skill should at least as large as the disutility from effort, i.e.,

$$(\theta_0 - \theta_1)\, w_D \geq c \tag{3}$$

On the other hand the firm promotes a worker only when it finds it profitable to do so. This leads to the following incentive compatibility constraints for the firm:

$$y_D(\theta_1, 1) - y_E(\theta_1, 1) = w_D \tag{4}$$

$$y_D(\theta_0, 0) - y_E(\theta_0, 0) = w_D \tag{5}$$

Note that the contract that maximizes the profit of the firm is also the one that maximizes the joint surplus. The firm always extracts all the surplus from the worker, i.e., (2) always binds in equilibrium[7]. We will look for the contract that yields the highest expected profit subject to the above constraints.[8]

The problem is to choose a feasible contract that is associated with the smallest θ_1 such that $\theta_1 \geq \theta^{FB}$. Observe that there may not exist an equilibrium where the worker acquires skill. Equation (3) implies[9] that at the optimum, $w_D = c/(\theta_0 - \theta_1)$. So if $(\theta_0 - \theta_1)$ is small then the required w_D is large. This will be the case if the returns to training are similar in the two jobs, i.e., if the jobs are similar in nature. But then there may not exist any θ_1 (*or* θ_0) value for which (3) (*or* (4)) holds. In such a case the only SPNE is that the worker doesn't acquire skills and the firm offers $\left(\bar{\theta}, \theta^{FB}, 0, 0, 0, 0\right)$. Let the optimal contract where $e = 1$, if exists, be $\left(\theta_0^P, \theta_1^P, \omega^P, 0, w_D^P, 0\right)$. Denote the expected profit associated with the optimal contract in this game as $\underline{\pi}$ and the expected utility as $\underline{u}$. Further denote the expected surplus

[7]The base wage ω doesn't play any role in creating incentive for skill acquisition. If the profit maximizing surplus is different from the surplus maximizing one, the firm can move on to the contract that maximizes the joint surplus. It can then extract entire surplus from the worker by choosing an appropriate ω without destroying any incentive for skill acquisition and be better off.

[8]This will pin down ω in the equilibrium, which is not the case if we look for the contract that maximizes the social surplus only.

[9]Note that it must also bind in equilibrium.

as $\underline{s} = \underline{\pi} + \underline{u}$. Note that in an equilibrium with $e = 1$, $\theta_1^P > \theta^{FB}$ as $w_D^P > 0$. So all the workers with $\theta \in \left(\theta^{FB}, \theta_1^P\right)$ are inefficiently assigned. They are more productive in job D though are kept in job E. The firm promotes a worker only if her productivity difference in the two jobs is equal to the wage premium in the difficult job.

If we keep our attention limited to only subgame perfect pure strategy equilibria, the worst payoffs that the two parties can get in a repeated game are $\underline{\pi}$ and $\underline{u}$. Further observe that in a repeated game with no reputation concern the best SPNE is the one where this static solution is played in every period.

2.2.2. *Repeated game without explicit contract.* Here we modify our initial model by assuming that no formal wage contract can be written. Further suppose that both θ and e are unobserved by the future generations. So bonus payments conditional on skill acquisition is not sustainable in equilibrium. All we can sustain is a job specific wage. Wage in the two jobs are sustained by reputation only. Here the relevant constraints are the same as in the benchmark static model but with an additional constraints for dynamic incentive compatibility. As before, what matters is the difference w_D. The set of constraints are as follows:

$$\begin{array}{ll} w_D\left(\theta_0 - \theta_1\right) \geq c & (a) \\ y_D\left(\theta_1, 1\right) - y_E\left(\theta_1, 1\right) \geq w_D & (b) \\ y_D\left(\theta_0, 0\right) - y_E\left(\theta_0, 0\right) \geq w_D & (c) \\ \frac{\delta}{1-\delta}\left(\bar{\pi} - \underline{\pi}\right) \geq w_D & (d) \end{array}$$

The last constraints is derived from the dynamic incentive constraint $y_D\left(\theta, e\right) - w_D + \frac{\delta}{1-\delta}\bar{\pi} \geq y_D\left(\theta, e\right) + \frac{\delta}{1-\delta}\underline{\pi}$ $\forall \theta \geq \theta_1$ when $e = 1$ and $\forall \theta \geq \theta_0$ when $e = 0$. The solution delivers the minimum θ_1 value for which all these constraints are satisfied. This in turn, delivers the minimum feasible w_D value. The minimum value of θ_1 and w_D that is obtained from the constraints $(a) - (c)$ also relaxes (d) to the minimum feasible extent. Hence if δ is high enough so that the solution obtained from $(a) - (c)$ is also satisfied by (d) the we can support an equilibrium with skill acquisition. Let δ^* be the minimum δ for which (d) is satisfied with the minimum θ_1 value that $(a) - (c)$ permits. The solution is of the following form: *for $\delta < \delta^*$ no solution exists where the worker acquire skills; for $\delta \geq \delta^*$ we have Prendergast solution.*

3. Implicit Contracts

Drawing on Levin (2000), we define implicit or relational contract as a complete plan for the relationship between the firm and each generation of worker. In generation t, let $h^t = \left(\left(\omega_0, W_0, e_0, j_0\right), ..., \left(\omega_{t-1}, W_{t-1}, e_{t-1}, j_{t-1}\right)\right)$ be the history of the game that is observed by both generation-t worker and the firm at the beginning of the period t where j_τ denotes the job assigned to generation τ worker.

Given the sequential moves of the player, in each generation the relevant set of history on which each player conditions their actions are different. Firm takes its action conditioned on the entire past sequence of $\{h^t\}$. The workers also takes into account the action of the firm in the current period, i.e. conditions its action on $(\{h^t\}, B_t)$. Let H_t be the set of all possible history up to generation $t-1$ that is relevant for the firm in generation t, H_0 being the null history. Similarly let H_t^w be the set of all possible history up to generation t that is relevant for the worker in generation t. Clearly, $H_t^w = H_t \cup (\mathbb{R} \times \mathcal{B})$, $(\mathbb{R} \times \mathcal{B})$ being the set of offers (ω_t, B_t) that the generation t workers received at the beginning the period.

In generation $t, (\forall t)$ the strategy of the firm is the following: in period 0 of the worker's career choose an *explicit contract*, depending on the realized history h_t, that specifies wages for job E & D, i.e., ω_t & $\omega_t + w_{D_t}$; in period 1 after (θ_t, e_t) is realized, choose a *job placement* and and a *promise* of bonus depending on the realized history h_t and the initial contract.

Let $\mathcal{C}$ be the set of all explicit contracts. We represent the strategy of the firm in generation t as a mapping $\sigma_L = (\sigma_L^0, \sigma_L^1)$ where $\sigma_L^0 : H_t \to \mathcal{C}$ and $\sigma_L^1 : H_t \times \mathcal{C} \times \Phi \to \mathbb{R}_+ \times \{D, E\}$. The generation-$t$ worker's strategy is a map $\sigma_w^t : H_t \to \{0, 1\}$. The contract $\{\sigma_L, \sigma_w^t\}_0^\infty$ is *self-enforcing* if the set of strategies constitute a *perfect public equilibrium.* In any PPE the firm always keeps its promise, i.e., the b_{Et} and b_{Dt} that it pays and the job placement that it offers are the one specified in B_t. Following any deviation the players revert back to the static Nash equilibrium. This is to say that we restrict ourselves to trigger strategies.

Before we go into solving the model we first derive a few results that brings out some features of such implicit contract. These results are due to Levin (2000) who derived them in a slightly different environment.

We will first show that the problem of designing an optimal contract can be greatly simplified as we can look for only stationary contracts without any loss of generality. In what follows, the generic term *'contract'* will denote a combination of explicit and implicit contracts.

Definition 1. *A contract is stationary if for every generation t, $t = 0, 1, 2, ...,$ on the equilibrium path, $\omega_t = \omega$, $B_t(\theta_t, e_t) = B(\theta_t, e_t)$ and $e_t = e$ for some $\omega \in \mathbb{R}$ and $B : \Phi \to \mathbb{R}_+ \times \{D, E\}$.*

An implicit contract to be self enforcing the following conditions should be satisfied: i) individual rationality:

$$u_t \geq \underline{u} \text{ and } \pi_t \geq \underline{\pi} \ \forall t$$

ii) incentive compatibility for the workers:

$$e_t^* \in \arg\max_{e_t \in \{0,1\}} E_\theta \left[\omega_t + B_t^1(\theta_t, e_t)\right] - ce_t \ \forall t$$

iii) incentive compatibility for the firm:

$$y_t - \left[\omega_t + B_t^1(\theta_t, e_t^*)\right] + \delta\pi^{t+1} \geq \pi^0 + \frac{\delta}{1-\delta}\underline{\pi} \ \forall t$$

where π^0 is the current period gain from one shot deviation and y_t is the output of generation-t worker in the second period. Recall that π^{t+1} is the discounted value of the firm's future profit stream from period $t+1$ onward. Lastly, iv) Each continuation contract is self-enforcing.

Lemma 1. *If an optimal self enforcing contract exists, it must yield the same expected surplus from each generation.*

Proof. Let $\{s_t^*\}_0^\infty$ be the sequence of expected surplus associated with an optimal contract and $s_t^* \neq s_{t'}^*$ for some t and t'.

Define $s_\tau^* = \max_t \{s_t^*\}$.[10] Let the $\langle \omega_\tau^*, B_\tau^* \rangle$ be the firm's offer to generation τ while e_τ^* is the effort level chosen by generation τ. Consider the contract where this surplus maximizing contract $(\langle \omega_\tau^*, B_\tau^* \rangle, e_\tau^*)$ is repeated every period. This is contract satisfies $i) - iii)$ as the initial contract is self enforcing by definition. Moreover, $iv)$ is satisfied as it is the same contract that is repeated every period.

Hence $\{s_t^*\}_0^\infty$ can't represent the optimal contract. ∎

The following proposition drastically simplifies the problem of finding optimal contract.

Proposition 1. *If an optimal self enforcing contract exists, there are stationary self enforcing contracts that are optimal.*

Proof. Let $\{s_t^*\}_0^\infty$ be the sequence of maximum expected surplus from each generation. By lemma 1, $s_t^* = s_{t-1}^* = s^*$. In the initial contract let the effort level chosen by generation 0 worker be e^*.

Take $\pi^* \in [\underline{\pi}, s^* - \underline{u}]$. Set $B_t^{1*}(\theta_t, e_t) = B_0^1(\theta_t, e_t) + \delta\pi^1 - \frac{\delta}{1-\delta}\pi^* \ \forall t$, $\omega^* = u^* - E_\theta\left[\omega^* + B_0^1(\theta_t, e_t^*) - c\right]$ where $u^* = s^* - \pi^*$.

Consider the stationary contract where the firm offers generation t worker, $\forall t$, a wage ω^* for job E and an incentive package of $B_t^* = B_0$ while the effort level is set at e^*.

We claim that such a contract is also optimum. Observe that in generation 0, there is no surplus destruction in such contract. It doesn't alter incentives in condition ii) as the only ω is changed. Hence $e = e^*$ is incentive compatible for the worker in the new contract. Moreover the job assignment rule is also unchanged. So the total surplus is s^* in this contract.

All that we need to show now is that this contract is self enforcing. The worker's payoff is $\omega^* + E_\theta\left[\omega^* + B_0^1(\theta_t, e_t^*) - ce_t^*\right] = u^*$. So the firm's payoff is $\pi^* = s^* - u^*$. As $\pi^* \leq s^* - \underline{u}$, $u^* \geq \underline{u}$. So i) is satisfied. Further observe that iii) doesn't depend on the ω_t as it gets cancelled from both sides (π^0 includes ω_t as firm can't renege on wage payments). As $B_t^{1*}(\theta_t, e_t) + \frac{\delta}{1-\delta}\pi^* = B_0^1(\theta_t, e_t) + \delta\pi^1$, iii) is kept unaltered as well. Moreover as the stationary contract repeats itself in each period, the continuation contract is self-enforcing as it is in the first period itself.

Hence the proposed stationary contract is self-enforcing. ∎

[10]We assume that the maximum exists.

Proposition 1 allows us concentrate on the class of stationary contracts only. This observation reduces the dynamic optimization problem of finding an optimal contract to a static one. Optimal contract is the one that maximizes the joint surplus for each generation subject to the self-enforcement constraint. By the virtue of Proposition 1 it is now enough to find a contract that maximizes the surplus from an arbitrary generation subject to the relevant incentive compatibility and individual rationality constraints. This contract being repeated in every period is then the optimal contract.

Another feature of such a contract is that efficiency and distribution issues can be separated to quite an extent. If there is a self-enforcing contract that generates a sequence of expected surplus $\{s_t\}_0^\infty$, such that $s_t > \underline{s}\ \forall t$, many other 'within generation' distribution of this surplus can be supported by a relational contracts as well. Proposition 2 makes this point precise:

Proposition 2. *If there is a self enforcing contract that generates a sequence of expected surplus* $\{s_t\}_0^\infty$ *, such that* $s_t > \underline{s}\ \forall t$*, then* $\exists$ *self-enforcing contracts that gives as expected payoff any sequence* $\{u_t, \pi_t\}_{t=0}^\infty$ *such that* $u_t + \pi_t \leq s_t$*,* $u_t \geq \underline{u}$ *and* $\pi_t \geq \underline{\pi}\ \forall t$ *and the incentive compatibility constraint of the firm is not violated.*

Proof. Let $\left\{u_t^{\prime}, \pi_t^{\prime}\right\}_{t=0}^\infty$ be the sequence of payoff associated with the initial equilibrium. Consider a new contract that entails only a new sequence $\{\hat{\omega}_t\}_0^\infty$ keeping all other aspect of the initial contract intact. Let $\{u_t, \pi_t\}_{t=0}^\infty$ be the the sequence of payoffs if we replace the sequence $\{\omega_t\}_0^\infty$ by $\{\hat{\omega}_t\}_0^\infty$ in the initial contract. Note that ii) is kept unaltered as they don't depend on ω_t values. Choose the $\{\hat{\omega}_t\}_0^\infty$ sequence so that $u_t \geq \underline{u}$ and $\pi_t \geq \underline{\pi}\ \forall t$ and iii) is not violated. Now the new contract is self enforcing by construction.

Hence the proof. ■

As the firm can extract all the surplus from the worker by choosing an appropriate value of ω, the contract that maximizes the joint surplus is identical to the one that maximizes the firm's payoff up to the value of ω. In the subsequent analysis we will solve for the contract that gives the highest profit to the firm. The nature of this contract coincides with that of surplus maximizing contract and also pins down the value of ω which the latter one doesn't. The following section takes up this problem.

4. The Optimal Contract

The optimal contract, as mentioned in the context of Proposition 1, is a self enforcing one that maximizes the surplus from each generation. As skill acquisition is efficient, we look for an optimal contract where the worker acquires the skill. Proposition 2 further simplifies the problem. We need not have to find a contract the maximizes each element of the sequence $\{s_t\}$ but it is enough to find a 'stage-game' contract that maximizes s_t for any arbitrary t. The optimal contract is this 'stage-game' contract being repeated every period. Hence from now on, unless it is necessary, we will drop the t subscript.

Note that the tuple $(\theta_0, \theta_1, \omega, b_E, w_D, b_D)$ completely characterizes the firm's compensation offer in the 'stage-game' contract. Consider the following strategies for the two players.

σ_L : σ_L : σ_L : *i)* At H_0, firm offers $\sigma_L^* = (\theta_0, \theta_1, \omega, b_E, w_D, b_D)$.

ii) At H_t, for any t, firm offers σ_L^* if $e_\tau = 1\ \forall \tau \leq t-1$. If there is deviation in the past, the firm offers the compensation plan associated with the static benchmark model. This is to say that the offer is $\left(\theta_0^P, \theta_1^P, \omega^P, 0, w_D^P, 0\right)$ if an equilibrium with skill acquisition exists and $\left(\bar{\theta}, \theta^{FB}, 0, 0, 0, 0\right)$ otherwise.

σ_w^t : *i)* At H_0^w, $e_t = 1$ if $\sigma_L = \sigma_L^*$ is incentive compatible and individually rational, else $e = 0$.

ii) At H_t^w, for any t, $e_t = 1$ if $\sigma_L = \sigma_L^*$ is incentive compatible & individually rational and there is no deviation in the past, else $e = 0$. If there is any deviation in past by the firm $e_t = 1$ only if the firms offer is incentive compatible even if payment of b_E and b_{Dt} are set to 0; $e_t = 0$ otherwise.

Recall that the stage game can have two possible outcomes. In one case there may not exist any solution where the worker acquires skill. In that case the continuation game after any deviation has the following feature: in every generation the firm offers $\sigma_0 = \left(\bar{\theta}, \theta^{FB}, 0, 0, 0, 0\right)$ and the worker chooses $e = 0$. In the other case there may exists equilibrium where the firm can induce skill acquisition through offering promotion as incentive. In such a case the continuation game after any deviation is: in every generation the firm offers $\sigma_P = \left(\theta_0^P, \theta_1^P, \omega^P, 0, w_D^P, 0\right)$ and the worker chooses $e = 1$. Let $\sigma_E = \left(\theta_0^E, \theta_1^E, \omega^E, 0, w_D^E, 0\right)$ and $e = e^E$ be the equilibrium outcome in the stage game. Obviously $\sigma_E \in \{\sigma_0, \sigma_P\}$ and $e^E \in \{0, 1\}$.

Individual rationality constraint of the worker (WIR) ensures that the payoff of the worker on the equilibrium under the implicit contract is at least as large as the cost of training. This leads to the following equation:

$$2\omega + (1-\theta_1)(w_D + b_D) + \theta_1 b_E \geq c \tag{6}$$

Incentive compatibility constraint of the worker requires $\theta_1 b_E + (1-\theta_1)(w_D + b_D) - (1-\theta_0) w_D \geq c$, i.e.,

$$(1-\theta_1) b_D + (\theta_0 - \theta_1) w_D + \theta_1 b_E \geq c \tag{7}$$

Before we describe the constraints of the firm, define the followings:

$$\begin{aligned} \bar{\pi} = \quad & \int_0^1 y_E(\theta, 1)\, d\theta + \int_0^{\theta_1} y_E(\theta, 1)\, d\theta + \int_{\theta_1}^1 y_D(\theta, 1)\, d\theta \\ & - \left(2\omega + \theta_1 b_E + (1-\theta_1)(w_D + b_D)\right) \end{aligned}$$

Further denote $\bar{y} = \int_0^{\theta_1} y_E(\theta, 1)\, d\theta + \int_{\theta_1}^1 y_D(\theta, 1)\, d\theta$. Note that $\bar{y}$ and hence $\bar{\pi}$, are decreasing (increasing) functions of θ_1 for $\theta_1 < \theta^{FB}$ $\left(\theta_1 \geq \theta^{FB}\right)$ and attain there maximum at $\theta_1 = \theta^{FB}$.

Here

$$\underline{\pi} = \int_0^{\theta'} y_E\left(\theta, e^E\right) d\theta + \int_{\theta'}^1 y_D\left(\theta, e^E\right) d\theta - \left(2\omega^E + (1-\theta_1)\, w_D^E\right)$$

where $\theta' = \bar{\theta}$ if $e^E = 0$; θ_1^P otherwise. $\bar{\pi}$ is the payoff of the firm when there has been no deviation up to the current generation. $\underline{\pi}$ is the payoff of the firm in the punishment phase.. Firm's individual rationality constraint requires non-zero profit when worker exerts effort, i.e.,

$$\bar{\pi} \geq 0 \tag{8}$$

Firm's incentive compatibility constraint is slightly more complicated. We need to consider two cases: $e = 1$ and $e = 0$. Both cases have two subcases: *i)* $\theta \leq \theta_1$ and *ii)* $\theta \geq \theta_1$ when $e = 1$; for $e = 0$, *i)* $\theta \leq \theta_0$ and *ii)* $\theta \geq \theta_0$. The optimal one stage deviations are different in these four cases. With $e = 0$ the firm should be willing to promote the worker in job D and pay the wage premium if the observed $\theta \geq \theta_0$. But even if the firm renege on promotion such deviation can't be detected by the future workers. The firm can always report a false θ to justify its action and that will not initiate the trigger. So the relevant incentive constraint are:

$$\begin{gathered} y_D(\theta, 0) - w_D \geq y_E(\theta, 0) \;\; \forall\, \theta > \theta_0 \\ y_E(\theta, 0) \geq y_D(\theta, 0) - w_D \;\forall\, \theta \leq \theta_0 \end{gathered}$$

i.e.,

$$y_D(\theta_0, 0) - w_D = y_E(\theta_0, 0) \tag{9}$$

For $e = 1$ we have the following two constraints:

$$\begin{gathered} y_E(\theta, 1) - b_E + \tfrac{\delta}{1-\delta}\bar{\pi} \geq \\ \max \left\{ \begin{array}{c} y_D(\theta, 1) - w_D + \frac{\delta}{1-\delta}\underline{\pi},\; y_E(\theta, 1) + \frac{\delta}{1-\delta}\underline{\pi}, \\ y_D(\theta, 1) - (w_D + b_D) + \frac{\delta}{1-\delta}\bar{\pi} \end{array} \right\} \forall \theta < \theta_1 \end{gathered} \tag{10}$$

Observe that here there are three possible deviations that the firm may adopt. It may renege on bonus payment and/or job assignment (this is captured in the first two terms inside the parenthesis in (10)) or it might renege on job assignment but pay the promised bonus in the new job (the last terms inside the parenthesis in (10) stands for this). The first two cases will initiate the trigger but the last one will not. Clearly, this deviation is not detected by future generations.

For all $\theta \geq \theta_1$ the firm can deviate in three ways as well. It may renege on the bonus payment and/or the promotion offer. In both cases the trigger will be initiated (the first two terms in the parenthesis in (11) stand for this). Else, it may not promote the worker but pay bonus and avoid future punishment (the last term in the parenthesis in (11) stand for this).

This leads to the following constraint:

$$y_D(\theta,1)-(w_D+b_D)+\tfrac{\delta}{1-\delta}\bar{\pi}\geq \max\left\{\begin{array}{c} y_D(\theta,1)-w_D+\tfrac{\delta}{1-\delta}\underline{\pi},\ y_E(\theta,1)+\tfrac{\delta}{1-\delta}\underline{\pi}, \\ y_E(\theta,1)-b_E+\tfrac{\delta}{1-\delta}\bar{\pi}\end{array}\right\}\ \forall\theta\geq\theta_1 \tag{11}$$

The optimal contract solves the following problem:

$$\begin{array}{c}\max\limits_{(\theta_0,\theta_1,\omega,b_E,w_D,b_D)}\pi \\ s.t.\ equation\ (6)-(11)\end{array} \tag{*}$$

As skill acquisition is optimal, the solution to the problem yields the lowest $\theta_1\geq\theta^{FB}$ that is supportable in a self-enforcing equilibrium.

The following observations simplifies the problem:

i) (6) should always bind in the equilibrium when the contract is designed to maximize the firm's profit.

As the difference y_D-y_E is increasing in θ for all e we conclude the followings:

ii) If (10) and (11) hold at $\theta=\theta_1$ they hold for all $\theta<\theta_1$ and $\theta\geq\theta_1$ respectively. So (10) and (11) can be replaced by the following constraints:

$$y_E(\theta_1,1)-b_E+\tfrac{\delta}{1-\delta}\bar{\pi}\geq \max\left\{\begin{array}{c} y_D(\theta_1,1)-w_D+\tfrac{\delta}{1-\delta}\underline{\pi},\ y_E(\theta_1,1)+\tfrac{\delta}{1-\delta}\underline{\pi}, \\ y_D(\theta_1,1)-(w_D+b_D)+\tfrac{\delta}{1-\delta}\bar{\pi}\end{array}\right\}$$

$$y_D(\theta_1,1)-(w_D+b_D)+\tfrac{\delta}{1-\delta}\bar{\pi}\geq \max\left\{\begin{array}{c} y_D(\theta_1,1)-w_D+\tfrac{\delta}{1-\delta}\underline{\pi},\ y_E(\theta_1,1)+\tfrac{\delta}{1-\delta}\underline{\pi}, \\ y_E(\theta_1,1)-b_E+\tfrac{\delta}{1-\delta}\bar{\pi}\end{array}\right\}$$

One can further note that these constraints are satisfied *iff* the following constraints hold:

$$\begin{array}{c} y_D(\theta_1,1)-(w_D+b_D)+\tfrac{\delta}{1-\delta}\bar{\pi}=y_E(\theta_1,1)-b_E+\tfrac{\delta}{1-\delta}\bar{\pi} \\ y_E(\theta_1,1)-b_E+\tfrac{\delta}{1-\delta}(\bar{\pi}-\underline{\pi})\geq y_E(\theta_1,1) \\ y_D(\theta_1,1)-(w_D+b_D)+\tfrac{\delta}{1-\delta}(\bar{\pi}-\underline{\pi})\geq y_D(\theta_1,1)-w_D\end{array}$$

iii) (8) can be ignored by the virtue of (10) and the fact that $\underline{\pi}\geq 0$.

Incorporating the above observations problem $(*)$ boils down to:

$$(*')\qquad \begin{array}{cc} \max_{(\theta_0,\theta_1,\omega,b_E,w_D,b_D)}\pi & \\ s.t.\ 2\omega+(1-\theta_1)(w_D+b_D)+\theta_1 b_E=c & (6') \\ (1-\theta_1)b_D+(\theta_0-\theta_1)w_D+\theta_1 b_E\geq c & (7') \\ y_D(\theta_0,0)-w_D=y_E(\theta_0,0) & (9') \\ y_D(\theta_1,1)-(w_D+b_D)=y_E(\theta_1,1)-b_E & (10') \\ \tfrac{\delta}{1-\delta}(\bar{\pi}-\underline{\pi})\geq b_E & (11') \\ \tfrac{\delta}{1-\delta}(\bar{\pi}-\underline{\pi})\geq b_D & (4.7')\end{array}$$

Any solution to $\left(*'\right)$, say $(\theta_0, \theta_1, \omega^*, b_E^*, w_D^*, b_D^*)$, is an optimal contract. The following lemmas illuminate the nature of an optimal contract.

Lemma 2. *For any δ, if the optimal contract exist, it must have $\theta_1 \geq \theta^{FB}$.*

Proof. Suppose not. If $\theta_1 < \theta^{FB}$ then $y_D(\theta_1, 1) < y_E(\theta_1, 1)$. From $\left(10'\right)$ we have $(w_D^* + b_D^*) < b_E^*$. Hence $b_D^* < b_E^*$ and therefore $\left(4.7'\right)$ can't bind. Now raise θ_1 by ϵ (> 0). Note that $\left(7'\right)$ can be written as

$$b_D^* + \theta_0 w_D^* - \theta_1 \left((w_D^* + b_D^*) - b_E^*\right) \geq c$$

So with $(w_D^* + b_D^*) < b_E^*$ such rise in θ_1 relaxes $\left(7'\right)$. Increase b_D^* to satisfy $\left(10'\right)$. This will further relax $\left(7'\right)$. Moreover, for a sufficiently small ϵ such manipulation of b_D^* is feasible as $\left(4.7'\right)$ can't bind. But now the value of π is increased. Therefore the initial contract can't be optimal.

Hence the proof. ■

This lemma implies the following conclusions:

Corollary 1. *For any δ, if the optimal contract exist, it must have $(w_D^* + b_D^*) \geq b_E^*$ with equality holding at the first best.*

Proof. This follows directly from $\left(10'\right)$. ■

Lemma 3 states another important observation about the nature of the optimal contract.

Lemma 3. *For any δ, if the optimal contract yields $\theta_1 > \theta^{FB}$ it must also have $\theta_1 < \theta_0$.*

Proof. Suppose not. Let $\theta_1 \geq \theta_0$. We will show that in that case the first best is feasible.

Observe that in this case we must have $(1 - \theta_1) b_D^* + \theta_1 b_E^* \geq c$, by $\left(7'\right)$. Hence $\max\{b_D^*, b_E^*\} = c$ and therefore $\frac{\delta}{1-\delta}(\bar{\pi} - \underline{\pi}) \geq c$. Consider the solution where $\theta_0 = \theta_1, \theta_1 = \theta^{FB}, b_E^* = b_D^* = c, w_D^* = 0$ and ω^* is so chosen such that $\left(6'\right)$ is satisfied. One can simply check that this solution is feasible. Therefore the initial contract can't be optimal.

Hence the proof. ■

Equipped with these observations we are ready for our next proposition. Proposition 3 characterizes the optimal contract as δ varies.

Proposition 3. *i) $\exists$ a minimum value of δ, say $\bar{\delta}$ (< 1) such that $\forall \delta \in [\bar{\delta}, 1)$, the first best is attained.*

ii) $\exists$ a $\underline{\delta} \geq 0$ such that $\forall \delta \in \left(\underline{\delta}, \bar{\delta}\right)$, π is strictly increasing.

Proof. i) Define $\bar{\delta}$ (< 1) such that $\frac{\bar{\delta}}{1-\bar{\delta}}\left(\bar{\pi}|_{\theta_1 = \theta^{FB}} - \underline{\pi}\right) = c$. We claim that $\bar{\delta}$ is the minimal δ such that $\forall \delta \in [\bar{\delta}, 1)$ the first best level of $\theta_1 = \theta^{FB}$ is attained. The proof is as follows.

At the optimum for $\delta = \bar{\delta}$, WIC must bind. Otherwise one can reduce b_E & b_E equally. This will not violate $\left(10'\right)$ but can be sustained for a smaller δ value. Hence $\bar{\delta}$ can't be the minimal such δ. So we must have $b_D + \theta_0 w_D - \theta_1 \left((w_D + b_D) - b_E\right) = c$.

From corollary (1) we know that in the first best $w_D + b_D = b_E$. Therefore $b_D + \theta_0 w_D = c$. The minimal reputational capital is required when $w_D + b_D$ is minimal subject to the fact that $b_D + \theta_0 w_D = c$. So we must have $b_D = c$ and $w_D = 0$ if the associated $\theta_0 < 1$. If $\theta_0 = 1$ any combination of w_D & b_D is optimal as long as $w_D + b_D = c$. Moreover $b_E = c$. By construction $\bar{\delta}$ is the minimum δ that makes these b_E & b_E values feasible.

ii)The prove this part we will show that $\exists$ a $\underline{\delta} \geq 0$ such that if θ_1 is associated with the optimal contract where $\delta = \delta_0 > \underline{\delta}$, then for $\delta_1 \in \left(\delta_0, \bar{\delta}\right)$, the solution yields $\theta_1 \in [\theta^{FB}, \theta_1)$.

Let $D = \{\delta \in [0,1] \mid \bar{\pi} - \underline{\pi} > 0 \text{ in equilibrium}\}$; $D \neq \emptyset$ as $\bar{\delta} \in D$. Let $\underline{\delta} = \inf D$. As δ_0 increases to δ_1, one can increase both b_E and b_D as $\left(11'\right)$ and $\left(4.7'\right)$ are now relaxed. Observe that the minimum θ_1 value is associated with the minimum feasible value for $(w_D + b_D) - b_E$. We argue that it is optimal to increase b_E and b_D to their maximum feasible level and reduce w_D up to the level where WIC is binding. The argument is as follows: it is trivial that b_E must be increased at its maximal feasible level as it enters with a negative sign in the expression $(w_D + b_D) - b_E$. b_D should also be increased as for an ϵ increment in b_D we can reduce w_D by $(1-\theta_1)\,\epsilon/\left(\theta_0 - \theta_1\right) \geq \epsilon$ ($> \epsilon$ when $\theta_0 < 1$) and keep WIC unaltered. Therefore $(w_D + b_D)$ reduces and $(w_D + b_D) - b_E$ attains its feasible minimum. Now as $(w_D + b_D) - b_E$ decreases θ_1 is reduced to maintain $\left(10'\right)$. Moreover, such increment will not violate $\left(7'\right)$ as its $R.H.S.$ is decreasing in θ_1.[11] Hence the solution yields $\theta_1 \in [\theta^{FB}, \theta_1)$.and surplus increases. ■

We make a few remarks in the context of this proof.

Remark 1. $\left(11'\right)$ *and* $\left(4.7'\right)$ *always binds until the first best is reached. This implies that the implicit incentive is always the used at its maximum feasible power.*

Remark 2. *Note that when first best is attend there is efficiency even in the off-the-equilibrium path.* $b_D^* = b_E^* = c,\ \theta_1 = \theta^{FB}$ *and* $\theta_0 = \bar{\theta}$ *is feasible* $\forall \delta \in [\bar{\delta}, 1)$.

Remark 3. $\underline{\delta} > 0$ *if the explicit incentive alone is too weak to induce skill acquisition. Then the implicit incentive has to be sufficiently large to provide enough incentive for skill acquisition. With small* δ *this need not be the case.*

Two important observations can be made on the basis of proposition 3. First, for a sufficiently patient firm the inefficiency issue a la Prendergast (1993) disappears and it is not essential for the jobs to be significantly different in terms of their underlying technology. At the optimum the firm need not have to use explicit incentive at all. This shows that implicit incentive can induce skill acquisition even if explicit incentive by itself is insufficient to induce skill acquisition. Second, when coupled with explicit incentives, implicit incentive increases total surplus. This is due to efficiency gain that implicit contract delivers over explicit contract. This observation leads to the main result of our paper: here explicit incentive through promotion and implicit incentive through bonus are *substitutes*. As the

[11]As $(w_D + b_D) \geq b_E$.

reputational concern increases firm may rely more on bonus and less on promotion and a source of incentives. This is further highlighted by remark 1. As δ increases the firm's 'relational capital' increases. The maximum feasible level of implicit incentives, namely the bonus payments, is limited by the size of 'relational capital'. With a rise in δ, until the first best is achieved, the firm will substitute explicit incentive (promotion offer) by implicit incentive (bonus payments). An obvious implication of this phenomenon is wage compression across jobs. Proposition 4 makes this point more precisely.

Proposition 4. *The compensation differential between the two jobs and the explicitly contracted wage differential are strictly decreasing in* δ *for* $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$ *and attains its minimum value* $0 \; \forall \delta > \bar{\delta}$.

Proof. Consider the constraint $\left(10'\right)$. For $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$ the fall in compensation differential follows as θ_1 is strictly decreasing in δ and $y_D\left(\theta_1, 1\right) - y_E\left(\theta_1, 1\right)$ is strictly increasing in θ_1. Moreover, from the proof of Proposition 3 we know that for $\delta \geq \bar{\delta}$ we have $w_D + b_D - b_E = 0$.

Again, from the proof of Proposition 3 *ii*), we know that as δ increases in $\left(\underline{\delta}, \bar{\delta}\right)$ the explicitly contracted wage differential w_D is substituted by b_D. Proof of Proposition 3 *i*) shows that at the first best, i.e. when $\delta = \bar{\delta}$, $w_D = 0$.

Hence the proof. ■

Corollary 2. *With wage compression the proportion of bonus payment in the total pay package of a skilled worker increases monotonically.*

Proposition 4 predicts that the more the firm cares about the future payoff the less is the difference in compensation in the two jobs for a skilled worker. The intuition behind this result is as follows. If promotion is to be used as an incentive device, there has to be a wage premium in the difficult job. For low δ the incentives generated from future gain or reputation is not sufficient for the firm to pay enough to induce the worker to acquire skill. Hence promotion scheme has to add more incentives on top of reputational incentives. So compensations has to be different in the two jobs. As δ increases, reputational concern gets stronger. Now promotion, as a device of creating incentives, need not have to be as high powered as before. Moreover, it creates efficiency loss. Therefore the firm finds it feasible and optimal to reduce the power of incentive created by the promotion offer. But this implies an decrease in explicit wage differential.

5. Conclusion

Our model captures the interaction of explicit and implicit incentives in the context of a dual moral hazard problem of firm specific skill acquisition. The explicit incentive is generated through the up-or-stay promotion offer while the reputational concern of the firm provides implicit incentives. For a risk neutral worker, these two incentives are perfect substitutes.

Promotion as an incentive device creates inefficiency. As it requires a wage premium, *too few* workers are promoted. Promotion is offered only when a worker's productivity difference justifies the premium. Moreover, for such premium to be feasible, the value of the firm specific skill in the two jobs have to be sufficiently different. The shadow cost of implicit incentive is less than that of explicit incentive. In the former case there is no efficiency distortion. Therefore, firm would prefer to use the latter as much as possible reducing the usage of explicit contract - substituting explicit incentive by implicit incentive.

The more patient the firm is, the more powerful is the implicit incentive. Hence the firm can afford to reduce the incentive provided through promotion offer without destroying incentives for skill acquisition. The required premium is reduced and we approach towards the first best as the difference in compensation across jobs reduces. If the reputational concern is too high then one can do away with promotion altogether. Only bonus payments, supported by reputational concern, can induce skill acquisition. Here we don't even need the jobs to be different (with respect to the value of skill in each of these jobs).

The prediction of reward compression is in contrast with the evidence from personnel data. It is found that usually promotions are attached with a substantial jump in compensation. In our environment that can never be the case as in the first best the cheapest way to provide incentive is to set the two compensation values equal. Perhaps this observation calls for a richer model.

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Northwestern University
E-mail address: `a-mukherjee@northwestern.edu`